Module 6: Basic Homology Theory  
Lecture 34: Small Simplices

Exercises:

1. Show that the map defined by (34.1) is the restriction to $\Delta_p$ of an affine map. Note: An affine map is the composition of a linear map and a translation.

2. Suppose $T: \mathbb{R}^{n+1} \rightarrow \mathbb{R}^{m+1}$ is an affine map such that $T(\Delta_n) \subset \Delta_m$, then $T_\#$ maps the subgroup $A_p(\Delta_n)$ into $A_p(\Delta_m)$ and is a chain map from the complex $\{A_p(\Delta_n)\}$ to $\{A_p(\Delta_m)\}$. Further prove the following:

(i) If $b \in \Delta_n$ and $\sigma \in A_p(\Delta_n)$ then $T_\#(K_b, \sigma) = K_{Tb}(T_\#\sigma)$

(ii) If $b$ is the barycenter of $\sigma$ then $b$ is the barycenter of $T_\#\sigma$.

What happens if we consider a degenerate two simplex where the points $v_1, v_2, v_3$ are not affinely independent? Discuss the case of the two simplex $[v_1, v_2, v_2]$.

3. Examine what happens if the term referred to as junk in equation (34.7) is retained.

4. Complete the details of the proof of theorem (34.4).

5. Show that $B^k$ is chain homotopic to the identity map. What is the chain homotopy?

6. Suppose that the maps $g$ and $h$ in the exact sequence

$$A \xrightarrow{g} B \xrightarrow{g} C \xrightarrow{h} D \xrightarrow{h} E$$

are replaced by the composites

$$\tilde{g}: B \xrightarrow{g} C' \xrightarrow{\lambda} X, \quad \tilde{h}: X \xrightarrow{\lambda^{-1}} C' \xrightarrow{h} D$$

the result is again an exact sequence.

7. Fill in the details in the proof of theorem (34.8). See exercise 6 of lecture 29.