Mathematical Methods in Science and Engineering

Quiz Questions

[Answers are **not** given.
The learner should seek out
the appropriate context
in the textbook or in the lectures
to verify his/her answers.]

compiled by

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Level 1

1. Under what condition, the ODE forms

   $$F(x, y, y') = 0 \text{ and } y' = f(x, y)$$

   are equivalent?

2. What is a singular solution of an ODE?

3. What is the order of a differential equation?

4. For constant input voltage E in

   $$L\frac{dI}{dt} + RI = E(t),$$

   what response do you expect for *arbitrary* initial condition?

5. How do you reduce the order of the equation

   $$y'' + e^y y'^3 = 0?$$

6. What functional form of the solution of the vibration equation

   $$my'' + cy' + ky = 0$$

   will result in an overdamped response?

7. Given three linearly independent functions $y_1(x)$, $y_2(x)$ and $y_3(x)$, how will you form an ODE satisfied by each of the three?

8. What correction is needed in the expression

   $$y_p(x) = -y_1 \int \frac{y_1 r}{W} dx + y_2 \int \frac{y_2 r}{W} dx$$

   to get the formula for variation of parameters for a particular solution of an ODE with RHS $r(x)$, where $y_1(x)$ and $y_2(x)$ are solutions of the corresponding HE?

9. What is the order of the ODE in the modelling of an RLC circuit?

10. What are the linearly independent solutions of

    $$[\{D - (1+i)\}\{D - (1-i)\}]^2 y = 0?$$

11. Under what condition, resonance occurs in an oscillating system?

12. What is a phase plane?

13. What is a phase portrait?

14. What is a critical point?

15. Name different kinds of critical points.

16. Which kinds of critical points may be stable or unstable?

17. If a system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ possesses a Lyapunov function, then which point is concluded as stable?

18. How do we extend the analysis if some point other than the origin is a critical point?

19. A positive definite candidate Lyapunov function was tried for stability analysis of a system around the origin. If its rate was found positive definite, then what can we conclude about the system?

20. Assume

$$y(x) = x^r(a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots)$$

and proceed. What is the *name* of this method?

21. Under what conditions, Frobenius method works for the solution of the equation

$$y'' + p(x)y' + q(x)y = 0?$$

22. What is the indicial equation in the context of the solution of a linear ODE?

23. Under what conditions the power series solution

$$y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

is valid for solving the equation

$$h(x)y'' + p(x)y' + q(x)y = r(x)?$$

24. Identify the equation

$$[r(x)y']' + [q(x) + \lambda p(x)]y = 0.$$

25. If the equation is available as

$$a(x)y'' + b(x)y' + c(x)y + \lambda d(x)y = 0,$$

then how do you convert it into the standard "Sturm-Liouville" form?

26. What are the boundary conditions for a periodic "Sturm-Liouville" problem?

27. What is the relationship between the functions $f_1(x)$ and $f_2(x)$ expressed by the equation

$$\int_a^b f_1(x)f_2(x)dx = 0?$$

28. What extension does Fourier integral effect on Fourier series?

29. What is the physical meaning of

$$\int_\omega^{\omega+\epsilon} |\hat{f}(w)|^2 dw?$$

30. What is the implication of Shannon's sampling theorem?

31. What is Fast Fourier Transform?

32. From which domain to which domain a function is converted by Laplace transform?

33. State the frequency shifting rule of Laplace transforms.

34. State the time shifting rule of Laplace transforms.

35. In which kind of problems, application of Laplace transform method is most vital?

36. Which kind of functions are mostly used as basis functions in finite element method?

37. With reference to a linear transformation, name the subspace whose dimension is called 'nullity'. Of which vector space it is a subspace?

38. What is the matrix-multiplication equivalent of an elementary column transformation?

39. What are the conditions for the system $\mathbf{Ax} = \mathbf{b}$ to have infinite solutions?

40. Combination of which two properties of a square matrix allow its Cholesky factorization?

41. Give two reasons for *not* using pivoting while solving a tridiagonal system.

42. If we are interested in the eigenvalues of largest and least magnitude of a large matrix, then which method should we prefer, generally?

