Module 4 : Unbalance in Multicylinder Engines – In-line, V-twin and Radial Engines; Balancing Techniques.

Lecture 7 : Unbalance in Multicylinder Engines – In-line, V-twin and Radial Engines; Balancing Techniques.

Objectives

In this lecture you will learn the following

- Typical arrangements of multiple cylinders
- State of balance of typical multi-cylinder engines

Consider a single cylinder engine (Fig 4.1) such as is used in a typical scooter or motor bike. We have analyzed the inertia forces in this case based on our approximate dynamic equivalent link model of connecting rod – replacing it by two masses one at crank pin and another at the piston end. Thus effectively there are reciprocating masses which included the mass of the piston and that part of the connecting rod which was lumped on the piston pin; rotating parts which included the effective mass of crank at its pin and part of connecting rod lumped at crank pin.

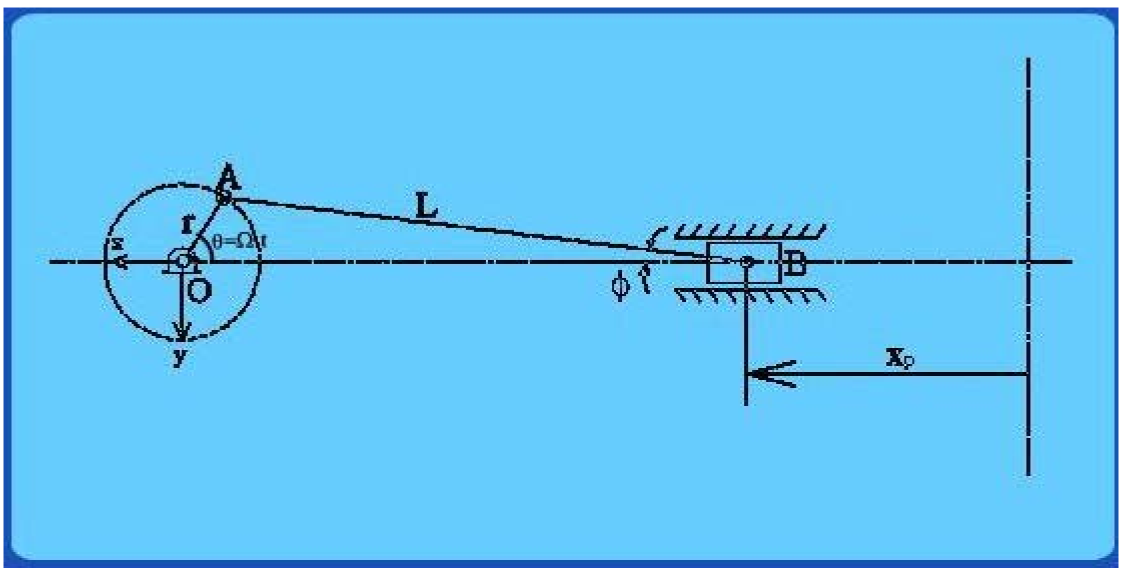

Figure 4.1 Typical Single Cylinder Engine.

From our earlier discussion on balancing it is easily conceived that the rotating parts can be readily balanced out by keeping a balancing mass at an appropriate radial location diametrically opposite m_{rot}. Thus we are left with only the inertia forces due to reciprocating masses. These forces are given by:

$$I.F. = m_{rec}\,\ddot{x}_p \qquad (4.1)$$

Recall that, with reference to Fig. 4.1, and eq(3.7), an approximate expression for the acceleration of the reciprocating parts is given by:

$$\ddot{x}_p = r\Omega^2\left(Cos\theta + \frac{r}{\ell}Cos2\theta\right) \qquad (4.2)$$

Thus an approximate expression for the net unbalanced inertia force in a single cylinder IC engine is given by:

$$I.F. = m_{rec} r \Omega^2 \left(Cos\theta + \frac{r}{\ell} Cos2\theta \right) \tag{4.3}$$

It is important to note that this force is directed ALONG the line of reciprocation of the masses m_{rec} and θ is the orientation of the crank measured from this line of reciprocation.

For example, as shown in Fig. 4.2, if the line of reciprocating motion (axis of cylinder) is arbitrarily directed, we can resolve this force into components along Cartesian coordinate axes X- and Y. In the subsequent discussion of the state of balance of multi-cylinder engines, with multiple cylinders arranged in a particular configuration, it is therefore only required to find the X- and Y- components of inertia forces of each cylinder and perform a vector addition of all the component forces and their moments to get the resultant unbalance force/moment for the whole engine.

