Battle of Sexes

- Between Boy, Girl
  - Cricket (C)
  - Movie Harry Potter (H)
<table>
<thead>
<tr>
<th></th>
<th>Girl</th>
<th>C</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boy</td>
<td>C</td>
<td>10,5</td>
<td>0,0</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>0,0</td>
<td>5,10</td>
</tr>
</tbody>
</table>

(C, C) and (H, H) are NE

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Boy is employing mixed strategy \( p, 1-p \). 

i.e. Choosing C with prob \( p \) 
Choosing H with prob \( 1-p \) 

\[
U_G(C) = 5p + 0(1-p) = 5p. \\
U_G(H) = 0 \times p + 10(1-p) = 10(1-p)
\]
\[ U_6(c) = 5p \]
\[ U_6(H) = 10(1 - p) \]

Therefore, girl will randomly choose between C, H only when payoffs are equal:
\[ 5p = 10(1 - p) \]
\[ 5p = 10(1-p) \]
\[ \Rightarrow 15p = 10 \]
\[ \Rightarrow p = \frac{2}{3} \Rightarrow 1-p = \frac{1}{3} \]

Therefore, probability mixture of boy in \( p = \frac{2}{3}, 1-p = \frac{1}{3} \)
i.e. Boy is watching C with prob \( \frac{2}{3} \), and A with prob \( \frac{1}{3} \).
Let the mixed strategy of girl be $q, 1-q$.

i.e. Girl is randomly choosing to watch $C$ fraction $q$ of time and watch $H$ fraction $1-q$ of time.

$$U_b(C) = 10q + 0(1-q) = 10q$$

$$U_b(H) = 0q + 5(1-q) = 5(1-q)$$
Boy will randomly choose between C & H only when

\[ U_B(C) = U_B(H) \]

\[ 10q = 5(1-q) \]

\[ 10q = 5(1-q) \]

\[ \Rightarrow 15q = 5 \]

\[ \Rightarrow q = \frac{1}{3} \]

\[ 1 - q = \frac{2}{3} \]
Mixture of Girl is
\( q^2 \frac{2}{3}, 1-q^2 \frac{2}{3} \)
nie randomly choosing C
prob \( \frac{1}{3} \) of time and choosing H randomly fraction \( \frac{2}{3} \) of
time.

Therefore, the mixed strategy Nash Equilibrium of Boys

\[
\begin{pmatrix}
\frac{2}{3} & \frac{1}{3} \\
\frac{1}{3} & \frac{2}{3}
\end{pmatrix}
\]

Mixed strategy of Boy

Mixed strategy of Girl.