`OUR NOT`

Duopoly

Competition between 2 firms.

Market - Strategic Interaction

2 Firms—producing 2 goods which are closely related
Strategic Substitutes

Quantities $s_1, s_2$
actions of the two firms
let the price function be given by 
\[ p(s_1, s_2) = \sqrt{A - B(s_1 + s_2)} \]

- price per unit
- price is decreasing with quantity - inverse demand function
- constants

let the cost per unit be given by \( C \),

- Total cost of \( F_1 = C s_1 \)
- Total cost for \( F_2 = C s_2 \)
$U_1(s_1, s_2) = \text{Total Revenue} - \text{Total cost}$

$= \text{price/\text{unit} \times q_{\text{by}} - Total cost}$

$= (A - B(s_1+s_2))s_1 - Cs_1$

$= (A-C-B(s_1+s_2))s_2$
To find best response $s_1^*$ of Firm 1, differentiate with respect to $s_1$, and set equal to 0.

\[
U_1(s_1, s_2) = s_1 \left( A - C - B(s_1 + s_2) \right) \\
= A s_1 - C s_1 - B s_1^2 - B s_1 s_2
\]

\[
\frac{\partial U_1}{\partial s_1} = A - C - 2B s_1 - B s_2
\]

\[
\frac{\partial U_1}{\partial s_1} = 0 \implies s_1^* = \frac{A - C - B s_2}{2B}
\]
\[ S_1^* = \frac{A - C - B S_2}{2B} \quad \text{BR}_1(S_2) \]
\[ S_2^* = \frac{A - C - B S_1}{2B} \quad \text{BR}_2(S_1) \]

\[ \text{NE - Best Responses intersect} \]
\[ S_1^* = \text{BR}_1(S_2^*) \]
\[ S_2^* = \text{BR}_2(S_1^*) \]
\[ S_1^* = \frac{A-C - 2B s_2^*}{2B} \]
\[ S_2^* = \frac{A-C - 2B s_1^*}{2B} \]

Only at NE

\[ s_1^* = \frac{A-C}{2B} - \frac{1}{2} s_2^* \]
\[ = \frac{A-C}{2B} - \frac{1}{2} \left( \frac{A-C}{2B} - \frac{1}{2} s_1^* \right) \]
\[ \Rightarrow \frac{3}{4} s_1^* = \frac{A-C}{4B} \]
\[ s_1^* = \frac{A-C}{3B} \]
\[ s_1^* = \frac{A - C}{3B} \]
\[ s_2^* = \frac{A - C}{3B} \]

Nash Equilibrium Quantities.

\[ \text{NE} = \left( \frac{A - C}{3B}, \frac{A - C}{3B} \right) \]

Cournot Duopoly

\[ s_1^* \quad s_2^* \]
\[ A = 10 \quad B = C = 1 \]
\[ s_1^* = \frac{A - C - BS_2}{2B} = 4.5 - \frac{1}{2} s_2 \]
\[ s_2^* = 4.5 - \frac{1}{2} s_1 \]