Auctions: Auctions as Bayesian Games.

Random variable which is uniformly distributed in \([0, 1]\).
Probability density function of the uniform random variable in [0, 1] is defined as

\[ f_X(x) = \begin{cases} 
1 & \text{if } 0 \leq x < 1 \\
0 & \text{otherwise.} 
\end{cases} \]
The probability that this random variable takes a value in the interval \([a, b]\) is given as:

\[
\int_{a}^{b} f_X(x) \, dx
\]

Example: What is the probability that the uniform random variable takes a value between \(\left[ \frac{1}{4}, \frac{1}{2} \right]\)?

\[
\frac{1}{b} \int_{a}^{b} f_X(x) \, dx = \int_{\frac{1}{4}}^{\frac{1}{2}} 1 \, dx = \left. x \right|_{a}^{b} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}
\]
The probability that the uniform random variable in $[0,1]$ takes a value between $[\frac{1}{4}, \frac{1}{2}]$ is $\frac{1}{4}$.

What is the probability that it lies in the interval $[0,\frac{1}{2}]$?

\[
\frac{1}{2} = \int_{0}^{\frac{1}{2}} f_X(x) \, dx = \int_{0}^{\frac{1}{2}} 1 \, dx = \frac{1}{2}.
\]
Consider any interval \([a, b]\) which lies in \([0, 1]\). The probability that the random variable takes any value in \([a, b]\) is,

\[
\int_a^b f_X(x) \, dx = \int_a^b 1 \, dx = x \bigg|_a^b = b - a
\]

Therefore, the probability that it lies in any interval \([a, b]\) fully contained in \([0, 1]\) is \(b - a\) i.e. the length of the interval.
Therefore, the probability that it takes a value in \([0, 1]\)
\[
\int_{0}^{1} f_X(x) \, dx = \int_{0}^{1} 1 \, dx = 1
\]

For example, consider a discrete probability event such as the tossing of a fair die. The probability of occurrence of any face is \(\frac{1}{6}\).
Random variable which is distributed uniformly in the interval $[0, 1]$. 

\[
\begin{array}{c}
0 & 1 \\
\hline
F_X(x) & \\
\end{array}
\]