Portfolio Management

Game

Investment Game:

Investor (I)

Stock 1 ($s_1$)

Stock 2 ($s_2$)
past return of stocks.

\[ r_1 \rightarrow r_2 \]

<table>
<thead>
<tr>
<th>Nature Market</th>
<th>( r_1 )</th>
<th>( r_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_1 )</td>
<td>5, -5</td>
<td>6, -6</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>10, -10</td>
<td>3, -3</td>
</tr>
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\( \Rightarrow \) No pure strategy NE.
<table>
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<th>$\pi_1$</th>
<th>$\pi_2$</th>
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<tbody>
<tr>
<td>Market</td>
<td>$a$</td>
<td>$1-a$</td>
</tr>
<tr>
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<td>$\pi_1$</td>
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$$U_M(\pi_1) = -5\pi + (10)(1-\pi) = 5\pi - 10$$

$$U_M(\pi_2) = -6\pi + (-3)(1-\pi) = -3\pi - 3$$
\[5p - 10 = -3p - 3\]
\[\Rightarrow 8p = 7\]
\[p = \frac{7}{8}\]

Mixed strategy of investor = \((\frac{7}{8}, \frac{1}{8})\)

implies invests \(\frac{7}{8}\) funds in stock 1 and \(\frac{1}{8}\) funds in stock 2.
\[ U_I(\Sigma_1) = 5q + 6(1-q) \]
\[ = 6 - q \]
\[ U_I(\Sigma_2) = 10q + 3(1-q) \]
\[ = 7q + 3 \]

\[ 6 - q = 7q + 3 \]
\[ \Rightarrow 8q = 3 \]
\[ \Rightarrow \boxed{q = \frac{3}{8}} \]
\[ \Rightarrow 1 - q = \frac{5}{8} \]
Therefore, mixed strategy employed by market

\[= \left( \frac{3}{8}, \frac{5}{8} \right)\]

Mixed strategy NE of this investment or portfolio management game is

\[
\begin{pmatrix}
\left( \frac{7}{8}, \frac{1}{8} \right), & \left( \frac{3}{8}, \frac{5}{8} \right)
\end{pmatrix}
\]

Investor \quad Market