Module 2
Selection of Materials and Shapes
Lecture 4
Case Studies - I
Instructional objectives

This is a continuation of the previous lecture. By the end of this lecture, the student will further learn how to develop and use typical material indices for the selection of material for common engineering parts. Two examples are illustrated here with regard to the development of material indices for corresponding applications.

Example 7: Selection of Material for an efficient flywheel

Figure 2.4.1 shows the schematic picture of a typical flywheel that is used to store rotational energy in applications such as automotive transmissions. An efficient flywheel should be able to store the maximum energy per unit volume or per unit mass at a specified angular velocity. The task is to search for a suitable material index for the selection of material for an efficient flywheel that should have adequate toughness and can store the maximum kinetic energy per unit volume or mass. The above problem can be translated into functional requirement, objective, constraints to be considered, and the free variables that the designers are allowed to change as follows.

**Function:** Flywheel for energy storage

**Objective:** Maximize kinetic energy per unit mass.

**Constraints:**
(i) Outer radius, R, may be fixed, (ii) Must not burst, and (iii) Should have adequate toughness to avoid catastrophic failure.

**Free variable:** Choice of material
Performance Equation

The **Performance Equation** can be developed as follows. The energy \( U \) stored in a flywheel can be estimated as

\[
U = \frac{1}{2} J \omega^2 \quad \text{where} \quad J = \frac{\pi \rho R^4 t}{2} \tag{1}
\]

where \( \rho \) is the density of the material, \( \omega \) is the angular speed of the flywheel, \( R \) is the radius and \( t \) is the thickness of the flywheel disc. The mass \( m \) of the flywheel disc can be given as

\[
m = \pi \rho R^2 t \tag{2}
\]

Hence, the energy per unit mass can be given as,

\[
\frac{U}{m} = \frac{1}{4} R^2 \omega^2 \tag{3}
\]

The maximum principal stress \( \sigma_{\text{max}} \) on the flywheel disc as a function of the rotational velocity can be expressed as,

\[
\sigma_{\text{max}} = \frac{1}{2} \rho R^2 \omega^2 \tag{4}
\]

Since the maximum principal stress should not exceed the failure strength \( \sigma_f \) of the material, we can develop the material index, \( M_1 \), to maximize the energy stored per unit mass by rearranging equations (3) and (4) as,

\[
\frac{U}{m} = \frac{1}{2} \left( \frac{\sigma_f}{\rho} \right) \approx \frac{1}{2} M_1 \tag{5}
\]

It is clear from equation (5) that greater values of \( M_1 \) will tend to maximize the energy stored per unit mass for a given angular speed, radius and thickness of the flywheel disc. A second material index, \( M_2 \), can be considered in terms of fracture toughness \( (K_{\text{IC}}) \) of the material.

Figure 2.4.2 depicts a typical chart of material properties (strength, \( \sigma_f \), vis-à-vis density, \( \rho \)) in a log-log scale. The advantage of log-log scale over decimal scale is that the constant material index lines will appear as a straight line which makes the selection and representation easier. The black line represents the constant material index line, for \( M_1 \). Since the chart is plotted in log-log scale, the black line confirms to a straight line \([\text{denoted as: } \log \sigma_f = \log M + \log \rho]\). Thus, any engineering material falling around the line will confirm to a similar value of material index and by sliding the line, it is possible to select a set of suitable candidate materials considering \( M_1 \).

Figure 2.4.2 depicts that aluminum alloys, titanium alloys, engineering composites and
engineering ceramic would provide excellent values of $M_1$. However, the engineering ceramics would provide very low fracture toughness [$M_2$] and hence, may be eliminated. Further selection must be made based on the cost and the energy storage capacity of specific materials.

![Schematic material chart of strength vis-à-vis density of engineering materials](image)

**Figure 2.4.2** Schematic material chart of strength vis-à-vis density of engineering materials

**Example 8: Selection of Material for pressure vessel**

Figure 2.4.3 shows the schematic picture of the cross-section of a spherical pressure vessel and typical circumferential stresses experienced by the vessel wall with a presumed crack. The safe design of a small sized pressure vessel would require that the material yields before a final fracture. Similarly, the safe design of large pressure vessels typically calls for a criterion that any small crack opens as a leak prior to a catastrophic failure. The above problem can be translated into *functional requirement*, *objective*, *constraints to be considered*, and the *free variables* that the designers are allowed to change as follows.

