NPTEL Online Certification

Candidate Name :
Roll Number :
Date/ Shift : 26th March 2017/AN
Duration : 3 hours
Total Marks : 100

ATTENTION CANDIDATES!
All question papers must be tied to the answer sheets. This is to ensure all the answers written are evaluated.

Number of pages in the question paper : 05
Number of questions in the question paper : 48

Modeling Transport Phenomena of Micro-particles

Note: Follow the notations used in the lectures. Symbols have their usual meanings. Variable typed in bold represent vector.
Use the following electrokinetic parameters: $\phi_0 = RT/F = k_BT/e = 0.02586$ V; permittivity, $\varepsilon_e = 695.39 \times 10^{-12}$ C/Vm; elementary charge, $e = 1.602 \times 10^{-19}$C; dynamic viscosity, $\mu = 10^{-3}$ Pa s; Faraday constant, $F = 96500$ C/mol; diffusivity of Na^+ ion, $D_{Na^+} = 1.33 \times 10^{-9}$ m^2/s and diffusivity of Cl^- ion, $D_{Cl^-} = 2.03 \times 10^{-9}$ m^2/s. Also, 1μm=10^{-6}m and 1 nm= 10^{-9}m.

SECTION-1 [20 x 1 = 20 marks]

Questions 1 to 12: Fill the blanks with appropriate answer

1. In case of Newtonian fluids, the fluid stress and strain obey the relation ....

2. Dimension of permeability of a porous medium is ....

3. If the total number of dimensional parameters is 6 of which 3 are independent, then the number of dimensionless groups is ....

4. Let $g(x)$ be a function defined by $g(x) = 1 + x + x^2,\ x \in \mathbb{R}$. Then the value of the integral $\int_{-\infty}^{\infty} g(x)\delta(x-5)dx$ is equal to...........

5. If the streamlines are given by $\psi = xy$ then the resultant velocity at $(1, 1)$ is ....

6. In case of a rigid spherical object swimming at very small Reynolds numbers, in the absence of any external forces, the normal velocity condition on the boundary is ....

7. If a fluid with velocity $\mathbf{u}$ is in contact with an impermeable boundary $x = 0$, having a velocity $\mathbf{v} = (3, -2)$, then the dynamic boundary condition is given by ....

8. In case of Stokes flow past a sphere, if one third of the drag force is due to the pressure forces, then two third is due to ....

9. Consider a uni-directional flow between two parallel plates under non-zero pressure gradient. If both the plates are at rest, then the velocity is always maximum at ....

10. Consider the Brinkman equation governing flow inside a porous medium. If the viscous forces are negligible, then, the Brinkman equation reduces to ....

11. Coulomb's law provides the _____ force between two point charges.

12. The electric force on a charge q under an electric filed $\mathbf{E}$ is _____.

13. In Cartesian coordinates if $\mathbf{E} = (A, 0, 0)$, where A is a constant, then the electric potential ϕ is _____.

14. The electric permittivity of a medium is _____ than the electric permittivity of vacuum.

15. If $\mathbf{u}$ is the velocity field of an incompressible fluid then $\nabla.\mathbf{u} =$ _____.

16. In steady-state, if $\mathbf{N}_i$ is the molar flux of the i^{th} ionic species then $\nabla.\mathbf{N}_i =$ _____.

17. If ions are obeying the Boltzmann distribution then the convective transport of ions are _____.

18. If n_1 and n_2 be the molar concentration of two ionic species with valence z_1 and z_2, respectively, then the charge density is __________.

19. If $\mathbf{u}$ is the electrophoretic velocity of a particle under an electric field $\mathbf{E}$ then mobility is ________.