43. What does a full cycle of Jacobi rotations produce?

44. What does a full cycle of Givens rotations produce?

45. Geometrically, what is the effect of Householder transformation on position vectors of particles in a body?

46. What is QR decomposition?

47. What is the expresion for div(grad f)?

48. For what kind of vector functions, the line integral depends only on end-points and not on the path?

49. Green's theorem in a plane is a special case of which other theorem?

50. Which theorem relates a surface integral to a volume integral?

51. What is the prime objection to a simple Lagrange interpolation through all the data points?

52. What is the highest order of continuity that can be achieved with cubic segments interpolating given points?

53. Name the first four members in the family of Newton-Cotes quadrature formula?

54. List out the terms in the Simpson's one-third rule.

55. Why there is no theory developed for unconstrained linear optimization?

56. What is the second order necessary condition for a local minimum point in a multivariate unconstrained optimization problem?

57. By the combination of which two methods we get the Levenberg-Marquardt method?

58. What is the strategy of this combination?

59. What is the conceptual difference of "derivative" of a complex function as compared to derivative in ordinary calculus sense?

60. When do you call a complex function analytic at a point, and in a domain?

61. What are Cauchy-Riemann conditions for

$$f(z) = u(x, y) + iv(x, y)?$$

62. At which points an analytic function gives a mapping that is *not* conformal?

63. Which equation a harmonic function necessarily satisfies?

64. How do you find the conjugate harmonic function of a given harmonic function?

65. For $f(z) = u(x, y) + iv(x, y)$, correct the formula

$$\int_C f(z)dz = \int_C (udx + vdy) - i \int_C (vdx - udy)$$

66. What is the value of

$$\oint_C \frac{dz}{z},$$

C being a circle with centre (5,5) and radius 6?

67. State the principle of path deformation, as applicable to complex integration.

68. How can you expand the function $\frac{e^z}{z^2}$ in Taylor's series?

69. Name and elaborate the power series that you can use to expand $\frac{e^z}{z^2}$ in powers of z.

70. Which kind of singularity $e^{1/z}$ has at the origin?

71. What is the order of the pole of $\frac{1-\cos z}{z^4}$ at the origin?

72. With reference to its Laurent series, what is the residue of a function at z_0?

73. State the residue theorem.

74. What contour you will start with to develop an improper (real) integral of the form

$$\int_{-\infty}^{\infty} f(x)dx \ ?$$

75. How is "generalized Fourier series" different from Fourier series?

76. What information can you gather about a function $f(x)$ from the statement that it has the Fourier series

$$f(x) = \sum_{n=1}^{\infty} a_n \cos 4nx?$$

77. What is the supremum of $\sum_{n=1}^{N} a_n^2$ here?

78. In $\mathcal{F}\{f'(x)\} = \alpha\mathcal{F}\{f(x)\}$, what is α?

79. What is a quasilinear partial differential equation?

80. If a heat flow phenomenon has been modelled in the form of an elliptic equation, then in what manner does the flow evolve in time?

81. For what domain in the xy plane, the PDE

$$yu_{xy} + x^2u_x = \sin(xy)$$

is hyperbolic?

82. For which domain is the equation

$$u_{xx} + (1 - x^2 - y^2)u_{yy} = f(x, y)$$

hyperbolic?

83. When you apply Fourier transform with respect to one variable on a PDE with two independent variables, what is the resulting equation?

84. A square matrix $\mathbf{P}$ satisfies the equation $\mathbf{P}^2 = \mathbf{P}$. What transformation does it represent?

85. What is the number of non-zero singular values of a matrix?

86. What is the singular value of a column vector?

87. Define Dirac's delta function.

88. If $\mathbf{A}$ and $\mathbf{B}$ are $n \times n$ matrices, $\mathbf{A}$ non-singular, and $\mathbf{c}$ is an n-d vector, then what is the efficient way to compute $\mathbf{A}^{-1}\mathbf{B}\mathbf{c}$?

89. State two reasons why it is easier to diagonalize symmetric matrices compared to non-symmetric ones (i.e. the diagonalizable ones).

90. What will be structure of the final system of equations resulting from relaxation formulation for the BVP with a second order linear ODE?

91. Under what conditions, a square matrix is LU-decomposable?

92. Why small value of determinant is *not* a very good measure for near-singular nature of a matrix?

93. What is meant by diagonalization of a matrix?

94. What is the concept of diagonalization of a linear transformation?

95. Why is the QR decomposition algorithm for the eigenvalue problem usually applied after making it tridiagonal or Hessenberg rather than on the original matrix?