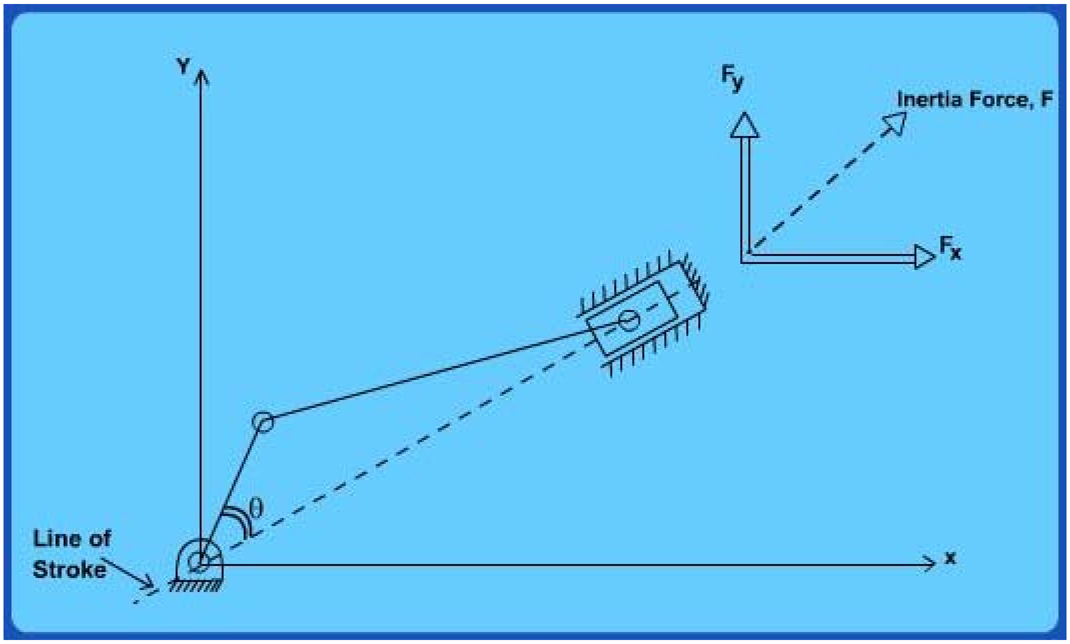

Figure 4.2 Arbitrarily oriented single cylinder engine

INERTIA FORCES

Let us designate the inertia forces due to the reciprocating parts of cylinder "i" as $I.F.|_i$. Thus,

$$\begin{aligned}
I.F.|_1 &= m_{rec}\, r\, \Omega^2 \left(Cos\theta_1 + \frac{r}{\ell} Cos2\theta_1 \right) \\
I.F.|_2 &= m_{rec}\, r\, \Omega^2 \left(Cos\theta_2 + \frac{r}{\ell} Cos2\theta_2 \right) \\
&= m_{rec}\, r\, \Omega^2 \left(Cos(\theta_1 + \beta_2) + \frac{r}{\ell} Cos2(\theta_1 + \beta_2) \right) \\
I.F.|_i &= m_{rec}\, r\, \Omega^2 \left(Cos(\theta_1 + \beta_i) + \frac{r}{\ell} Cos2(\theta_1 + \beta_i) \right)
\end{aligned} \tag{4.5}$$

These forces are all acting along their respective cylinder axes and are therefore axially separated as indicated in Fig. 4.3. Thus the total inertia force due to the reciprocating parts is given by:

(4.6)

$$I.F. = \sum_{i=1}^{n} I.F.\Big|_i$$
$$= m_{rec}\, r\, \Omega^2 \left[\sum_{i=1}^{n} Cos(\theta_1 + \beta_i) + \frac{r}{\ell} \sum_{i=1}^{n} Cos2(\theta_1 + \beta_i) \right]$$

where "n" stands for the number of cylinders. From standard trigonometric relations, we can write,

$$\sum_{i=1}^{n} Cos(\theta_1 + \beta_i) = Cos\theta_1 \sum_{i=1}^{n} Cos\,\beta_i \quad - \quad Sin\theta_1 \sum_{i=1}^{n} Sin\,\beta_i$$
$$\sum_{i=1}^{n} Cos2(\theta_1 + \beta_i) = Cos2\theta_1 \sum_{i=1}^{n} Cos2\,\beta_i \quad - \quad Sin2\theta_1 \sum_{i=1}^{n} Sin2\,\beta_i \tag{4.7}$$

