Paying Taxes:

Game between Tax Payers (T) and Auditors (A)
Taxpayers
\[ \rightarrow \text{Honest (H)} \rightarrow \text{Cheat (C)} \]

Auditors
\[ \rightarrow \text{Audit (A)} \rightarrow \text{Not Audit (N)} \]
No intersection of best response in pure strategies

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>N</th>
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<tbody>
<tr>
<td>H</td>
<td>0,20</td>
<td>0,40</td>
</tr>
<tr>
<td>C</td>
<td>-100</td>
<td>40,0</td>
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No pure strategy NE

\[
\begin{array}{c|cc}
\text{Auditor} & V & 1 - q \\
\hline
\text{Tax Payer} & & \\
\hline
H & 0,20 & 0,40 \\
C & -100 & 40,0 \end{array}
\]
\[ U_A(A) = 20p + 40(1-p) = 40 - 20p \]
\[ U_A(N) = 40p + 0(1-p) = 40p \]

Auditor will employ a mixed strategy only when
\[ 40 - 20p = 40p \]
40 - 20p = 40p
⇒ 60p = 40
⇒ p = \frac{40}{60} = \frac{2}{3}

Mixture of Tax Payer is p = \frac{2}{3}, 1-p = \frac{1}{3}

Pay tax with prob p = \frac{2}{3}
Chat prob 1-p = \frac{1}{3}
We can think of this mixed strategy NE as a cross a population, i.e. if you randomly pick a person, with prob $p=\frac{2}{3}$ you encounter a honest tax payer and prob $1-p=\frac{1}{3}$ you encounter a person cheating on taxes.
Therefore, the mixture across the population is:

\[ p = \frac{2}{3}, \quad 1 - p = \frac{1}{3} \]

\[
\begin{align*}
U_T(H) &= 0q + 0(1 - q) \\
&= 0 \\
U_T(C) &= -100q + 40(1 - q) \\
&= 40 - 140q
\end{align*}
\]
\begin{align*}
0 &= 40 - 140q \\
q &= \frac{40}{140} = \frac{2}{7} \\
1 - q &= \frac{5}{7}.
\end{align*}

Therefore, Auditor is using the mixed strategy \((\frac{2}{7}, \frac{5}{7})\).
Therefore, mixed strategy NE of the game is:

\[
\left( \frac{2}{3}, \frac{1}{3} \right), \left( \frac{2}{7}, \frac{5}{7} \right)
\]

Taxpayer \quad Auditor