NPTEL Online Certification

Candidate Name :
Roll Number :
Date/ Shift : 26th March 2017/ FN
Duration : 3 hours
Total Marks : 100

ATTENTION CANDIDATES!
All question papers must be tied to the answer sheets. This is to ensure all the answers written are evaluated.

Number of pages in the question paper : 06
Number of questions in the question paper : 48

Modeling Transport Phenomena of Micro-particles

Note: Follow the notations used in the lectures. Symbols have their usual meanings. Variable typed in bold represent vector.
Use the following electrokinetic parameters: $\phi_0 = RT/F = k_BT/e = 0.02586$ V; permittivity, $\varepsilon_e = 695.39 \times 10^{-12}$ C/Vm; elementary charge, $e = 1.602 \times 10^{-19}$C; dynamic viscosity, $\mu = 10^{-3}$ Pa s; Faraday constant, $F = 96500$ C/mol; diffusivity of Na^+ ion, $D_{Na^+} = 1.33 \times 10^{-9}$ m^2/s and diffusivity of Cl^- ion, $D_{Cl^-} = 2.03 \times 10^{-9}$ m^2/s. Also, 1μm=10^{-6}m and 1 nm= 10^{-9}m.

[20 x 1 = 20 marks]

SECTION-I

Questions 1 to 12: Fill the blanks with appropriate answer

1. The no-slip boundary condition is applicable only inside a thin layer above a surface where the viscous forces are effective. This layer is referred as ....

2. Dimension of dynamic viscosity of a fluid is ....

3. If the total number of dimensional parameters is n and r is the minimum number of independent dimensions, then the number of dimensionless groups is ....

4. The condition for a free-surface $F(x, y, z, t) = 0$ to be a bounding surface is ....

5. If the streamlines are represented by $\psi = x^2 + y^2$, then the resultant velocity at a point $(2, 2)$ is..............

6. A rigid spherical object can swim at very small Reynolds numbers, in the absence of any external forces due to ....

7. Consider the Brinkman equation governing flow inside a porous medium. If the permeability approaches a very large value, then the Brinkman equation reduces to ....

8. The points where the velocity becomes zero are known as stagnation points. If the streamlines are represented by $\psi = x^2 - y^2 - x - y$, then the stagnation point is ....

9. Consider a uni-directional flow between two parallel plates under zero pressure gradient. If the lower plate is at rest and the upper plate is moving, the velocity is always maximum at ....

10. If a fluid with velocity $\mathbf{u}$ is in contact with an immiscible fluid, the kinematic boundary condition in terms of $\mathbf{u}$ is given by ....

11. If the electric field $\mathbf{E} = (E_0, 0, 0)$ and ϕ is the electric potential then $\nabla^2\phi =$ ....

12. If $\frac{dy}{dx}|_i = \frac{y_{i+1}-y_{i-1}}{2h}$ then the order of accuracy is ....

Questions 13 to 17: Complete the following statements appropriately by choosing the word either directly or inversely

13. The Debye length is _____ proportional to the ionic concentration.

14. Electroosmotic flow is _____ proportional to the surface potential.

15. Surface charge density is _____ proportional to the surface potential.

16. Current density is _____ proportional to the electric field.

17. Electrophoretic velocity of a particle is _____ proportional to its surface charge density.

Questions 18 to 20: State whether the following statements are TRUE or FALSE

18. The Helmholtz-Smoluchowski velocity is independent of the channel height.

19. Dielectric permittivity of a medium is higher than that of the air.

20. The Thomas algorithm for solving a system of equations is an iterative method.

SECTION-II [20 X 2 = 40 marks]

1. Consider a flow inside a wavy walled channel bounded by two impermeable plates located at $y = 1 + \cos(2\pi x)$ and $y = -(1 + \cos(2\pi x))$. If the upper plate is moving with a velocity $\mathbf{u} = (V, 0)$, then obtain the no-slip boundary condition at the upper plate.

