Stackelberg Model

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Market Structure and Oligopoly

• Markets differ along following criterion
  ◦ number of firms
  ◦ barrier of entry and exit
  ◦ ability of firms to differentiate their products

• Oligopoly
  ◦ small number of firms in a market with relatively high barriers to entry

• because relatively few firms compete in an oligopoly,
  ◦ each firm faces a downward-sloping demand curve
  ◦ each firm can set its price: $p > MC$
  ◦ each affects rival firms
Duopoly as a Special Case of Oligopoly

• Basic Duopoly Model
  ◦ Only 2 firms (no other firms can enter)
  ◦ firms sell identical products
  ◦ market that lasts only 1 period (product or service cannot be stored and sold later)

• Two Types
  ◦ firms choose quantities: Cournot model, Stackelberg model
  ◦ firms set prices: Bertrand model
Stackelberg model

• **Cournot model**: both firms make their output decisions simultaneously

• **Stackelberg's model**: firms act sequentially
  ◦ A firm sets its output first [Leader]
  ◦ then its rival sets its output [Follower]
  ◦ Once the two quantities are chosen, price is set to clear the market. For example, take \( P = a - b(q_L + q_F) \)
How do we solve this game?

• Work backwards -- use backward induction
• Start at the last step: \( P = a - b(q_L + q_F) \), setting price to clear the market
• Next step before that -- follower chooses quantity to maximize profit given leader’s choice.
  • \( \pi_F = (a - b(q_L + q_F) - c) \cdot q_F \)
  • Take derivative and set = 0 to get BR
  • \( a - bq_L - 2bq_F - c = 0 \)
  • \( q_F^* = \frac{(a - bq_L - c)}{2b} \)
Now go the first step -- leader chooses quantity to maximize profit

\[ \pi_L = (a - b(q_L + q_F) - c) q_L \]

However, leader knows how follower will respond -- leader can figure out follower’s BR, so:

\[ \pi_L = (a - b(q_L + (a - bq_L - c)/2b) - c) q_L \]

Simplify to get \[ \pi_L = (a - bq_L - c)/2 q_L \]

Take derivative and set equal to 0 to get BR:

\[ (a - 2bq_L - c)/2 = 0 \to q_L = (a - c)/2b \]

And \[ q_F^* = (a - bq_L - c)/2b = (a - b(a - c)/2b - c)/2b = (a - c)/4b \]
• Leader has the advantage -- he sets higher quantity and gets a higher profit than the follower

• Often called the “first-mover” advantage

• Total output = (a-c)/2b + (a-c)/4b = 3(a-c)/4b

• Greater than total Cournot output of 2(a-c)/3b
Depicting the Stackelberg outcome

quantities in a Stackelberg equilibrium