Module 2
Selection of Materials and Shapes
Instructional objectives

By the end of this lecture, the student will learn how to implement compound material indices for a few practical applications.

Example 18: Spars for man-powered plane

Most of the engineering designs consist of conflicting demands, multiple objectives and a number of constraints. In designing a spar for typical man-powered plane, the objective is to make the spar light and stiff to maintain the aerodynamic efficiency of the wings and to keep the overall weight of aircraft sufficiently light. So, the above problem can be translated into functional requirement, objective, constraints to be considered, and the free variables that the designers are allowed to change. Figure 2.7.1 schematically shows a man-powered aircraft structure with two spars one spanning the wings and the other linking the wings to the tail.

**Figure 2.7.1** Schematic picture of a man-powered plane with two spars. Both are to be designed for stiffness at minimum weight. [2]

<table>
<thead>
<tr>
<th>Function:</th>
<th>Wing Spar of man-powered aircraft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective:</td>
<td>Minimise mass.</td>
</tr>
<tr>
<td>Constraints:</td>
<td>(i) Length L is specified, (ii) Bending stiffness is specified.</td>
</tr>
<tr>
<td>Free variables:</td>
<td>(i) Material, (ii) Section shape and scale</td>
</tr>
</tbody>
</table>
A material-shape combination is required such that the mass is minimised for a given bending stiffness. Thus, the combined material index ($M_1$) to be maximised can be given as

$$M_1 = \frac{(\phi^s B E)^{1/2}}{\rho}$$  \hspace{1cm} (1)

Figure 2.7.2 depicts a combination of Young’s Modulus (E) vis-à-vis Density ($\rho$) for several materials in a graphical form. Some initial choice of material can be made from Figure 2.7.2 for the wing spar. The dashed line in the figure represents the constant value of the material index $(E)^{1/2}/\rho$, which is equal to 10. Figure 2.7.2 also demonstrates how consideration of an efficient shape for a typical material can indicate the superiority of the same material for the given application. For example, typical tube sections of aluminum alloy confirm to a shape factor of 20 i.e. $\phi^e_B = 20$. If we now consider a modified material index with $E^* = E/\phi^e_B$ and $\rho^* = \rho/\phi^e_B$, and with $\phi^e_B = 20$, the position of the same aluminum alloy shifts to a more preferred location in the graph as indicated by the arrow. Similarly, if we consider typical box sections in CFRP (carbon fibre reinforced polymer) with typical $\phi^e_B = 10$, the same material also shifts to a more preferred location as indicated by the arrow in Figure 2.7.2.

**Figure 2.7.2** Graphical representation of Young’s Modulus (E) vis-à-vis Density (\(\rho\)) for engineering materials [2]
Table 2.7.1 presents the analytically computed data of the material indices for a set of seven chosen materials. If all the materials confirm to the same shape or in other words, no contribution from the shape factor is considered, Balsa and spruce are significantly superior from the rest of the materials as can be observed in column 4. However, when the corresponding shape factors are considered, aluminium becomes marginally better or similar to balsa and spruce while CFRP appears to the best of all. Obviously, beryllium cannot be considered due to its toxicity although it yields a very high value of combined material index. Advances in the technology of drawing thin-walled aluminum tubes allows today highly efficient shape factor that cannot be reproduced in wood, giving aluminum a performance edge. Further more, CFRP meets the requirements of lower density and higher modulus. So, CFRP outperforms all other suitable materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>E (GPa)</th>
<th>ρ (Mg/m³)</th>
<th>( \frac{E^{1/2}}{\rho} )</th>
<th>( \phi_B^e )</th>
<th>( \frac{(E\phi_B^e)^{1/2}}{\rho} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balsa</td>
<td>4.20 – 5.20</td>
<td>0.17 – 0.24</td>
<td><strong>10</strong></td>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>Spruce</td>
<td>9.80 – 11.90</td>
<td>0.36 – 0.44</td>
<td>8</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>Steel</td>
<td>200 – 210</td>
<td>7.82 – 7.84</td>
<td>1.8</td>
<td>25</td>
<td>9</td>
</tr>
<tr>
<td>AA 7075 T6</td>
<td>71 – 73</td>
<td>2.80 – 2.82</td>
<td>3</td>
<td>20</td>
<td>14</td>
</tr>
<tr>
<td>CFRP</td>
<td>100 – 160</td>
<td>1.50 – 1.60</td>
<td>7</td>
<td>10</td>
<td><strong>23</strong></td>
</tr>
<tr>
<td>Beryllium</td>
<td>290 – 310</td>
<td>1.82 – 1.86</td>
<td>9.3</td>
<td>15</td>
<td><strong>36</strong></td>
</tr>
<tr>
<td>Borosilicate Glass</td>
<td>62 – 64</td>
<td>2.21 – 2.23</td>
<td>3.7</td>
<td>10</td>
<td>11</td>
</tr>
</tbody>
</table>