2. Let us consider the following simplified Navier-Stokes equations in case of a specific geometry

$$0 = -\frac{\partial p}{\partial r} + \rho\omega^2 r,$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial \theta}, \quad 0 = -\frac{\partial p}{\partial z} - \rho g.$$

Integrate these to obtain the corresponding pressure $p(r,z)$ upto a constant. If this pressure satisfies $p(r,z) = 0$ at $(r,z) = (0,h)$ then eliminate the constant and write down the final expression for the pressure.

3. Let a solid sphere of radius a be held fixed in a uniform stream U flowing steadily in the positive x-direction. Let the fluid be incompressible viscous with viscosity μ. Compute the corresponding Stokes drag when (in SI units) $a = 2$ m, $\mu = 2$ Ns/m^2, $U = 7$ m/s.

4. Consider the steady state heat conduction exterior to a rigid impermeable sphere of radius a (with azimuthal axi-symmetry). We are seeking a solution of the form

$$T(r,\theta) = \sum_{n=0}^{\infty} \left(a_n r^n + \frac{b_n}{r^{n+1}} \right) P_n(\cos\theta).$$

If the far field temperature is given by $T(r,\theta) \sim T_\infty + T_\infty r^2(3\cos^2\theta - 1)$, then determine the coefficients a_n, where T_∞ is constant. (Hint: $P_2(x) = \frac{1}{2}(3x^2 - 1)$)

5. Consider the Brinkman equation governing fluid flow inside a porous medium

$$0 = -\nabla p + \mu \nabla^2 \mathbf{u} - \frac{\mu \mathbf{u}}{K}, \quad \nabla \cdot \mathbf{u} = 0$$

where μ denote viscosity and K denote permeability, both being constants. If the stream function is introduced in case of two-dimensions, obtain the corresponding equation satisfied by the stream function.

Answer questions 6 and 7 based on the following information:

Consider a unidirectional flow (along x-direction) through a porous medium bounded by two parallel plates located at $y = h$ and $y = 0$ under zero pressure gradient. Assume that the flow is governed by Brinkman equation. The lower plate is stationary and the upper plate is moving with a velocity U. The effective viscosity and the dynamic viscosity are equal.

6. Find the corresponding velocity.

7. Obtain the maximum velocity and the location where this maximum occurs.

Answer questions 8 and 9 based on the following information

Consider a sphere of radius 3 cm which is made of gravel. The volume occupied by the void is 60 cm^3. The mean grain diameter of the gravel is 2 mm. Then,

8. Find the porosity of the sphere.

9. Find the permeability of the sphere using Carman-Kozney relation, $K = \frac{D_p^2 \phi^3}{180(1-\phi)^2}$.

10. If the velocity field of a fluid flow is given by $\mathbf{u} = -x\vec{i} + (y + t)\vec{j}$, then compute the corresponding stream function at $t = 2$.

11. Calculate the Debye length for 1 mol/m3 NaCl solution.

Answer questions 12 - 14 for the following problem:

Consider the electroosmotic flow of 10 mol/m^3 NaCl solution in a slit microchannel with half height, h=25 nm and wall zeta-potential 0.05 V under the influence of an external electric field 200 V/m acting along the axis of the channel (x-axis). Consider the mid-plane of the microchannel as the x-axis. Then,

12. Find the electric potential, ψ at $y = 2$ nm.

13. Find the velocity along the x-axis i.e., u at $y = 2$ nm.

14. Calculate the volumetric flow rate at any cross-section.

Answer questions 15 and 16 for the following problem:

Consider the boundary value problem

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 8y = 0,\ y(0) = 1,\ y(1) = 0.$$

15. Obtain the discretized form of the BVP at any grid point x_i by using the central difference scheme.

16. Derive the discretized linear system of equations of the BVP by using the boundary conditions when the step size is $h = 0.25$.

17. Consider the electrophoresis of a charge spherical particle of radius 0.1 μm with ζ-potential 0.05127 V in an aqueous solution with Debye length λ such that $a/\lambda << 1$. Find the electrophoretic velocity due to an externally imposed electric field E_0=1000 V/m.

18. Calculate the electrophoretic velocity of a charged spherical particle of radius $a = 100$ nm with surface potential ζ = 0.0456 V suspended in an aqueous NaCl solution with Debye length λ such that $a/\lambda >> 1$ when an electric field $E_0 = 1000$ V/m is applied.