20. Debye-Hückel approximation is valid for ________ surface charge density.

SECTION-II **[20 x 2 = 40 marks]**

1. Consider the mass conservation equation in (r, θ, z) cylindrical coordinates,(where the flow is axi-symmetric) given by

$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{\partial v_z}{\partial z} = 0.$$

If the velocity vector is only along the radial direction, then show that the radial velocity component v_r is of the form $\frac{1}{r}f(z)$ for some arbitrary function f that depends only on z.

2. Consider the mass conservation equation given in (r, θ, z) cylindrical coordinates,(where the flow is axi-symmetric). The corresponding velocity components are given in as $u_r = \frac{r^2 z}{3}, u_z = -\frac{rz^2}{2}$. Then, compute the corresponding stream function.

Answer questions 3 and 4 based on the following information:

Consider a sphere of radius 10 cm which is made of limestone. The volume occupied by the void is 40 cm^3. The mean grain diameter of the limestone particles is 1 mm. Then,

3. Find the porosity of the sphere.

4. Find the permeability of the sphere using Carman-Kozney relation, $K = \frac{D_p^2 \phi^3}{180(1-\phi)^2}$.

5. Consider the steady state heat conduction interior to a rigid impermeable sphere of radius a (with azimuthal axi-symmetry). We are seeking a solution of the form

$$T(r,\theta) = \sum_{n=0}^{\infty} \Big(a_n r^n + \frac{b_n}{r^{n+1}} \Big) P_n(\cos\theta).$$

If a heat source is located at the origin given by $T_S(r,\theta) = \frac{50}{r^2}\cos\theta$, then determine the coefficients b_n. (Hint: $P_1(x) = x$).

6. Consider the free-space Green's function of unit strength, $G(\mathbf{x}, \mathbf{x}_0) = \ln r$, where $\mathbf{x} = (x, y), \mathbf{x}_0 = (1, 2)$ and $r = |\mathbf{x} - \mathbf{x}_0|$, then compute $\frac{\partial G}{\partial x}$.

Answer questions 7 and 8 based on the following information:

Consider Stokes equations governing viscous incompressible flow at very low Reynolds number (two-dimensional), where $\mathbf{v}$ represents the velocity, p represents the corresponding pressure, ψ denote the stream function. Then,

7. Decide whether following quantities are harmonic or bi-harmonic, (i). $\mathbf{v}$, (ii). $\nabla \times \mathbf{v}$.

8. Decide whether following quantities are harmonic or bi-harmonic, (i). p, (ii). ψ.

9. If the velocity vector corresponding to a three-dimensional viscous incompressible flow is given by $\mathbf{u} = (yz - x, y + z, 0)$, then compute the tangential stress components $(\tau_{xy}, \tau_{yz}, \tau_{zx})$.

10. Show that $u = 2cxy,\ v = c(a^2 + x^2 - y^2)$ are the velocity components of a possible fluid motion. Determine the corresponding stream function (c is a constant)

11. Consider a point charge $q = 0.005$ C is placed within the center of volume enclosed by a sphere. Find the flux of the electric field through the surface of the sphere.

Answer questions 12 - 16 based on the following statement:

A planar surface of ζ-potential as 2.586×10^{-2} V is in contact with a NaCl solution with inverse of Debye length $\kappa = 1.073 \times 10^{8}\ \mathrm{m}^{-1}$, then:

12. Find the surface charge density.

13. Use the Debye-Hückel approximation to find the electric potential at a point 0.5 nm from the planar surface

14. Use Boltzmann distribution to obtain the ionic concentration of Na^+ at the point 0.5 nm from the planar surface.

15. Calculate the ionic concentration of Cl^- at the point 0.5 nm form the planar surface when the ions obey the Boltzmann distribution.

16. Find the charge density at the point 0.5nm from the planar surface.

Answer questions 17 and 18 for the problem:

Consider a combined electroosmosis and pressure driven flow of 1 mol/m^3 NaCl electrolyte solution in a slit microchannel of half channel height $h = 50$ nm under the influence of an external electric field of 10^4 V/m acting along the axis of the channel with a constant pressure gradient (dp/dx) along the length of the channel. Consider the mid-plane of the microchannel as the x-axis. The wall ζ-potential is 0.1 V.