In order that the total inertia force be zero for all positions of cranks (i.e., for all values of θ_1), we therefore have the following conditions:

$$\begin{aligned} \sum_{i=1}^{n} Cos\,\beta_i &= 0 \\ \sum_{i=1}^{n} Sin\,\beta_i &= 0 \\ \sum_{i=1}^{n} Cos2\,\beta_i &= 0 \\ \sum_{i=1}^{n} Sin2\,\beta_i &= 0 \end{aligned} \tag{4.8}$$

PITCHING MOMENTS

Due to the axial separation of the cylinders, there is a pitching moment developed about a lateral axis as given by $\sum_{i=1}^{n} I.F.\Big|_i z_i$ where z_i stands for the axial distance of i^{th} cylinder from a common reference frame. When the cylinders are arranged along the length direction of the vehicle, such a moment about a lateral axis of the vehicle tends to induce pitching motion of the vehicle and hence this is known as pitching moment. For the pitching moment to be zero for all crank positions, we can similarly derive the following conditions:

$$\begin{aligned} \sum_{i=1}^{n} z_i\, Cos\,\beta_i &= 0 \\ \sum_{i=1}^{n} z_i\, Sin\,\beta_i &= 0 \\ \sum_{i=1}^{n} z_i\, Cos2\,\beta_i &= 0 \\ \sum_{i=1}^{n} z_i\, Sin2\,\beta_i &= 0 \end{aligned} \tag{4.9}$$

Thus for a given crank shaft profile (i.e., z and β_s) we could determine the resultant force/moment on the frame.

We will illustrate the calculations through an example.

Example: Inline Four Cylinder Four Stroke Engine

Consider an inline four cylinder four stroke marine engine arrangement as shown in Fig. 5.4. Let the reciprocating masses in each cylinder be 500kg. Let the crank length be 200mm and the connecting rod length 800mm. Engine speed is 100 RPM. Investigate its state of balance.

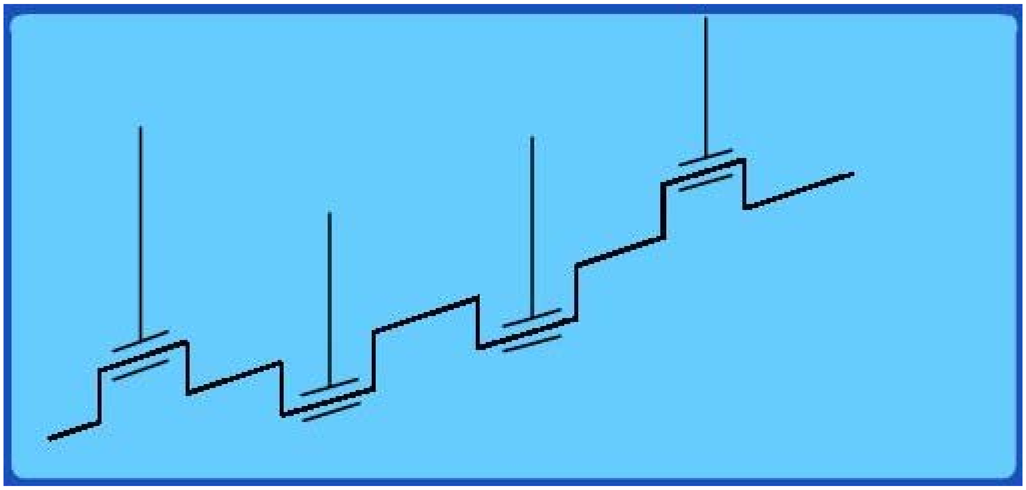

Figure 4.4 Four cylinder In-line engine

Solution:

Let the crank position of the first crank be θ. Then the remaining cylinders' cranks are at (θ+180), (θ +180) and θ respectively. In other words, $\beta_1 = 0;\ \beta_2 = 180^0;\ \beta_3 = 180^0;\ \beta_4 = 0$. Thus,

$$\sum Cos\ \beta_i = 0$$
$$\sum Sin\ \beta_i = 0$$
$$\sum Cos\ 2\beta_i = 4$$
$$\sum Sin\ 2\beta_i = 0$$

