Module 3: Fundamental groups & its basic properties
Lecture 9: Functiorial properties of the fundamental group

Exercises:

1. Show that the sphere $S^2$ retracts onto one of its longitudes. If $X$ is the space obtained from $S^2$ by taking its union with a diameter, there is a surjective group homomorphism $\pi_1(X) \longrightarrow \mathbb{Z}$.

2. Prove that $A$ is a retract of $X$ if and only if every space $Y$, every continuous map $f : A \longrightarrow Y$ has a continuous extension $\tilde{f} : X \longrightarrow Y$.

3. Show that the fundamental group respects arbitrary products.

4. Construct a retraction from $\{(x, y) : x \text{ or } y \text{ is an integer}\}$ onto the boundary of $I^2$.

5. Show that every homeomorphism of $E^2$ onto itself must map the boundary to the boundary.

6. Given that there exists a functor $T$ from the category $\textbf{Top}$ to the category $\textbf{AbGr}$ such that $T(X)$ is the trivial group for every convex subset $X$ of a Euclidean space and $T(S^n)$ is a non-trivial group, prove that $S^m$ is not a retract of the closed unit ball in $\mathbb{R}^{n+1}$. 