Example 19: Forks for a racing bicycle

Figure 2.7.3 schematically outlines the forks of typical bicycle. The significant consideration in design of forks is the strength to avoid yielding during normal use. The loading on the forks are predominantly bending in nature. Also, the forks should be as light as possible for comfortable riding. Thus, the design problem is translated into functional requirement, objective, constraints to be considered, and the free variables that the designers are allowed to change as given below.
Function: Forks for a racing bicycle.
Objective: Minimise mass
Constraints: (i) Length $L$ is specified, (ii) Bending stiffness is specified.
Free variable: (i) Material, (ii) Section shape and size

Figure 2.7.3  Schematic picture of a bicycle and the forks [2]

The forks of a bicycle can be envisaged as a beam of length $L$ that must carry a maximal load $F$ without failure. In other words, the forks can be designed as light and strong beams. The combined material index ($M_1$) to be maximised is given below

$$M_1 = \left( \frac{\phi_B \sigma_f}{\rho} \right)^{2/3}$$

[2]

where $\sigma_f$ is failure strength (ultimate tensile strength or yield strength) of the material, $\rho$ is the material density, and $\phi_B$ is the shape factor for failure in bending. Table 2.7.2 enlists seven possible candidate materials with the corresponding properties. Table 2.7.2 shows that spruce performs the best when the shape factor is not considered i.e. the material index $\left( \frac{\sigma_f^{2/3}}{\rho} \right)$
without considering the shape factor is the highest. If the shape factor \( \phi_B^f \) is considered, the ranking of the materials is changed with CFRP (carbon fibre reinforced polymer) as the best performing material followed by titanium alloy (Ti-6Al-4V) and steel. In strength-limited applications the performance of magnesium alloys is expected to be poor despite its low density.

In spite of the best performance of CFRP, bicycles are commonly made of steel with costlier variants in aluminium and titanium alloys. This is primarily due to the fact that steel has good fracture resistance and excellent manufacturability. The racing bicycles are often made of CFRP with interleaving of the carbon fibers with layers of glass or Kevlar to improve the fracture-resistance. Mountain bicycles, for which strength and impact resistance are particularly important, usually consist of steel or titanium forks.

**Table 2.7.2** Candidate materials and corresponding material indices for bicycle forks

<table>
<thead>
<tr>
<th>Material</th>
<th>( \sigma_f ) (MPa)</th>
<th>( \rho ) (Mg/m³)</th>
<th>( \phi_B^f )</th>
<th>( (\sigma_f)^{2/3} \frac{f}{\rho} )</th>
<th>( (\phi_B^f \sigma_f)^{2/3} \frac{f}{\rho} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spruce</td>
<td>70 – 80</td>
<td>0.46 – 0.56</td>
<td>1</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>Bamboo</td>
<td>80 – 160</td>
<td>0.60 – 0.80</td>
<td>2.20</td>
<td>34</td>
<td>59</td>
</tr>
<tr>
<td>Steel</td>
<td>770 – 990</td>
<td>7.82 – 7.83</td>
<td>7.50</td>
<td>12</td>
<td>48</td>
</tr>
<tr>
<td>AA 6061–T6</td>
<td>240 – 260</td>
<td>2.69 – 2.71</td>
<td>5.90</td>
<td>15</td>
<td>48</td>
</tr>
<tr>
<td>Ti-6Al–4V</td>
<td>930 – 980</td>
<td>4.42 – 4.43</td>
<td>5.90</td>
<td>22</td>
<td>72</td>
</tr>
<tr>
<td>Magnesium AZ 61</td>
<td>160 – 170</td>
<td>1.80 – 1.81</td>
<td>4.25</td>
<td>17</td>
<td>46</td>
</tr>
<tr>
<td>CFRP</td>
<td>300 – 450</td>
<td>1.50 – 1.60</td>
<td>4.25</td>
<td>33</td>
<td>88</td>
</tr>
</tbody>
</table>

**Example 20: Floor Joists**

Figure 2.7.4 schematically shows the cross-section of typical floor-joists. The floors are supported by the joists that should withstand a specified bending load without sagging, failure and sudden fracture. The floor joist materials should be cheap to avoid cost escalation. So, the above problem can be translated into functional requirement, objective, constraints to be considered, and the free variables that the designers are allowed to change as given below.
**Function:** Floor Joists  
**Objective:** Minimise material cost  
**Constraints:** (i) Length $L$ is specified, (ii) Bending stiffness is specified, and (iii) Strength specified  
**Free variable:** (i) Material, (ii) Section shape

![Figure 2.7.4 Schematic picture of floor joist cross-sections [2]](image)

The floor joist can be envisaged as a beam of length $L$ that must carry a maximul load $F$ without sagging (i.e. considering stiffness limited design) that leads to a *compound material index* ($M_1$) including the shape factor, which should be maximized, as

$$M_1 = \frac{(\phi_B \sigma_f)^{1/2}}{C_m \rho} \quad [3]$$

where $\rho$ is the density of the material, $C_m$ is the material cost (per unit mass). In a similar manner, the joist can be envisaged as a beam of length $L$ that must carry a maximul load $F$ without failure (i.e. considering strength limited design) that leads to a *compound material index* ($M_2$) including the shape factor, which should be maximized, as

$$M_2 = \frac{(\phi_B \sigma_f)^{2/3}}{C_m \rho} \quad [4]$$

Table 2.7.3 enlists three possible candidate materials with the corresponding properties. Table 2.7.3 shows that wood can be processed to sections that may perform better even compared to the most efficient I-beam sections in steel. However, wood is a hygroscopic material and degrades with time. Hence, wood has been used widely in small buildings. However, in large commercial building, the steel I-beam sections are used as floor joist.
Table 2.7.3  Candidate materials and corresponding material indices for floor joists

<table>
<thead>
<tr>
<th>Material</th>
<th>$\sigma_f$ (MPa)</th>
<th>$\rho$ (Mg/m³)</th>
<th>$\phi_B^e$</th>
<th>$\phi_B^f$</th>
<th>$C_m$</th>
<th>$\frac{(\sigma_f)^{2/3}}{C_m\rho}$</th>
<th>$\frac{(\phi_B^f\sigma_f)^{2/3}}{C_m\rho}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood</td>
<td>37 – 45</td>
<td>0.44 - 0.54</td>
<td>2</td>
<td>1.4</td>
<td>0.8-1.2</td>
<td>8.8</td>
<td>38</td>
</tr>
<tr>
<td>Bamboo</td>
<td>38 – 42</td>
<td>0.60 - 0.80</td>
<td>3.2</td>
<td>2.2</td>
<td>1.8-2.1</td>
<td>5.4</td>
<td>14.5</td>
</tr>
<tr>
<td>Steel (I-Section)</td>
<td>350 – 360</td>
<td>7.90 - 7.91</td>
<td>10</td>
<td>4</td>
<td>0.6-0.7</td>
<td>8.8</td>
<td>24</td>
</tr>
<tr>
<td>Al-alloys</td>
<td>240–260</td>
<td>3.26-3.33</td>
<td>31</td>
<td>10</td>
<td>2.05-2.06</td>
<td>5.8</td>
<td>26.8</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>770–990</td>
<td>7.82–7.84</td>
<td>25</td>
<td>13</td>
<td>3.22-3.41</td>
<td>3.33</td>
<td>18.44</td>
</tr>
</tbody>
</table>

Exercise

(1) Select a suitable material for a typical bridge crane. Explain the *function*, *constrain*, *objective*, *free variable* clearly and select material index along with shape factor. Use the material chart given in the modules.

Hint: Figure of a typical bridge crane is given below.

References

Example 20: Materials and Process Selection for Rowing Oars
Participants: Sushrut Pande (08011008), Mukul Saha (08010036)

Figure 2.7.4 schematically shows the parts of a rowing oar and its maximum allowable deflection in operation, and also a typical finished pair of rowing oars. The rowing oars should facilitate: (a) high stiffness to avoid deflection under load, (b) minimum mass to be light enough and hence, to reduce effort by the rower, (c) enough strength to sustain bending moments in service, (d) good fracture toughness so as to sustain oar clashes and collision with rocks, and obviously (e) low cost.