96. What is the relevance of eigenvalue problem for polynomial root finding?

97. What is the value of

$$\oint_C \frac{\sin z + e^z}{z} dz,$$

C being a unit circle with centre at origin?

98. How will you determine

$$\oint_C \frac{\sin z + e^z}{z^5} dz$$

for the same contour?

99. What is the integrating factor of the Leibnitz equation $y' + p(x)y = r(x)$?

100. What substitution will transform the Bernoulli Equation $y' + p(x)y = r(x)y^k$ into Leibnitz (linear) equation?

101. What is the convolution of two functions?

102. What is the Laplace transform of a convolution?

103. If $y_1(x)$ and $y_2(x)$ are solutions of

$$y'' + p(x)y' + q(x) = 0,$$

then what is the function

$$y_1(x)y_2'(x) - y_2(x)y_1'(x)$$

called?

104. If the Wronskian function has a value 4 at $x = 4$, then at what value of x it is zero?

105. If $\text{Rank}(\mathbf{A}) = m$ and $\text{Rank}(\mathbf{B}) = n$, then what can we say about $\text{Rank}(\mathbf{AB})$?

106. For a vector function $\mathbf{V}$, what is

$$\text{grad}(\text{div}\mathbf{V}) - \nabla^2\mathbf{V}?$$

107. Under what condition, a fixed-point iteration scheme $x_{k+1} = g(x_k)$ is guaranteed to converge?

108. What is the significance of the singular solution of a first order ordinary differential equation?

Level 2

1. What does it mean for a numerical ODE method to be of order p?

2. What is the dimension of state space for the system of equations

$$y_1''' = x - y_1'' + y_2, \quad y_2' = xy_1''?$$

3. What is the general solution of the equation

$$y'' - 2y' + y = 0?$$

4. What should we add to that to get the general solution of the equation $y'' - 2y' + y = e^x$?

5. What kinds of critical points a damped simple pendulum may have at its lowermost point?

6. Name a feature of a phase portrait that only a nonlinear system may have and not a linear one.

7. How many critical points are there for the system

$$y'' - y + y^2 = 0?$$

8. What is a Lyapunov function?

9. What is the physical implication of the term "asymptotically stable" or "stable and attractive"?

10. Name the well-known method that gives you the formula

$$\mathbf{y} = \mathbf{y}^{(h)} + \mathbf{y}^{(p)} = \mathbf{Y}\mathbf{c} + \mathbf{Y}\int \mathbf{Y}^{-1}(t)\mathbf{g}(t)dt.$$

11. Identify each term in the above formula.

12. How does the method of diagonalization simplify the evaluation for a particular solution of $\mathbf{y}' = \mathbf{A}\mathbf{y} + \mathbf{g}(t)$?

13. What is meant by saying that function $f(x)$ is *analytic* at $x = x_0$?

14. Name a continuous function in the interval $-\pi \leq x \leq \pi$ that is orthogonal to each of the functions 1, $\cos nx$, $\sin nx$, where $n = 1, 2, 3 \cdots$?

15. After we find the best possible approximation of $T_5(x)$ as a linear combination of $T_0(x)$, $T_1(x)$, $T_2(x)$, $T_3(x)$ and $T_4(x)$ (Chebyshev polynomials), what will be the resulting error function?

16. Next, we find its best possible approximation as linear combination of Legendre polynomials $P_0(x)$, $P_1(x)$, $P_2(x)$, $P_3(x)$ and $P_4(x)$. In which case, the maximum error will be less?

17. In a two-point boundary value problem of state space dimension 8, five conditions are given at the initial point and three at final. In shooting method employing Newton-Raphson method, how many initial value problems have to be solved at each iteration?

18. How is the condition number of a matrix defined?

19. What is its implication on the solution of a linear system?

20. At least how many eigenvectors a 6×6 non-singular matrix *must* have?

21. What is Sylvester's criteria for negative definiteness of a matrix?

22. Which relationship among vectors is preserved by an orthogonal transformation?

23. What is singular value decomposition?

24. Under what conditions a real matrix has a singular value decomposition?

25. State a *conceptually* straightforward process to compute the SVD of a matrix?

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26. Clearly specify a single scalar function that characterizes a plane curve completely?

