Module 3: Fundamental groups & its basic properties
Lecture 11: Homotopies of maps. Deformation retracts

Exercises:

1. Check that the map \( \phi \) constructed in the proof of theorem 11.3 is continuous and is indeed a homotopy. Work out the proof of theorem 11.5.

2. Show that the boundary \( \partial M \) of the Möbius band \( M \) is not a deformation retract of \( M \) by taking a base point \( x_0 \) on the boundary and computing explicitly the group homomorphism
   \[
   i_* : \pi_1(\partial M, x_0) \longrightarrow \pi_1(M, x_0).
   \]

3. Show that the boundary of the Möbius band is not even a retract of the Möbius band.

4. Fill in the details on the continuity of the map \( G \) in the example preceding corollary 11.9.

5. Show that the space \( \mathbb{R}^3 - \{ (x, y, z) | x^2 + y^2 = 1, z = 0 \} \) deformation retracts to a sphere with a diameter attached to it.

6. Let \( X \) be the union of \( S^2 \) and one of its diameters \( D \), \( Y = S^2 \vee S^1 \) and \( Z \) be the union of \( S^2 \) with a punctured half disc contained in a half with edge along \( D \). Show that \( X \) and \( Y \) are both deformation retracts of \( Z \) and so they have the same homotopy type.