1. If \( f(z) \) is defined for \( |z| > R > 0 \), define when \( \lim_{z \to \infty} f(z) = \infty \). 2 marks

2. How will you treat a meromorphic function on a domain as a continuous function on that domain? Why? 2 marks

3. What is the fundamental point of difference, in studying spaces of functions with respect to convergence, between continuous functions and analytic functions? 2 marks

4. Show that the spherical metric on the extended complex plane is invariant under the inversion \( z \mapsto z^{-1} \). 2 marks

5. State and prove the Casorati-Weierstrass Theorem. 5 marks

6. State Marty’s Theorem. Explain why it is stronger than its holomorphic avatar viz., Montel’s theorem. 6 marks

7. Find \( f(0 < |z| < 10^{-2014}) \) if \( f(z) = e^{1/z} + e^{-1/z} \). 3 marks

8. Consider the family 
   \[ \mathcal{F} := \{ f_\epsilon(z) = \frac{z}{z + \epsilon}; \ 0 < \epsilon \leq 1 \} \].

   a) Compute the spherical derivatives of the functions of \( \mathcal{F} \).
   b) Check \( \mathcal{F} \) for normality at infinity.
   c) Does \( \mathcal{F} \) have a non-normal point? Justify your answer. 10 marks

9. State and prove Zalcman’s Lemma. 10 marks

10. State the Fundamental Normality Criteria (Fundamental Normality Tests) of Montel for meromorphic and for analytic functions on a domain. 2 marks

11. Show that the family of univalent (one-to-one) analytic functions on the open unit disc that never vanish is a normal family. 6 marks
12. Show that a family of analytic functions $f$ on a domain satisfying
\[ |f'| \leq |f|^3 \]
is normal. \hspace{1cm} 3 marks

13. State and prove Schottky’s Theorem. Explain where each of the hypotheses of the theorem have been used in the proof. \hspace{1cm} 7 marks