19. Consider the flow of a viscous fluid of viscosity $\mu = 10^{-3}$ Pa s in a parallel plate microchannel of height $h = 50$ nm. The lower wall ($y = 0$) of the channel is hydrophobic with slip length β=5 nm. The flow is driven by a constant pressure gradient, $\frac{dp}{dx} = -0.36$ Pa/m along the length of the channel. Find an expression for the fluid velocity.

20. Solve

$$u_{xx} = 32u_t,$$
$$u(x,0) = 0,\ 0 < x < 1,$$
$$u(0,t) = 0,\ u(1,t) = t,\ t > 0,$$

by an explicit scheme with $r = \delta t/\delta x^2 = 1/2$, $\delta x = 1/2$. Compute the solution for the first two time steps.

SECTION-III **[8 x 5 = 40 marks]**

Answer questions 1 and 2 based on the following information:

Consider flow past a porous object with a far-field (ambient) velocity having a magnitude V. Correspondingly, we have the length of the object: L, pressure: P, viscosity of the fluid: μ and permeability of the porous medium: K.

1. Find the number of non-dimensional groups involved using Buckingham-π theorem. Further, find one of the non-dimensional groups choosing the form $f(P, L, V, \mu) = 0$.

2. Find another non-dimensional group choosing the form $f(L, V, \mu, K) = 0$.

3. Consider a flow through porous medium that is governed by the extended Brinkman equation

$$\rho\Big(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla\mathbf{u}\Big) = -\nabla p + \mu\nabla^2\mathbf{u} - \frac{\mu\mathbf{u}}{K},$$

where $\mathbf{u} = (u, v)$ represent the velocity. In order to non-dimensionalize, the following dimensionless variables are used:

$$x' = \frac{x}{L}, y' = \frac{y}{L}, t' = \frac{Ut}{L}, u' = \frac{u}{U}, v' = \frac{v}{U}, p' = \frac{p}{\mu U/L}, Da = \frac{K}{L^2}.$$

If x-component of the above governing equation is given by

$$\alpha\Big(\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'}\Big) = -\frac{1}{\Lambda}\frac{\partial p'}{\partial x'} + \beta\Big(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2}\Big) - \gamma u',$$

then find the parameters α, β, γ and Λ.

4. Consider a channel filled with anisotropic porous material. Let the flow inside the porous medium be governed by the equation of the form

$$-\nabla p + \mu\nabla^2\mathbf{u} - \mu\mathbf{K}^{-1}\mathbf{u} = 0,$$

where $\mathbf{K}$ is the permeability of the medium that is given by the matrix $\mathbf{K} = \begin{pmatrix} K_1 & K_2 \\ K_2 & \lambda K_1 \end{pmatrix}$, where λ is a constant and $\lambda {K_1}^2 > {K_2}^2$, then derive the x- and y-components of the momentum equation.

5. Write the Nernst-Planck equation for ion transport.Derive the Boltzmann distribution of ions from this equation.

6. Describe the governing equations for the fully developed electroosmotic flow of an aqueous solution between two infinite parallel plates of surface potential $\zeta(>\phi_0)$. The electric field E_0 is applied along the length of the channel and the transport of ions are governed by the Nernst-Planck equations. Derive the non-dimensional form of the governing equations by stating clearly the scaling parameters and the boundary conditions.

7. Describe an iterative method to solve the following transport equation by the Crank-Nicolson scheme. Derive the system of linear algebraic equations which need to be solved at each iteration level when the step size $\delta x = 1/4$ and $\delta t = 1/32$.

$$u_t + uu_x = \nu u_{xx}$$

where ν is a constant parameter. Numerical solutions are not required.

8. Consider the electrophoresis of a spherical particle of radius a with surface potential ζ in an electrolyte saturated hydrogel medium with permeability k_p. Obtain the non-dimensional form of the equation describing the fluid flow. The volumetric charge density at any point of the electrolyte medium can be assumed as ρ_e.

END OF THE QUESTION PAPER