ME-662 CONVECTIVE HEAT AND MASS TRANSFER

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LECTURE-21 NATURE OF TURBULENT FLOWS

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1. Introduction
2. Characteristics of Turbulent Flows
3. Reynolds's Averaged (RANS) Equations

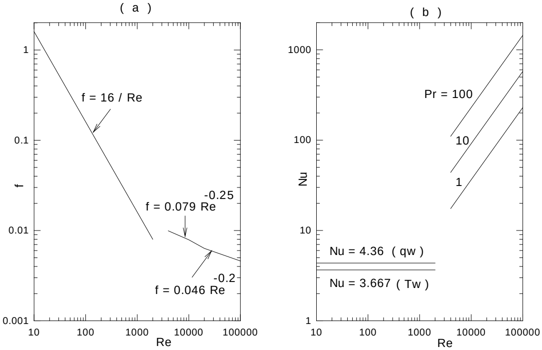
Introduction L21($\frac{1}{13}$)

1. The phenomenon of turbulence is associated with high fluid velocities characterised principally by the Reynolds number
2. For the same temperature difference, turbulent flows achieve much greater rate of heat transport than would be possible with laminar flows.
3. A turbulent flow is always a 3 dimensional and unsteady (time-dependent) phenomenon.
4. Experimentally, this was observed by Reynolds in his celebrated **pipe flow** experiment in which a laminar flow at low water velocities *was turned into a flow with irregular fluid motion* when the velocity was increased beyond a threshold value.
5. The main objective of the Theory is to develop capability for predicting f and Nu

Approaches to Understanding - L21($\frac{2}{13}$)

1. Formal Aspects
   1. How does laminar flow turn into turbulent motion ?
   2. How does turbulence, once generated, sustain itself ?
   3. What are the most convenient methods for mathematical representation of the complexities of turbulent flows ?
   4. What do the mathematical representations mean in terms of the physical mechanisms that govern the sustenance ?
2. Predictive Aspects
   1. How to make problem of predicting turbulent flow tractable ? That is, how to bring the problem of prediction **in line with that of predicting laminar flows** ?
   2. This requires generation of **universally valid** equations governing main variables characterising turbulence such that each of the convective, diffusive and dissipative effects are accurately captured in each flow situation.
   3. f and Nu characteristics will then differ due to boundary conditions as was the case with laminar flows

Special Features - 1 - L21($\frac{3}{13}$)



PIPE FLOW - f and Nu characteristics of turbulent flow differ greatly from laminar flow

Special Features - 2 - L21($\frac{4}{13}$)

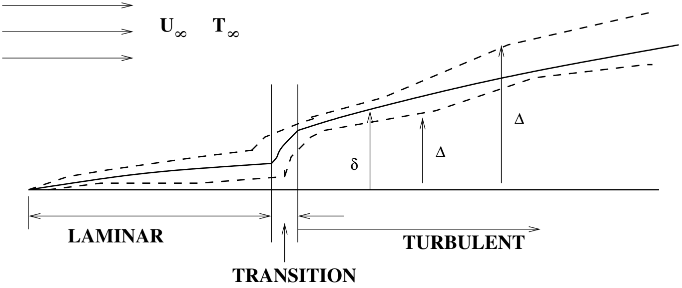
a) Velocity Profiles   b) Temperature Profiles

PIPE FLOW - Vel (Pitot tube) and Temp (Thermocouple) profiles of turbulent flow differ greatly from laminar flow.

$$\frac{u_{cl}}{\overline{u}} = 2 \ \ \text{(lam)} \quad \simeq 1.05 \quad \text{to} \quad 1.3 \ \ \text{(turb)}$$

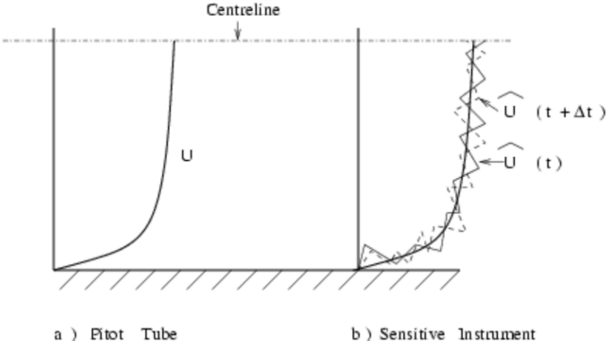
Similar ratios are observed for $(T_{cl} - T_w)/(T_b - T_w)$. Vel and Temp gradients at the wall in turb flow $>$ laminar flow

Special Features - 3 - L21($\frac{5}{13}$)



External Boundary Layers - $\delta \propto x^{0.5}$ (lam) $\delta \propto x^{0.8}$ (turb)
Similar dependence is observed for thermal boundary layer thicknes Δ (Pr). Rates of momentum and hence heat transport (normal to the main flow direction) in turbulent flows are greater than those found in laminar flows.

