Bayesian Games
with mixed strategies

Bayesian BoS:

<table>
<thead>
<tr>
<th></th>
<th>( q_1 )</th>
<th>1 - ( q_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>10,5</td>
<td>0,0</td>
</tr>
<tr>
<td>H</td>
<td>0,0</td>
<td>5,10</td>
</tr>
</tbody>
</table>

\( P(C) = \frac{1}{2} \)

\( P(H) = \frac{1}{3} \)

\( P(U) = \frac{1}{2} \)

<table>
<thead>
<tr>
<th></th>
<th>( q_2 \leq 0 )</th>
<th>1 - ( q_2 \leq 1 )</th>
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<td>C</td>
<td>10,0</td>
<td>0,10</td>
</tr>
<tr>
<td>H</td>
<td>0,5</td>
<td>5,20</td>
</tr>
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\( P(C) = \frac{2}{3} \)

\( P(H) = \frac{1}{3} \)
Payoff to girl of type I for always choosing C is:

\[ 5p + 0(1-p) = 5p \]

Payoff to girl of type I for always choosing H is:

\[ 0p + 10(1-p) = 10(1-p) \]

Therefore, girl of type I will employ mixed strategy, when,

\[ 5p = 10(1-p) \]

\[ 15p = 10 \]

\[ p = \frac{2}{3} \]
Therefore, mixed strategy employed by boy is $\frac{2}{3}, \frac{1}{3}$.

Mixing C, H with probabilities $\frac{2}{3}, \frac{1}{3}$ respectively.

Payoff to girl of type U for choosing C is

$$0 \times \frac{2}{3} + 5 \times \frac{1}{3} = \frac{5}{3}$$

Payoff to girl of type U for choosing H is

$$10 \times \frac{2}{3} + 0 \times \frac{1}{3} = \frac{20}{3}$$
For girl of type U, choosing H yields a strictly greater payoff than choosing C.
Therefore $q_2 = 0$
$1 - q_2 = 1$

Payoff to boy for choosing C is,
$\frac{1}{2}(10q_1 + 0(1 - q_1))$
$+ \frac{1}{2} \times 0 = 5q_1$
Payoff to boy for always choosing $H$ is

\[
\frac{1}{2} \left( 0q_1 + 5(1-q_1) \right) + \frac{1}{2} \times 5 = \frac{5}{2} (1-q_1) + \frac{5}{2}
\]

Since boy is using a mixed strategy, payoff to $C, H$ must be equal. Therefore,

\[
5q_1 = \frac{5}{2} (1-q_1) + \frac{5}{2} \\
10q_1 = 5(1-q_1) + 5
\]
\[ 15q_1 = 10 \]
\[ \Rightarrow q_1 = \frac{2}{3} \]
\[ 1 - q_1 = \frac{1}{3} \]

Mixture of girl of type I is \( \left( \frac{2}{3}, \frac{1}{3} \right) \).

Therefore, the mixed strategy Bayesian Nash Equilibrium of the game is
\[
\left( \left( \frac{2}{3}, \frac{1}{3} \right), \left( \frac{2}{3}, \frac{1}{3} \right), (0, 1) \right)
\]

Mixture of boy Mixture of girl of type I Mixture of girl of type II.
Let girl of type U be mixing. Her payoff to always choosing C is \[ 0p + 5(1-p) = 5(1-p) \]
Her payoff to always choosing H is \[ 10p + 0(1-p) = 10p. \]
Therefore
\[ 5(1-p) = 10p \]
\[ \Rightarrow 15p - 5 \]
\[ \Rightarrow p = \frac{1}{3} \]
\[ 1 - p = \frac{2}{3} \]

Payoff to girl of type T for always choosing C is
\[ 5 \times \frac{1}{3} + 0 \times \frac{2}{3} = \frac{5}{3} \]
Her payoff to always choosing H is
\[ 0 \times \frac{1}{3} + 10 \times \frac{2}{3} = \frac{20}{3} \]
Therefore girl of type \( I \)
is always choosing \( H \).

\[ \Rightarrow q_1 = 0 \]

\[ 1 - q_1 = 1 \]

Payoff to Boy corresponding to
\( C \) is

\[ \frac{1}{2} \times 0 + \frac{1}{2} \left( 10q_2 + 0(1-q_2) \right) \]

\[ = 5q_2 \]
Payoff to Boy for always choosing A is:

\[
\frac{1}{2} \times 5 + \frac{1}{2} \times 5 \left(1 - q_2\right)
\]

\[= \frac{5}{2} + \frac{5}{2} \left(1 - q_2\right)\]

\[5q_2 = \frac{5}{2} + \frac{5}{2} \left(1 - q_2\right)\]

\[\Rightarrow 10q_2 = 5 + 5 \left(1 - q_2\right)\]

\[\Rightarrow 9q_2 = \frac{2}{3}\]

\[1 - q_2 = \frac{1}{3}\]
Another mixed strategy
Bayesian Nash Equilibrium
\[
\left(\frac{1}{3}, \frac{2}{3}\right), \left(0, 1\right), \left(\frac{2}{3}, \frac{1}{3}\right)\]

Mixture of Baye's Mixture of Mixture of girl of type E girl of type E girl of type U.