Thus the primary forces are completely balanced but not the secondary forces. The magnitude of the unbalanced secondary forces can be estimated as follows:

I.F. = (4) (500) (0.2) (2* π*100/60) 2 (0.2/0.8) = 10966 N

Let us take the mid-plane as the reference plane. Then,

z_1 = - 1.5m

z_2 = - 0.5m

z_3 = 0.5m

z_4 = 1.5m

Thus, we have,

$$\sum z_i\ Cos\ \beta_i = 0$$
$$\sum z_i\ Sin\ \beta_i = 0$$
$$\sum z_i\ Cos\ 2\beta_i = 0$$
$$\sum z_i\ Sin\ 2\beta_i = 0$$

Therefore the primary and secondary moments are completely balanced.

STATE OF BALANCE OF A RADIAL ENGINE

A radial engine is one in which all the cylinders are arranged circumferentially as shown in Fig. 4.5. These engines were quite popularly used in aircrafts during World War II. Subsequent developments in steam/gas turbines led to the near extinction of these engines. However it is still interesting to study their state of balance in view of some elegant results we shall discuss shortly. Our method of analysis remains identical to the previous case i.e., we proceed with the assumption that all cylinders are identical and the cylinders are spaced at uniform interval $\left(\frac{2\pi}{n}\right)$ around the circumference. Thus the crank angle for an i^{th} cylinder will be given by:

$$\theta_i = \theta_1 + (i-1)\left(\frac{2\pi}{n}\right) \tag{4.10}$$

Inertia force due to reciprocating parts of the i^{th} cylinder is given by:

$$I.F.\big|_i = m_{rec} r \Omega^2 \left(Cos\theta_i + \frac{r}{\ell} Cos 2\theta_i \right) \tag{4.11}$$

Resolving this along global Cartesian X-Y axes, we have,

$$I.F.\big|_{iX} = m_{rec} r \Omega^2 \left(Cos\theta_i + \frac{r}{\ell} Cos 2\theta_i \right) Cos\left[(i-1)\frac{2\pi}{n} \right] \tag{4.12}$$

$$I.F.\big|_{iY} = m_{rec} r \Omega^2 \left(Cos\theta_i + \frac{r}{\ell} Cos 2\theta_i \right) Sin\left[(i-1)\frac{2\pi}{n} \right] \tag{4.13}$$

When summed up for all the cylinders, we get the resultant inertia forces for the whole radial engine.

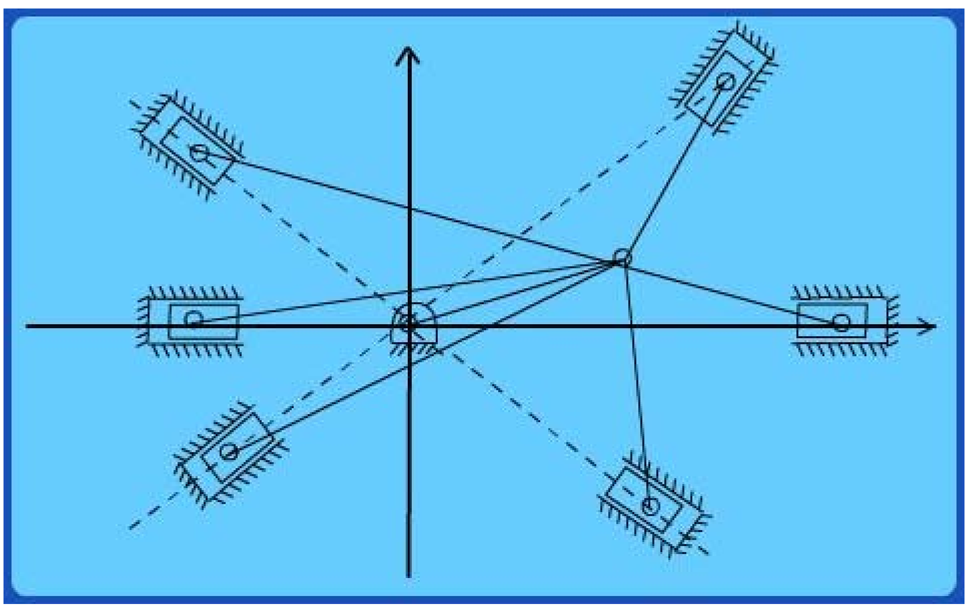

Fig. 4.5 Typical Radial Engine (Not to scale)

