Exercises:

1. Prove that the map $\eta$ in the five lemma is surjective.

2. Show that the map (38.2) is indeed an isomorphism. To prove that it is surjective use the decompositions $S^U(X) = S(X - U) + S(\text{int} \ A)$ and $S^V(A) = S(A - U) + S(\text{int} \ A)$.

3. Prove the Barrett-Whitehead lemma.

4. Calculate the local homology groups $H_2(X, X - \{p\})$ in the following cases:

   (i) The space $X$ is the cylinder $S^1 \times [0, 1]$ and $p$ a point on its boundary.

   (ii) The space $X$ is the Möbius band and $p$ is a point on its boundary.

   Deduce that the cylinder and the Möbius band are not homeomorphic.

5. A topological manifold is a Hausdorff space in which each point has a neighborhood homeomorphic to an open ball in $\mathbb{R}^n$. Show that if $p$ is a point on a topological manifold $M$, 

$$H_n(M, M - \{p\}) \cong \mathbb{Z}.$$