1. Both valuations $v_1, v_2$ are distributed uniformly in the interval $[0,1]$ i.e. probability density $f_{v_1}(v) = 1$ if $0 \leq v_i \leq 1$ and 0 otherwise. The expected value of the valuation $V_1$ can be calculated as,

$$E(V_1) = \int_0^1 v_1 f_{v_1}(v_1) dv_1 = \int_0^1 v_1 dv_1 = \frac{v_1^2}{2} \bigg|_0^1 = \frac{1}{2}$$

Similarly, we have, $E(V_2) = \frac{1}{2}$

Therefore, expected value of the sum of the valuations is,

$$E(V_1 + V_2) = E(V_1) + E(V_2) = \frac{1}{2} + \frac{1}{2} = 1$$

Ans b)

2. Expected payoff to player 1 in a first price auction as a function of his bid $b$ is

$$\Pr(\text{win}) \times (V_1 - b)$$

Since player 2 is bidding $V_2$, probability of winning for player 1 is the probability that his bid is greater than $V_2$ i.e.

$$\Pr(\text{win}) = \Pr(V_2 < b) = b$$

Therefore, expected payoff is $b(V_1 - b)$. Differentiating this with respect to $b$ and equating to 0, the best response bid $b$ can be found as,

$$V_1 - 2b = 0 \Rightarrow b = \frac{V_1}{2}$$

Ans d)

3. As shown in class lecture, the Bayesian Nash equilibrium bids are

$$b_1 = \frac{V_1}{2}, b_2 = \frac{V_2}{2}$$

Ans b)

4. As shown in class lecture, since Nash equilibrium bids as $b_1 = \frac{V_1}{2}, b_2 = \frac{V_2}{2}$ and the person with the highest bid wins and pays equal to his bid amount, the revenue to the auctioneer is
½ max\{V_1,V_2\}
Ans b)
5. As shown in class, expected value of the revenue to the auctioneer is 1/3
Ans c)
6. Given V_1 > V_2 and that player 2 is bidding V_2/2. Let bid of player 1 be b. To
win the auction he needs to bid b > V_2/2. Further, his payoff on winning is
V_1 - V_2/2 > 0, since he pays the second highest bid i.e. V_2/2. Therefore, it can
be seen that his payoff does not depend on the specific value of b as long
as b > V_2/2. However, if b < V_2/2, then he loses the auction and his payoff is
0. Therefore, the best response can be seen to be any bid b > V_2/2
Ans d)
7. As shown in class, the Bayesian Nash equilibrium bids are b_1 = v_1 and b_2 = v_2
Ans a)
8. As shown in class, since the player with the highest bid wins and pays the
second highest bid, revenue to auctioneer at Nash equilibrium is
\min\{V_1, V_2\}
Ans c)
9. As shown in class lecture, the probability that the minimum of the two
valuations V_1, V_2 lies in [v,v+dv] is 2(1-v)dv
Ans d)
10. Expected value of the revenue to the auctioneer is 1/3 as shown in class
Ans c)
11. Expected Revenue in Bayesian first and second price auctions are equal
Ans b)
12. The principle governing the revenues is the "Revenue Equivalence
Principle" as stated in class
Ans d)
13. Consider a bid b for player 1. In an ALL-PAY auction, the bid amount b is
paid irrespective of the outcome. Therefore, the payoff as a function of the
bid b is,
Pr(win) X (V_1 - b) + Pr(lose) X (-b)
Since player 2 is bidding b_2 = V_2, probability of player 1 winning is,
Pr(win) = Pr(b_2 < b) = Pr(V_2 < b) = b
Therefore, expected payoff as a function of the bid $b$ is

$$b \times (V_1 - b) + (1 - b) \times (-b)$$

Ans a)

14. Simplifying the above payoff as a function of the bid $b$, it can be seen that the expected payoff is given as,

$$b \times V_1 - b = b(V_1 - 1)$$

Since $V_1 < 1$, $V_1 - 1 < 0$. Therefore, $b(V_1 - 1) < 0$. Therefore, this is maximum for $b = 0$. Thus the best response bid is $b = 0$

Ans d)

15. As seen in the class lecture, the Bayesian Nash equilibrium for the ALL-PAY auction above is $b_1 = \frac{v_1^2}{2}$ and $b_2 = \frac{v_2^2}{2}$.

Ans b)