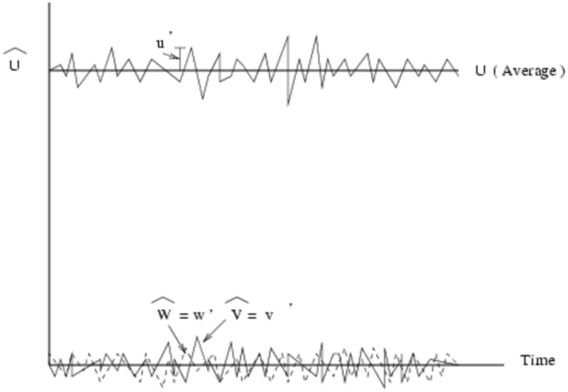
Special Features - 4 - L21($\frac{6}{13}$)



Severe non-linearities in a turbulent flow can be appreciated by comparing Pitot Tube measurements with hot-wire or LDA measurements .[1]

[1] It will be difficult to measure the irregular behaviour at all radii at the same time-instant. Rapid radial traverse is assumed.

Special Features - 5 - L21($\frac{7}{13}$)



Sensitive instrument held pointing in x (stream-wise), r and θ directions successively at *a fixed point at any radius* - Although the flow is fully developed, $\hat{v}_r$, $\hat{v}_\theta$ *are finite but mean values are zero* - Instantaneous value = mean + fluctuation ($\pm$)

Reynolds's Averaging Rules - L21($\frac{8}{13}$)

$$\begin{aligned}
\hat{\phi}\,(x, y, z, t) &= \phi\,(x, y, z) + \phi^{'}\,(x, y, z, t) \quad \text{(decomposition)} \\
\overline{\hat{\phi}} &= \frac{1}{t \to \infty} \int_0^t \hat{\phi}\, d\, t = \phi \;\; \to \;\; \frac{1}{t \to \infty} \int_0^t \phi^{'}\, dt = 0 \\
\overline{\hat{\phi}_1\, \hat{\phi}_2} &= \overline{(\phi_1 + \phi_1^{'})\,(\phi_2 + \phi_2^{'})} = \phi_1\, \phi_2 + \overline{\phi_1^{'}\, \phi_2^{'}}
\end{aligned}$$

where ϕ = u, v, w, p, T, ω_j. What should be the value of t_{max} in $t \to \infty$? This is determined from Auto-correlation coefficient to be introduced in next lecture

Transport Equations in $\hat{\Phi}$ variables will now be Time-averaged

RANS Equations - L21($\frac{9}{13}$)

WE assume uniform properties and neglect body forces

$$\frac{\partial \hat{u}_j}{\partial x_j} = 0 \quad \text{(Instantaneous)} \quad \frac{\partial u_j}{\partial x_j} = 0 \quad \text{(Time averaged)}$$

$$\rho_m \left[\frac{\partial \hat{u}_i}{\partial t} + \frac{\partial \hat{u}_j \, \hat{u}_i}{\partial x_j} \right] = - \frac{\partial \hat{p}}{\partial x_i} + \frac{\partial \hat{\tau}_{ji}}{\partial x_j} \quad \text{(Instantaneous)}$$

$$\rho_m \left[\frac{\partial u_i}{\partial t} + \frac{\partial u_j \, u_i}{\partial x_j} \right] = - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\tau_{ji} - \rho \, \overline{u_i' \, u_j'} \right) \quad \text{(Time averaged)}$$

$$\tau_{ij} = \mu \left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right] = \mu \, S_{ij} \quad \text{(Stokes's Law)}$$

Turbulent stresses ($-\rho_m \, \overline{u_i' \, u_j'}$) arise out of time averaging of non-linear convection terms $\rho_m \, \partial \hat{u}_j \, \hat{u}_i / \partial x_j$. Also, $\overline{\hat{\tau}_{ji}} = \tau_{ji}$