27. What condition restricts a curve to a plane?

28. Why is it important to use adaptive step size while solving a differential equation numerically?

29. How to find the least square solution of $\mathbf{Ax} = \mathbf{b}$, the underlying field being $\mathcal{C}$, the set of complex numbers?

30. What is the best approximation (in the sense of least deviation) of x^n by a polynomial of degree $\leq n-1$?

31. How do you express the general solution of the ODE

$$y = xy' - \frac{y'}{\sqrt{1+y'^2}}?$$

32. What is the method to find its singular solution?

Level 3

1. The linearized model of a nonlinear system has a stable critical point, P. But, from that it is not possible to predict the stability of the original nonlinear system. What kind of critical point it is (of the linear model)?

2. In case no basis of eigenvectors is available for $\mathbf{A}$ while analyzing $\mathbf{y}' = \mathbf{A}\mathbf{y}$, why do we need to consider

   $$\mathbf{y}^{(2)} = t\mathbf{y}^{(1)} + \mathbf{u}e^{\mu t}?$$

   Why not simply $\mathbf{y}^{(2)} = t\mathbf{y}^{(1)}$?

3. Identify this equation:

   $$x^2y'' + xy' + (x^2 - \nu^2)y = 0.$$

4. Identify this family of functions: $f_0(x) = 1$, $f_1(x) = x$, $f_2(x) = (3x^2 - 1)/2$, $f_3(x) = (5x^3 - 3x)/2$, $f_4(x) = (35x^4 - 30x^2 + 3)/8$, $\cdots$.

5. What is this inequality and what does it imply?

   $$|f(x, y_2) - f(x, y_1)| \leq M|y_2 - y_1|$$

6. What is the shape of the convolution $(f * g)(t)$, if

   $$\begin{aligned} f(t) &= 1 \text{ for } -2a \leq t \leq 2a, \ 0 \text{ otherwise}, \\ g(t) &= 1 \text{ for } -a \leq t \leq a, \ 0 \text{ otherwise?} \end{aligned}$$

7. What is the fundamental idea of convolution in mathematics?

8. In the predator-prey model

   $$y_1' = ay_1 - by_1y_2, \qquad y_2' = ky_1y_2 - ly_2,$$

   How many critical points are there?

9. What kind of critical point is the origin and what is its interpretation?

10. What kind of critical point is the other one and what is its interpretation?

11. What is this formula and what is its application?

    $$(\mathbf{A} + \mathbf{u}\mathbf{v}^T)^{-1}$$
    $$= \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{u}(1 - \mathbf{v}^T\mathbf{A}^{-1}\mathbf{u})^{-1}\mathbf{v}^T\mathbf{A}^{-1}$$

12. What is the generalized algebric eigenvalue problem?

13. Name the simplest form to which a matrix with real eigenvalues can always be reduced through similarity transformation and describe its structure.

14. What kind of a solution of $\mathbf{Ax} = \mathbf{b}$ do you get from pseudoinverse if the system has both conflict and redundancy at the same time?

15. If the pseudoinverse gives arguably the best kind of solutions in all possible cases, then why other methods of solving linear systems are still in use?

16. What is the maximum possible number of intersections between a quartic and cubic curves in a plane?

17. What are stiff differential equations and why their numerical solution poses particular difficulty?

18. Under what condition, a Newton's step

$$\mathbf{x}_{k+1} = \mathbf{x}_k - [\mathbf{H}(\mathbf{x}_k)]^{-1}\nabla f(\mathbf{x}_k)$$

is guaranteed to at least give a direction of descent?

19. How will you use the advantage of an efficient inversion of a tridiagonal matrix in inverting a matrix of the following form?

$$\begin{bmatrix} d_1 & e_2 & 0 & 0 & \cdots & 0 & p \\ f_2 & d_2 & e_3 & 0 & \cdots & 0 & 0 \\ 0 & f_3 & d_3 & \ddots & \cdots & 0 & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \ddots & d_{n-1} & e_n \\ q & 0 & 0 & 0 & \cdots & f_n & d_n \end{bmatrix}$$

20. What are the bare minimum requirements that a metric, or generalized distance function, has to satisfy?