![Schematic picture of parts of a rowing oar and a finished oar](image)

Figure 2.7.4  Schematic picture of parts of a rowing oar and a finished oar

Typically, a rowing oar can be considered as a beam loaded under bending with its collar [Fig. 2.7.4] constrained / attached to the boat. In service, the spoon that is immersed in water experiences pressure / bending load. Thus, the design problem is translated into functional requirement, objective, constraints to be considered, and the free variables that the designers are allowed to change as given below.

**Function:** Oars for rowing  
**Objective:** Minimum weight  
**Constraints:**
1. Specified minimum stiffness  
2. Suitable fracture resistance  
3. Low cost  
4. Specified length  
**Free variable:** Area of cross section (A) and material
Structural Analysis

Let us assume that the total length of the oar equals to \( L + d_1 + d_2 = nL \) as shown in Fig. 2.7.4 with a circular cross-sectional area of \( A \) through the loom. The maximum moment experienced on the collar of the *rowing oar* due to the applied load \( F \) on spoon will be equal to \( FL \). The second moment of area (I) of the *rowing oar* for the assumed section can be considered as \( A^2/2\pi \). Subsequently, the selection of material for a *rowing oar* can be taken further in three different directions.

1. **Light and Stiff material for a rowing oar**

   The material index in this case can be considered as \( M_1 = \sqrt{E/\rho} \) for a requisite stiffness (S) of the *rowing oar* given as \( S = C_1EA^2/2\pi L^3 \) and writing the mass (m) of the rowing oar as
   \[
   m = \rho A(nL) = f(\text{geometry}) \cdot f(\text{shape}) \cdot f(\text{material})
   \]
   and, further substituting the term A in terms of stiffness. The material index, \( M_1 \), needs to be maximized.

![Figure 2.7.5](image.png)

**Figure 2.7.5** Young’s Modulus vis-à-vis density of engineering materials [3]. The red arrow indicates the direction of increasing values of the material index (M1).
The best possible materials for a light, stiff <i)rowing oar</i>, we should attempt to pick the materials in Fig. (2.7.5) with higher values of $M_1$ i.e. materials lying near the top red line. The candidate materials are CFRP, parallel grain woods and engineering ceramics as apparent in Fig. (2.7.5).

[2] Light and Fracture Resistance material for a rowing oar

The fracture toughness ($K_{IC}$) can be expressed as $K_{IC} = C_2 \sigma$ where $\sigma$ is the applied stress and $C_2$ is a constant depending on geometry/shape and the initial crack length. Subsequently, the toughness ($K$) can be expressed as $K = 2\pi C_2 FL/A^2$. Next, substituting for $A$ in the following relation

$$m = \rho A(nL) = f(\text{geometry}) \bullet f(\text{shape}) \bullet f(\text{material})$$

leads to a material index, $M_2$, as $M_1 = K_{IC}^{2/3}/\rho$ and the same needs to be maximized. To achieve the same, we can consider a straight line $\log(K_{IC}) = (3/2)(\log M_2) + \log \rho$ in a typical log-log chart of fracture toughness vis-à-vis density as shown in Fig. 2.7.6.

![Fracture Toughness vis-à-vis Density of engineering materials](image)

**Figure 2.7.6** Fracture Toughness vis-à-vis Density of engineering materials [3]. The red arrow indicates the direction of increasing values of the material index ($M_2$)
The best possible materials for a light and tough rowing oar, we should attempt to pick the materials in Fig. (2.7.6) with higher values of M₂ i.e. materials lying near the top red line in Fig. 2.7.6 and the candidate materials are CFRP, parallel grain woods and alloy steels.

[3] Cheap and Stiff material for a rowing oar
The cost can be presumed as \( V \cdot C_r \) where \( V \) is the volume and \( C_r \) is the cost per unit volume. The corresponding material index can be given as in case [1] with little rearrangement as \( M_3 = \frac{\sqrt{E}}{(\rho \times C_r)} \) and the same needs to be maximized. To achieve the same, we can consider a straight line \( \log(E) = 2(\log M_3) + 2 \log(\rho \times C_r) \) in a typical log-log chart of elastic modulus vis-à-vis density as shown in Fig. 2.7.7.