Energy Equation - L21($\frac{10}{13}$)

$$\rho_m\, c_{pm}\, \left[\frac{\partial \hat{T}}{\partial t} + \frac{\partial \hat{u}_j\, \hat{T}}{\partial x_j}\right] = -\frac{\partial \hat{q}_j}{\partial x_j} + \mu\, \hat{\Phi}_v \quad \text{(Instantaneous)}$$

$$\rho_m\, c_{pm}\, \left[\frac{\partial T}{\partial t} + \frac{\partial u_j\, T}{\partial x_j}\right] = -\frac{\partial}{\partial x_j}\left(q_j + \rho_m\, c_{pm}\, \overline{u_j'\, T'}\right) + \mu\, \Phi_v + \rho_m\, \epsilon \quad \text{(Time averaged)}$$

$$q_j = -\, k_m\, \frac{\partial T}{\partial x_j} \quad \text{(Fourier's Law)}$$

$$\mu\, \Phi_v = \tau_{ij}\, \frac{\partial u_i}{\partial x_j} \qquad \rho_m\, \epsilon = \overline{\tau_{ij}'\, \frac{\partial u_i'}{\partial x_j}} \quad \text{(Turb Energy Dissipation)}$$

Turbulent heat fluxes ($-\rho_m c_{pm}\, \overline{u_i'\, T'}$) arise out of time averaging of non-linear convection terms $\rho_m c_{pm}\, \partial \hat{u}_j\, \hat{T}/\partial x_j$. Also, $\overline{\hat{q}_j} = q_j$

Mass Transfer Eqn - L21($\frac{11}{13}$)

For each species ω_k of the mixture ($\rho_m = \sum \rho_k$)

$$\rho_m \left[\frac{\partial \hat{\omega}_k}{\partial t} + \frac{\partial \hat{u}_j \, \hat{\omega}_k}{\partial x_j} \right] = - \frac{\partial}{\partial x_j} \left(\hat{\dot{m}}_{k\,,\,j} \right) \quad \text{(Instantaneous)}$$

$$\rho_m \left[\frac{\partial \omega_k}{\partial t} + \frac{\partial u_j \, \omega_k}{\partial x_j} \right] = - \frac{\partial}{\partial x_j} \left(\dot{m}_{k\,,\,j} + \rho_m \, \overline{u_j' \, \omega_k'} \right) \quad \text{(Time averaged)}$$

$$\dot{m}_{k\,,\,j} = - D \, \frac{\partial \omega_k}{\partial x_j} \quad \text{(Fick's Law)}$$

Turbulent mass fluxes ($-\rho_m \, \overline{u_i' \, \omega_k'}$) arise out of time averaging of non-linear convection terms $\rho_m \, \partial \hat{u}_j \, \hat{\omega}_k / \partial x_j$. Also, $\overline{\hat{\dot{m}}}_{k\,,\,j} = \dot{m}_{k\,,\,j}$

New unknowns - L21($\frac{12}{13}$)

1. The six turbulent stresses ($-\rho_m \overline{u_i' u_j'}$) are new unknowns. When i = j, we have *normal stresses* ($-\rho_m \overline{u_i'^2}$). The one-point correlations ($\overline{u_i'^2}$) are always positive. When i $\neq$ j, correlations ($\overline{u_i' u_j'}$) can be positive or negative.
2. In the energy eqn, the three turbulent heat fluxes, ($-\rho_m c_{pm} \overline{u_i' T'}$) likewise, can be both positive or negative. The same for three turbulent mass fluxes ($-\rho_m \overline{u_i' \omega_k'}$)
3. Thus, we have a closure problem . In order to render the number of equations equal to number of unknowns, we need to model the turbulent stresses and fluxes. This is known as turbulence modeling .

Summary - L21($\frac{13}{13}$)

1. Reynolds's time-averaging leads to the closure problem. The task of turbulence modeling is to recover information lost due to time averaging .
2. This *closure problem* is similar to the problem encountered when equations for laminar flow were derived. Then, the problem was overcome by postulating Stokes's stress-rate-of-strain law , Fourier's heat conduction law and Fick's mass-diffusion law . These laws enabled us to recover information lost due to the *continuum assumption* in which averaging is carried out over motions of groups of particles.
3. In the next lecture, we shall consider some *formal aspects* of turbulence which aid turbulence modeling.