Interesting result on the primary forces

The primary component of the forces above sum upto:

$$\sum_{i=1}^{n} I.F.\big|_{XP} = \sum_{i=1}^{n} m_{rec} r\Omega^2 \left(Cos\theta_i\right) Cos\left[\left(i-1\right)\frac{2\pi}{n}\right]$$

(4.14)

$$\sum_{i=1}^{n} I.F.\big|_{YP} = \sum_{i=1}^{n} m_{rec} r\Omega^2 \left(Cos\theta_i\right) Sin\left[\left(i-1\right)\frac{2\pi}{n}\right]$$

Using standard trigonometric relations, it can be shown that this summation simply reduces to (for n > 2):

$$\sum_{i=1}^{n} I.F.\big|_{XP} = \frac{n}{2} m_{rec} r\Omega^2 \left(Cos\theta_1\right)$$

$$\sum_{i=1}^{n} I.F.\big|_{YP} = \frac{n}{2} m_{rec} r\Omega^2 \left(Sin\theta_1\right)$$

(4.15)

Thus, the arrangement of the radial engine has resulted in a total resultant primary force of fixed magnitude viz., $\frac{n}{2} m_{rec} r\Omega^2$ and directed along the first crank. Thus this fixed magnitude force "rotates" along with the first crank. Such a resultant force can therefore readily be balanced out by an appropriate mass kept on the crank. Therefore it is **possible to get complete balance of the primary forces**.

Further analysis of the inertia forces reveals that for even number of cylinders (n > 2) i.e., for four, six, eight etc. cylinders the secondary forces are also completely balanced out.

Thus we can have an engine where in all the inertia forces can be completely balanced. Since the reciprocating engines can NOT be normally operated at high speeds because of the inertia forces due to the reciprocating masses, this gives an exciting possibility of operating the radial engine at high speeds. However when we think of high speeds, we should revisit our analysis of the inertia forces. Recall that eq. (5.3) involved an approximation as given in eq. (4.2) wherein we neglected the higher order terms in the series. Including these terms also, we get the more accurate expression for the acceleration of the reciprocating parts as:

$$\ddot{x}_p = r\Omega^2 \left(Cos\theta + c_2 Cos2\theta + c_4 Cos4\theta + c_6 Cos6\theta + ...\right) \qquad (4.16)$$

where,

$$c_2 = \left[\frac{1}{4}\left(\frac{r}{\ell}\right) + \frac{1}{16}\left(\frac{r}{\ell}\right)^3 + \frac{15}{512}\left(\frac{r}{\ell}\right)^5 + ...\right]$$

$$c_4 = \left[\frac{1}{64}\left(\frac{r}{\ell}\right)^3 + \frac{3}{256}\left(\frac{r}{\ell}\right)^5 + ...\right] \qquad (4.17)$$

$$c_6 = \left[\frac{5}{512}\left(\frac{r}{\ell}\right)^5 + ...\right]$$

Since crank radius "r" is usually less than one-fourth the length of the connecting rod, it is common practice to ignore the higher order terms. However in view of the possibility of higher speeds (i.e. large Ω^2), higher order terms may become significant for a radial engine. Fortunately for even number of cylinders (n > 2) i.e., for four, six, eight etc. cylinders it can be shown that all the higher order forces are also completely balanced out .

However as mentioned earlier, radial engines are normally not in use now-a-days. In view of the power-to-weight ratio advantage, gas turbine engines are common in modern aircrafts. However, V-engine configurations wherein the cylinders are arranged in two banks forming a "V" are commonly employed. For example, a six-cylinder engine may have two sets of (three, in-line) cylinders on a common crankshaft arranged as a "V". Similarly, two sets of the in-line four cylinders of previous example could be arranged on the arm of a "V" to give an eight-cylinder engine. Of course, the larger number of cylinders results in more power. The V-angle introduces additional phase shifts. Typical marine engines use the V-8 configuration.

Analysis of V-engines proceeds exactly same as that for radial engines.

Recap

In this modules, you have learnt the following

- Common arrangements of multiple cylinders
- Methods of analyzing the shaking forces/moments
- In particular you have studied the inline four cylinder engine configuration as this is the most common configuration used in the passenger cars you see on the road in India such as Maruti 800, Zen, Santro, Honda City, Toyoto Corolla etc.

Congratulations, you have finished Module 4. To view the next module select it from the left hand side menu of the page