Exercises:

1. Explicitly construct a homotopy between the loop \( \gamma(t) = (\cos 2\pi t, \sin 2\pi t, 0) \) on the sphere \( S^2 \) and the constant loop based at \((1, 0, 0)\). Note that an explicit formula is being demanded here.

2. Show that a loop in \( X \) based at a point \( x_0 \in X \) may be regarded as a continuous map \( f : S^1 \to X \) such that \( f(1) = x_0 \). Show that if \( f \) is homotopic to the constant loop \( \varepsilon_{x_0} \) then \( f \) extends as a continuous map from the closed unit disc to \( X \).

3. Show that if \( \gamma \) is a path starting at \( x_0 \) and \( \gamma^{-1} \) is the inverse path then prove by imitating the proof of the reparametrization theorem (that is by taking convex combination of two functions) that \( \gamma \ast \gamma^{-1} \) is homotopic to the constant loop \( \varepsilon_{x_0} \).

4. Prove theorems (7.2) and theorem (7.6) using Tietze's extension theorem.

5. Suppose \( \phi : [0, 1] \to [0, 1] \) is a continuous function such that \( \phi(0) = \phi(1) = 0 \) and \( \gamma \) is a closed loop in \( X \) based at \( x_0 \in X \). Is it true that \( \gamma \circ \phi \) is homotopic to the constant loop \( \varepsilon_{x_0} \)?

6. Show that the group isomorphism in theorem (7.8) is natural namely, if \( f : X \to Y \) is continuous and \( x_1, x_2 \in X \) then

\[
\pi_1(X, x_1) \xrightarrow{f_*} \pi_1(Y, y_1)
\]

\[
h_{[\sigma]} \circ f_* = h_{[\sigma]} \circ f''_*
\]

where, \( y_1 = f(x_1) \), \( y_2 = f(x_2) \) and \( \sigma \) is a path joining \( x_1 \) and \( x_2 \). The maps \( f'_* \) and \( f''_* \) are the maps induced by \( f \) on the fundamental groups. This information is better described by saying that the following diagram commutes: