Module 4: Theory of Covering Spaces
Lecture 18: The lifting criterion

Exercises:

1. For the map $S$ in example (18.3) show that $S^*$ is the map $\mathbb{Z} \to \mathbb{Z}$ given by
   $x \mapsto 2x$.

2. Suppose $G$ is a path connected topological group with unit element $e$ and
   $p : \tilde{G} \to G$ is a covering map. For any choice of $\tilde{e} \in p^{-1}(e)$ show that there is a
   group operation on $\tilde{G}$ with unit element $\tilde{e}$ that makes $\tilde{G}$ into a topological group and $p$
   is a continuous group homomorphism.

3. Show that if $\Omega$ is an open subset of $\mathbb{C} - \{0\}$ on which a continuous branch of the
   logarithm exists then this branch is automatically holomorphic. Likewise show that the
   continuous branch of $\sqrt{z(2z - 1)}$ on $\mathbb{C} - [0, 1/2]$ obtained in the lecture is
   holomorphic.

4. Use the fact that $S^{n-1}$ is not a retract of $S^n$ to prove that $\mathbb{R}P^{n-1}$ is not a retract of $\mathbb{R}P^n$.

5. Show that any continuous map $S^n \to S^1$ is homotopic to the constant map if $n \geq 2$.

   What about maps from the projective spaces $\mathbb{R}P^n \to S^1$ ($n \geq 2$)?