Bayesian Cournot Game.

Market competition between 2 firms $F_1, F_2$. 
Firm 1 has a production cost of \( C \) per unit.

Firm 2 of types:

- Low cost \( \frac{1}{2} C \) \( P(\text{low}) = \frac{1}{2} \)
- High cost \( \frac{1}{2} C \) \( P(\text{high}) = \frac{1}{2} \)

Inverse demand curve:
\[
\text{price } p = \left( a - (q_1 + q_2) \right)
\]

Quantity produced by Firm 1

Quantity produced by Firm 2
Payoff to each firm $j$

= price x quantity - cost of production

= $(a - (q_1 + q_2)) q_j - c_j q_j$

Firm 1 produces $q_1$, type $L$, $q_1^L$

Firm 2 type $H$, $q_2^H$
Payoff to Firm 2 of type L

\[(a - (q_1 + q_2^L))q_2^L - \frac{1}{2} C q_2^L = aq_2^L - q_1 q_2^L - (q_2^L)^2 - \frac{1}{2} C q_2^L\]

\[a - q_1 - 2q_2^L - \frac{1}{2} C = 0\]

\[(q_2^L)_{\text{best response}} = \frac{a - \frac{1}{2} C - q_1}{2}\]

Best response of Firm 2 of type L
Payoff to firm 2 of type H is
\[
\left( a - (q_1 + q_2^H) \right) q_2^H - C q_2^H = a q_2^H - q_1 q_2^H - (q_2^H)^2 - C q_2^H
\]
\[
a - q_1 - 2 q_2^H - C = 0
\]

\[
(q_2^H)^* = \frac{a - q_1 - C}{2}
\]

Best response of Firm 2 of type H.
Payoff to firm 1, corresponding to type low of firm 2 is 
\[(a-(q_1+q_2^L))q_1 - C q_1\]

Payoff to firm 1 corresponding to firm 2 of type H is 
\[(a-(q_1+q_2^H))q_1 - C q_1\]
average payoff of firm 1 is
\[ \frac{1}{2} \left( (a - (q_1 + q_2^{L})) q_1 - C q_1 \right) \]
\[ + \frac{1}{2} \left( (a - (q_1 + q_2^{H})) q_1 - C q_1 \right) \]

Differentiate w.r.t. \( q_1 \) and set equal to 0 to find the best response \( q_1 \).

\[ \frac{1}{2} \left( a - 2q_1 - q_2^{L} - C \right) \]
\[ + \frac{1}{2} \left( a - 2q_1 - q_2^{H} - C \right) = 0 \]
\[ 2q_1^* = \frac{1}{2} \left( a - c - q_2^L \right) + \frac{1}{2} \left( a - c - q_2^H \right) \]

\[ q_1^* = \frac{a - c}{2} - \frac{1}{4} \left( q_2^L + q_2^H \right) \]

\[ (q_2^L)^* = \frac{a - \frac{1}{2} c - q_1^*}{2} \]

\[ (q_2^H)^* = \frac{a - a_1^* - c}{2} \]
\[ q^*_1 = \frac{1}{2} (a-c) - \frac{1}{4} \left( (q^*_2)^* + (q^*_3)^* \right) \]

\[ = \frac{1}{2} (a-c) - \frac{1}{4} \left\{ \frac{a - \frac{1}{2} c - q^*_1}{2} + \frac{a - c - q^*_1}{2} \right\} \]

**equation for \( q^*_1 \)**

\[ q^*_1 = \frac{a - \frac{5}{4} c}{3} \]

*Best response quantity of Firm 1.*
\[
(q^L_2)^* = \frac{a - \frac{1}{2} c - a_1^*}{2} = \frac{a - \frac{1}{2} c - \frac{1}{3}(a - \frac{5c}{4})}{2} = \frac{a}{3} - \frac{c}{24}
\]

\[
(q^H_2)^* = \frac{a - c - a_1^*}{2} = \frac{a - c - \frac{1}{3}(a - \frac{5c}{4})}{2} = \frac{a}{3} - \frac{7c}{24}.
\]
Bayesian NE of the Bayesian Cournot Game is

$$\left( \frac{a - \frac{5c}{4}}{3}, \left( \frac{a - c}{3}, \frac{a - 7c}{24} \right) \right)$$

(quantity of Firm 1) = \( \frac{\text{quantity of type L} + \text{quantity of type H}}{2} \)