![Figure 2.7.7](image)

**Figure 2.7.7** Young’s Modulus vis-à-vis Relative Cost of engineering materials [3]. The red arrow indicates the direction of increasing values of the material index (M3).

The best possible materials for a cheap and stiff rowing oar, we should attempt to pick the materials in Fig. (2.7.7) with higher values of M₃ i.e. materials lying near the top red line in Fig.
2.7.7 and some of the candidate materials are porous ceramics, parallel grain woods, cast iron and steels. Following table (table 2.7.4) provides a comparative performance of several candidate materials that can be considered based on the above three criteria presumed for the selection material for a **good performing rowing oar**.

### Table 2.7.4  
A comparative assessment of different candidate materials for Rowing oars

<table>
<thead>
<tr>
<th>Material</th>
<th>$K_{IC}$ (MPa m$^{0.5}$)</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>E (GPa)</th>
<th>$\frac{E^{0.5}}{\rho}$</th>
<th>$C_r$ ($$/m^3$$)</th>
<th>$\frac{(K_{IC})^{2/3}}{\rho}$</th>
<th>$\frac{E^{0.5}}{\rho * C_r}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel-Grained Wood</td>
<td>5</td>
<td>750</td>
<td>10</td>
<td>133</td>
<td>2</td>
<td>3.9</td>
<td>66.67</td>
</tr>
<tr>
<td>CFRP</td>
<td>3</td>
<td>1800</td>
<td>100</td>
<td>176</td>
<td>83</td>
<td>1.16</td>
<td>2.11</td>
</tr>
<tr>
<td>Alloy Steel</td>
<td>50</td>
<td>7850</td>
<td>200</td>
<td>57</td>
<td>19</td>
<td>1.73</td>
<td>2.98</td>
</tr>
<tr>
<td>Ceramics (SiC)</td>
<td>4</td>
<td>3210</td>
<td>400</td>
<td>197</td>
<td>124</td>
<td>0.78</td>
<td>1.58</td>
</tr>
<tr>
<td>Polymers (PVC)</td>
<td>1.1</td>
<td>1400</td>
<td>2.5</td>
<td>36</td>
<td>2</td>
<td>0.76</td>
<td>16.67</td>
</tr>
</tbody>
</table>

The ceramics are eliminated because they are brittle and expensive. The CFRP (carbon-fibre-reinforced polymers) composites offer light, stiff and tough oars but are expensive. The **Parallel grained woods** appear to be one of the best choices. Typical **Balsa wood**, in particular, provides for highest $M_1$ and $M_2$, but is slightly costly. Assuming the relative importance of stiffness and toughness are higher than the cost, the Balsa wood offers the best properties for a rowing oar. The wooden rowing oars typically cost between USD 20 to USD 50. The composite oars, on the other hand, though very light, stiff and tough, can cost as high as USD 450.

**Manufacturing Process**

The wooden oars are typically manufactured by the hand craftsmen owing to difficulty in automating the process and due to the directionality of properties of wood. Moreover, wood, being hygroscopic, needs to be laminated and sealed with a water-resistant compound like urethane. The composite oars are typically made of CFRP. They have an elaborate manufacturing process route involving several processes that may include some of the following steps.
1. Body is manufactured by wrapping flexible sheets of carbon fibre around cylindrical mandrel, followed by curing in oven.
2. Spoon is manufactured by hot press molding, followed by CNC milling
3. Handle is made on a Lathe.
4. For the purpose of assembly, holes are drilled & epoxy is inserted for complete bonding.

REFERENCES:
Example 21: Selection of material, shape and process for blades of ceiling fan
Participants: Aniket Chaudhury (113109010), Ashutosh Yetalkar (123100039)

The blades of a typical ceiling fan contribute to its primary function i.e. the circulation of air. Figure 2.7.8 shows the schematic diagram of the assembly of a ceiling fan along with a typically finished product.

![Figure 2.7.8 Schematic presentation of a ceiling fan](image)

The main function of the fan blades is to provide the flow of air in the downward direction by creating a pressure difference on the two sides of the blade causing the air to flow from the high pressure to the low pressure side. Evidently, the blades are primarily under bending loads and hence, the blades may fail either due to excessive stress or deflection. Considering the fan blades similar to a cantilever beam, the design problem is translated into functional requirement, objective, constraints to be considered, and the free variables that the designers are allowed to change as given below.