**Function:** Pressure vessel to contain an internal pressure of $p$ safely.

**Objective:** Maximize kinetic energy per unit mass.

**Constraints:** (i) maximize safety using yield-before-break criterion (small vessel), or
(ii) maximize safety using leak-before-break criterion (large vessel)

**Free variable:**

(i) Choice of material

![Figure 2.4.3](image_url)  
**Figure 2.4.3**  Schematic presentation of the cross-section of a spherical pressure vessel

**Performance Equation:**

The **Performance Equation** can be developed as follows. Stress ($\sigma$) in the wall of a thin-walled spherical pressure vessel of radius $R$, wall thickness $t$ and with internal pressure, $p$, can be given as

$$\sigma = \frac{pR}{2t} \quad (6)$$

The minimum stress required for a presumed circumferential crack of diameter $2a_c$ to propagate can be given as

$$\sigma = \frac{CK_{1C}}{\sqrt{\pi a_c}} \quad (7)$$

where $C$ is a constant and $K_{1C}$ the plane-strain fracture toughness of the pressure vessel material. Hence, the largest pressure (for a given vessel radius, $R$, wall thickness, $t$, and initial circumferential crack diameter, $2a_c$) would be carried by the material with the greatest value of $K_{1C}$ and hence, we can write

$$M_1 = K_{1C} \quad (8)$$

However, the material index $M_1$ alone cannot ensure a fail-safe design, which further requires

$$\sigma = \sigma_f,$$

where $\sigma_f$ is the failure strength of the material, which translates to an appropriate material index, $M_1$, as
\[
\pi a_c \leq C^2 \left[ \frac{K_{IC}}{\sigma_f} \right] \quad \Rightarrow M_1 = \frac{K_{IC}}{\sigma_f}
\]  

(9)

For typical large pressure vessels, we can estimate the minimum stress required for a crack to penetrate the wall thickness, \( t \), thereby making a leak can be estimated as

\[
\sigma \approx \sigma_f = \frac{C K_{IC}}{\sqrt{\pi t / 2}}
\]  

(10)

Substituting for \( t \) using equations (1) and (10), we can further express the safe internal pressure as

\[
p \leq \frac{4C^2}{\pi R} \left( \frac{K_{IC}^2}{\sigma_f^2} \right)
\]  

(11)

Thus, the maximum pressure would be contained safely by the material with the largest value of

\[
M_2 = \left( \frac{K_{IC}^2}{\sigma_f^2} \right)
\]  

(12)

Lastly, we must ensure that the material with the minimum thickness would offer the maximum strength and hence, we must consider another material index, \( M_3 \), as

\[
M_3 = \sigma_f
\]  

(13)

Figure 2.4.4 depicts a typical chart of material properties (strength, \( \sigma_f \), vis-à-vis fracture toughness, \( K_{IC} \)) in a log-log scale with three black lines confirming to the material indices, \( M_1 \), \( M_2 \) and \( M_3 \). The safe region would be the one which is above all the three lines as indicated in the figure. It can be noticed that one of the most suitable material appears to be stainless steel [\( M_1 \sim 0.35 \text{ m}^{1/2}, M_3 \sim 300 \text{ MPa} \)], which is actually used for all critical pressure vessels. For example, some special grade of stainless steel is widely used for nuclear pressure vessels. A second candidate is low-alloy steel [\( M_1 \sim 0.20 \text{ m}^{1/2}, M_3 \sim 800 \text{ MPa} \)], which is a standard material used for manufacturing pressure vessels. A third candidate is copper [\( M_1 \sim 0.50 \text{ m}^{1/2}, M_3 \sim 200 \text{ MPa} \)], and hard drawn copper is often used to manufacture small boilers and pressure vessels. The pressure tanks of rockets and aluminum are often made of aluminum alloys which confirm to \( M_1 \sim 0.15 \text{ m}^{1/2} \) and \( M_3 \sim 200 \text{ MPa} \). Titanium alloys are another choice with \( M_1 \sim 0.13 \text{ m}^{1/2} \) and \( M_3 \sim 800 \text{ MPa} \), which are often used for light pressure vessels while they are relatively expensive.
Figure 2.4.4  Schematic chart of strength vis-à-vis fracture toughness of engineering materials

Exercise
1. Develop a suitable material index for the selection of material for Oars used for rowing.
2. Develop a suitable material index for the selection of material for Spatula used for cooking.

References