Auction: As a Normal Form Game

Dr. Vimal Kumar, Assistant Professor of Economics
Indian Institute of Technology Kanpur,
vimalk@gmail.com
Auction as a Game

• Players: n Potential Bidders.

• Strategy Set of Bidder \( i \): A set of all not negative bids

• Payoff: If she wins then amount equals to her valuation minus what she pays. And, 0 if she loses

• Rules of the game: Type of Auction, Type of valuation, how does winner get decided.

• Is the seller a player in the game? The seller typically gets to set the rules for the auction.
Modelling Vickrey Auction as a Game.

- **Vickrey Auction**: Second Price sealed bid auction & players have private valuation.
- **Players**: all the bidders.
- **Strategy Set of Bidder $i$**: A set of all not negative bids
- **Payoff**: If she submits the highest bid then amount equals to her valuation minus the highest among remaining bids. And, 0 if she loses
- **What is the Nash Equilibrium of this game?**
Three Strategies for Player i:

- **Strategy 1**: Bidding truthfully
- **Strategy 2**: Bidding more than the valuation
- **Strategy 3**: Bidding less than the valuation
Strategy 2: Bidding More than the true value.

- **Scenario 1**: Same as Strategy 1
- **Scenario 2**: Same as the Strategy 1
- **Scenario 3**: Worse than Strategy 1

Player $i$’s bid:

- **Player $i$’s valuation**

Other players’ bid:
Strategy 3: Bidding Less than the true value.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Player i’s bid</th>
<th>Other players’ bid</th>
<th>Player i’s valuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Same as the Strategy 1</td>
<td>Same as the Strategy 1</td>
<td></td>
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<tr>
<td>2</td>
<td>Same as the Strategy 1</td>
<td>Same as the Strategy 1</td>
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<tr>
<td>3</td>
<td>Worse than Strategy 1</td>
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</table>

- **Scenario 1:**
  - Player i’s bid: Same as the Strategy 1
  - Other players’ bid: Same as the Strategy 1
  - Player i’s valuation: Same as the Strategy 1

- **Scenario 2:**
  - Player i’s bid: Same as the Strategy 1
  - Other players’ bid: Same as the Strategy 1
  - Player i’s valuation: Same as the Strategy 1

- **Scenario 3:**
  - Player i’s bid: Same as the Strategy 1
  - Other players’ bid: Same as the Strategy 1
  - Player i’s valuation: Worse than Strategy 1
Nash Equilibrium

• Vickrey Auction: Equilibrium strategy is to bid truthfully.

• One Nash Equilibrium is a strategy profile in which all the bidders bid truthfully. [Weak Dominant Strategy Equilibrium]

• Any other Nash Equilibrium?: Consider a Vickrey Auction with only two players.

• In English Auction: what does it mean?

• How about First Price Sealed Bid Auction.

• Revisit the topic of Auction after the introduction of Bayesian Games.
Other Auction Formats

• All Pay Auction
• Double Auction
• Auction as a scheduling mechanism
• Combinatorial Auctions