**Function:** Fan blades (analogous to light, low cost beam, which is strong and stiff)

**Objective:** Minimum mass and cost

**Constraints:**

1. Length L is specified
2. Maximum Deflection is specified
3. Young’s Modulus, $E > 50$ GPa [considered as a prerequisite to avoid deflection in service]

**Free variable:** Material, Manufacturing processes
Structural Analysis

Let $F$ be the total force acting on the blade of length $L$, the cross-sectional area $A$, the second moment of area $I$. The mass ($m$) of a typical fan blade can be given as, $m = \rho AL$, where $\rho$ is the density of material and, the net cost ($C$) of the same can be given as, $C = C_m \rho AL$, where $C_m$ is the cost of material per unit weight (kg). The normal stress ($\sigma_f$) due to a bending moment ($M$) [because of the load ($F$)] can be considered as $\sigma_f = M/Z$, where $Z$ is the section modulus. The requisite stiffness ($S$) to avoid deflection can be estimated as $S = C_1 EI/L^3$, where $E$ is the elastic modulus of the material being considered.

[1] Light and Strong material for a fan blade

The material index ($M_1$) in this case can be considered as $M_1 = (\sigma_f^{2/3})/\rho$ that needs to be maximized. We can consider a straight line, $(2/3) \log(\sigma_f) = (\log M_1) + \log(\rho)$ as indicated by the blue line in Fig. 2.7.9, in a typical log-log chart of failure strength vis-à-vis density [Fig. 2.7.9] and select a material above the blue line in the direction of the red arrow.

Figure 2.7.9  Failure strength vis-à-vis density of engineering materials [1]. The red arrow indicates the direction of increasing values of the material index ($M_1$)
[2] Light, Strong and Economical material for a fan blade

The material index ($M_2$) in this case can be considered as $M_2 = \left(\frac{\sigma_T^2}{f_2}\right) / (C_m \rho)$ that needs to be maximized. We can consider a straight line, $(2/3) \log(\sigma_T) = (\log M_2) + \log(C_m \rho)$ as indicated by the blue line in Fig. 2.7.10, in a typical log-log chart of failure strength vis-à-vis relative cost of engineering material [Fig. 2.7.10] and select a material above the blue line in the direction of the red arrow.

![Failure strength vis-à-vis relative cost of engineering materials][1]

Figure 2.7.10 Failure strength vis-à-vis relative cost of engineering materials [1]. The red arrow indicates the direction of increasing values of the material index ($M_2$)

[3] Light and Stiff material for a fan blade

The material index ($M_3$) in this case can be considered as $M_3 = \left(\frac{E^{1/3}}{\rho}\right)$ [presuming the fan blade shape as a rectangular panel] that needs to be maximized. We can consider a straight line, $(1/3) \log(E) = (\log M_3) + \log(\rho)$ as indicated by the blue line in Fig. 2.7.11, in a typical log-log chart of failure strength vis-à-vis relative cost of engineering material [Fig. 2.7.11] and select a material above the blue line in the direction of the red arrow.
chart of elastic modulus vis-à-vis density [Fig. 2.7.11] and select a material above both the blue line and the yellow line (that indicates a minimum elastic modulus of 50 GPa).

Figure 2.7.11 Elastic modulus vis-à-vis density of engineering materials [1]. The yellow line indicates an elastic modulus of 50 GPa.

[4] Light, Stiff and Economical material for a fan blade

The material index \( M_4 \) in this case can be considered as \( M_4 = (E^{1/3})/(C_m \rho) \) [presuming the fan blade shape as a rectangular panel] that needs to be maximized. We can consider a straight line, \( (1/3) \log(E) = (\log M_4) + \log(C_m \rho) \) as indicated by the blue line in Fig. 2.7.12, in a typical log-log chart of elastic modulus vis-à-vis relative cost of engineering materials [Fig. 2.7.12] and select a material above both the blue and the yellow lines (that indicates a minimum elastic modulus of 50 GPa).
Figure 2.7.12 Elastic modulus vis-à-vis relative cost of engineering materials [1]. The yellow line indicates an elastic modulus of 50 GPa.

Considering the indicated preferred zones of selectable materials in Figs 2.7.9 to 2.7.12, the following materials can be shortlisted.

