Expected revenue of the first price auction:

Nash equilibrium

\[ b_1 = \frac{1}{2} v_1 \]
\[ b_2 = \frac{1}{2} v_2 \]

Since the player with the maximum bid wins the auction and pays an amount equal to the bid, revenue: \( \max \{ b_1, b_2 \} \).
\[ \text{revenue} = \max \left\{ \frac{1}{2} b_1, \frac{1}{2} b_2 \right\} \]
\[ = \max \left\{ \frac{1}{2} v_1, \frac{1}{2} v_2 \right\} \]
\[ = \frac{1}{2} \max \left\{ v_1, v_2 \right\} \]

\[ V_1, V_2 \text{ are independent valuations uniformly distributed in } [0,1]. \]

\[ \begin{array}{c}
0 & 1 \\
\hline
v & v + dv \\
\hline
0 & 1 \\
\end{array} \]
What is the probability that \( \max\{V_1, V_2\} \) lies in the infinitesimal interval \( [V, V + dv] \)?

**Scenario 1:** \( V_1 \) is the maximum

- \( V_1 \) lies in \( [V, V + dv] \)
- \( V_2 \) lies in \( [0, V] \).

\[
Pr = Pr(V_1 \in [V, V + dv]) \times Pr(V_2 \in [0, V]) \\
= dv \times V = V dv
\]
Scenario 2: $V_2$ is maximum

$V_2$ lies in $[v, v + dv]$

$V_1$ lies in $[0, v]$

$$P_r = Pr(V_1 \in [0, v])$$

$$\times Pr(V_2 \in [v, v + dv])$$

$$= v \times dv = v dv$$

Probability that $\max\{V_1, V_2\}$

lies in $[V, V + dv]$

$$= V dv + v dv$$

$$= 2v dv.$$
Average revenue corresponding to $\max\{v_1, v_2\} \in [v, v + dv]$

\[
= \frac{1}{2} v \times 2v \, dv \\
= v^2 \, dv
\]

Net average revenue to the auctioneer

\[
= \int_0^1 v^2 \, dv \\
= \left. \frac{v^3}{3} \right|_0^1 = \frac{1}{3}.
\]
The expected revenue of the auctioneer = $\frac{1}{3}$.  

Sealed bid first-price auction.