Game Tree and Information Sets

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Recap: Extensive Form Games

• In strategic interactions
  – Players [Decision making units] may move simultaneously or sequentially.
  – They may move once or many times. The interactions may get repeated.

• How should we represent such interactions?

• We should consider:
  1. List of the players participating in the strategic interactions
  2. When does a player get to move in the game? [Order of Moves]
  3. What are the actions available to the player when she gets to move?
  4. How much does a player know when he gets to move? [Information]
  5. Pay-offs

• Notice: Only 1, 3 and 5 were required in a normal form game.
Game Tree

• Game Tree: A simple and useful way of representing an extensive form game.
• A game tree is a graph.
• It consists of
  – Nodes
  – Branches
• Nodes -> Labels
  – Initial Nodes: beginning of the game
  – Decision Nodes: Player labels
  – Terminal Nodes: Payoffs
• Nodes -> Information
• Branches-> Actions.
Introducing Nature as a player

• How can we model uncertain outcomes?
• An Example

• Nature to model uncertain outcomes.
Nodes and Branches

• Each node indicates
  – either the beginning of the game, or
  – a player’s turn to make a decision, or
  – the end of the game.

• Each branch always indicates an action taken by one of the players.
  – Predecessor Node and Immediate Predecessor Node
  – Successor Node and Immediate Successor Node
Important Rules

Three important requirements

- Unique Initial Node
- Only one way to proceed: Notion of paths
- No Cycles: Unique Immediate Predecessor
Player Function

• A function which assigns each decision node to a player.

• Player function partitions the set of all decision nodes.
Knowledge

• Consider Prisoners’ Dilemma and variant:
  – Both prisoners move simultaneously.
  – Prisoner 1 moves first, prisoner 2 observes 1’s action and then decides his action.

• How to represent these two strategic interactions?
Information Sets

• An information set belongs to a particular player and contains decision nodes satisfying following criterion
  – the player gets to play/make a move at every node in that information set, and
  – when a node belonging to the information set is reached, the player does not know which node in the information set has been reached.

• Each decision node is in exactly one information set.

• At each decision node in an information set, the player must have
  – the same set of feasible actions, and
  – ultimately choose the same action.
Information Sets: Example
Perfect vs. Imperfect Information

• Common Knowledge: Players know the game structure

• Perfect Information
  – Players, when making any decision, know of all the events that have previously occurred.
  – All information sets are singleton.

• Imperfect Information:
  – Players when making any decision, may not be perfectly informed about some (or all) of the events that have already occurred.
  – At least one of the information sets is not singleton.