**Table 2.7.5** A comparative assessment of different candidate materials for fan blades

<table>
<thead>
<tr>
<th>Material</th>
<th>ρ (Mg/m$^3$)</th>
<th>E (GPa)</th>
<th>$\sigma_f$ (MPa)</th>
<th>$C_m$ ($/m^3$)</th>
<th>$\frac{\sigma_f^{2/3}}{\rho}$</th>
<th>$\frac{\sigma_f^{2/3}}{C_m\rho}$</th>
<th>$\frac{E^{1/3}}{\rho}$</th>
<th>$\frac{E^{1/3}}{C_m\rho}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>7 – 8</td>
<td>105 – 107</td>
<td>200 – 1100</td>
<td>0.9 – 1.2</td>
<td>4.8 – 13.3</td>
<td>5.3 – 11.1</td>
<td>0.6 – 0.5</td>
<td>0.7 – 0.4</td>
</tr>
<tr>
<td>Aluminium Alloys</td>
<td>2 – 3</td>
<td>70 – 80</td>
<td>30 – 400</td>
<td>3 – 4</td>
<td>4.8 – 18.1</td>
<td>1.6 – 4.5</td>
<td>2.1 – 1.4</td>
<td>0.7 – 0.35</td>
</tr>
<tr>
<td>CFRP (Carbon fibre reinforced composite)</td>
<td>1.6 – 1.7</td>
<td>80 – 105</td>
<td>600 – 1000</td>
<td>20 – 30</td>
<td>44.46 – 58.82</td>
<td>2.2 – 1.9</td>
<td>2.6 – 2.7</td>
<td>0.1 – 0.09</td>
</tr>
<tr>
<td>Titanium Alloys</td>
<td>4 – 5</td>
<td>100 – 104</td>
<td>300 – 1100</td>
<td>60 – 70</td>
<td>11.2 – 21.3</td>
<td>0.2 – 0.3</td>
<td>1.6 – 0.9</td>
<td>0.03 – 0.01</td>
</tr>
</tbody>
</table>
CFRP and Titanium alloys can be eliminated because of high cost as evident from table 2.7.5. Aluminium is about three times lighter in weight in comparison to steel and hence, the motor torque required to rotate the blades will be less. Secondly, aluminium is also a better corrosion-resistant material. Hence, aluminium alloys may be considered as the final material of choice for making the blades of the ceiling fan.

**Consideration of shape for a fan blade**

The blades of ceiling fans depict a certain shape so that it can slice the air during rotation resulting in the air above and below the blade to move at different speeds leading to a pressure difference for the air flow to occur. Considering a presumed amplitude of bending of the sheet as a, the second moment of area (I) can be given as, \((2a + t)^2 t/12\). The same \((I_0)\) for a flat sheet of thickness \(t\) can be given as, \(t^3/12\). Hence, the shape factor of the fan blade in elastic bending \((\phi^e_\text{B})\) can be estimated as, \(\phi^e_\text{B} = I/I_0 = (2a + t)^2/t^2\). Similarly, the shape factor of the fan blade considering failure in bending \((\phi^f_\text{B})\) can be estimated as, \(\phi^f_\text{B} = Z/Z_0 = (2a + t)/t\). Considering that the preferred engineering shape required for the functionality of the fan blades is mandatory and can be produced by simple bending processes both in steel and aluminum with equal ease, we decide the selection of material will be governed primarily by the material indices.

**Selection of Manufacturing Process for a fan blade**

We want to select the manufacturing process to produce blades of ceiling fan with material selected as aluminium alloys. The blades are made from flat sheets. The approximated weight of each blade would be about 0.2 kgs with the section thickness about 1 mm. To identify a suitable process, we look through the following charts [Figs 2.7.13 – 2.7.16]. The possible areas of selection are highlighted in Figs 2.7.13 to 2.7.16 indicating the most suitable process as sheet forming, and in particular, bending. Although the size of the fan blade is small, precaution must be taken to select the bending process and the process conditions to avoid *spring back* after bending.
Figure 2.7.13 Compatibility charts of process vis-à-vis engineering materials [1]

Figure 2.7.14 Compatibility charts of process vis-à-vis shape of engineering materials [1]
References

5) http://www.moorefans.com/pdfs/TMC_661P_.PDF
6) http://www-materials.eng.cam.ac.uk/mpsite/default.html