17. Calculate the Debye length.

18. Find the axial velocity,u at $y = 5$ nm when the constant axial pressure gradient is $\frac{dp}{dx} = -0.36$ Pa/m.

19. A spherical particle of radius a=10 nm with surface potential $\zeta = 0.02586$ V is suspended in a non-conducting liquid i.e., inverse Debye length κ is such that $\kappa a << 1$. Find the electrophoretic velocity due to an externally imposed electric field $E_0 = 100$ V/m.

20. Consider the boundary value problem (BVP)

$$(1+x^2)\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 2, \quad y(0) = 0, \quad y(1) = 1/2$$

Derive the discretized form of the BVP by using a central difference scheme when the step size $h = 1/3$.

SECTION-III [8 X 5 = 40 marks]

1. Consider a flow through porous medium that is governed by the extended Brinkman equation

$$\rho\Big(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla\mathbf{u}\Big) = -\nabla p + \mu\nabla^2\mathbf{u} - \frac{\mu\mathbf{u}}{K},$$

where $\mathbf{u} = (u, v)$ represent the velocity. In order to non-dimensionalize, the following dimensionless variables are used:

$$x' = \frac{x}{L}, y' = \frac{y}{L}, t' = \frac{\mu t}{\rho L^2}, u' = \frac{u}{U}, v' = \frac{v}{U}, p' = \frac{p}{\rho U^2}, Da = \frac{K}{L^2}.$$

If x-component of the above governing equation is given by

$$\alpha\frac{\partial u'}{\partial t'} + \beta\left(u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'}\right) = -\frac{1}{\Lambda}\frac{\partial p'}{\partial x'} + \Big(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2}\Big) - \gamma u',$$

then find the parameters α, β, γ and Λ.

2. Consider the governing equation for a viscous incompressible flow inside a porous medium

$$-\nabla p + \mu \nabla^2 \mathbf{u} - \mu \mathbf{K}^{-1}\mathbf{u} = 0,$$

where $\mathbf{K}$ is the permeability of the medium that is given by the matrix $\mathbf{K} = \begin{pmatrix} K_1 & 0 \\ 0 & K_2 \end{pmatrix}$, where K_1 and K_2 are constants. Then introduce stream function in two-dimensions and hence derive the corresponding equation satisfied by the stream function.

Answer questions 3 and 4 based on the following information:

Consider a fluid motion inside a porous circular cylinder. Given that the magnitude of the velocity is V and the radius of the cylinder is L, the pressure: P, density of the fluid: ρ and the permeability of the porous medium: K.

3. Find the number of non-dimensional groups involved using Buckingham-π theorem. Further, find one of the non-dimensional groups which is of the form $f(P, L, V, \rho) = 0$.

4. Find another non-dimensional group which is of the form $f(L, V, \rho, K) = 0$.

5. Write the equation for the electric field which relates the charge density ρ_e at any point in an electrolyte solution of permittivity ε_e. If the ions obey the Boltzmann distribution then derive the Poisson-Boltzmann equation for the electric field.

6. Find an expression for the electroosmotic flow velocity in a slit micro-channel with surface potential of both the walls as ζ and the electric field E_0 applied parallel to the channel walls.

7. Solve the following partial differential equation for the first time step through an implicit scheme.

$$u_t = u_{xx}$$
$$u(x, 0) = \sin \pi x, 0 < x < 1$$
$$u(0, t) = u(1, t) = 0, t > 0$$

with $\delta x = 1/3$ and $\delta t = 1/36$.

8. Derive the fluid transport equations in non-dimensional form for the electrophoresis of a charged spherical particle of radius a with surface potential ζ in an electrolyte medium of permittivity ε_e and charge density ρ_e.

END OF THE QUESTION PAPER