Solutions to Assignment 2:

The game table for the Hawk-Dove game is given below

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0,0</td>
<td>6,1</td>
</tr>
<tr>
<td>D</td>
<td>1,6</td>
<td>3,3</td>
</tr>
</tbody>
</table>

1. From the best responses in the table above it can be seen that both (H,D) and (D,H) are pure strategy Nash equilibria for the above game. Ans (c)

2. Since row player is mixing with 1/2, ½, payoff to column for choosing H,D respectively are
   Payoff to H is ½ x 0 + ½ X 6 = 3
   Payoff to D is ½ X 1 + ½ X 3 = 2
   Ans (b)

3. Column player is mixing H,D with probabilities 1/3,2/3 respectively. Payoff to row player for choosing H,D are
   Payoff to H is 1/3 X 0 + 2/3 X 6 = 4
   Payoff to D is 1/3 X 1 + 2/3 X 3 = 7/3
   Ans (a)

4. If row player is mixing (H,D) with (p,1-p) payoff to column player for choosing pure strategies H and D are
   Payoff to column player for H is 0X p + 6(1-p) = 6(1-p) = 6-6p
   Payoff to column player for D is 1 X p + 3 X(1-p) = 3-2p
For payoffs to be equal, \(6-6p = 3-2p \Rightarrow p = \frac{3}{4}\)  
Ans (c)

5. As seen in problem 4 above, \(p = \frac{3}{4}\) yields equal payoff to column player for H,D. Similarly, let column player mix (H,D) with probabilities (q,1-q) respectively.

Payoff to row player for H is \(0 \times q + 6 \times (1-q) = 6-6q\)
Payoff to row player for D is \(1 \times q + 3 \times (1-q) = 3-2q\)

At mixed strategy Nash equilibrium, both these payoffs have to be equal.
Therefore, \(6-6q = 3-2q \Rightarrow q = \frac{3}{4}\)
Therefore, mixed strategy Nash equilibrium is \((\frac{3}{4},\frac{1}{4}), (\frac{3}{4},\frac{1}{4})\) i.e. row player mixing with \((\frac{3}{4},\frac{1}{4})\) and column player also mixing with \((\frac{3}{4},\frac{1}{4})\).
Ans (d)

6. This game has 3 Nash equilibria. The two pure strategy Nash equilibria both (H,D) and (D,H) and the mixed strategy Nash equilibrium \((\frac{3}{4},\frac{1}{4}), (\frac{3}{4},\frac{1}{4})\)). Ans (b)

Game table for modified Battle of Sexes is shown below.

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>4,2</td>
<td>1,0</td>
</tr>
<tr>
<td>H</td>
<td>0,1</td>
<td>2,4</td>
</tr>
</tbody>
</table>

7. From the table above, it can be seen that both (C,C) and (H,H) are pure strategy Nash equilibria. However, the only (C,C) is given in the options. Therefore, ans (a)

8. Boy is mixing (C,H) with \((1/3,2/3)\) respectively. Payoffs to girl i.e. column player for C and H are given as,
Payoff to girl for C is \(1/3 \times 2 + 2/3 \times 1 = 4/3\)
Payoff to girl for H is \(1/3 \times 0 + 2/3 \times 4 = 8/3\)
9. Let the mixed strategies employed by the boy and girl at Nash equilibrium be \((p, 1-p)\) and \((q, 1-q)\)

Payoff to girl for C is \(2p + 1 \times (1-p) = 1+p\)
Payoff to girl for H is \(0 \times p + 4 \times (1-p) = 4-4p\)

Both above payoffs are equal at mixed strategy NE. Hence, \(1+p = 4-4p \Rightarrow p = 3/5\). Mixed strategy of Boy is \((3/5, 2/5)\).

Similarly, payoffs to boy are
Payoff to Boy for C is \(4q + 1 \times (1-q) = 1+3q\)
Payoff to Boy for H is \(0 \times q + 2 \times (1-q) = 2-2q\)

Both are equal for mixed strategy NE. Therefore, \(1+3q = 2-2q \Rightarrow q = 1/5\). Mixed strategy employed by girl is \((1/5, 4/5)\).

Hence, mixed strategy Nash equilibrium of game is \(((3/5, 2/5), (1/5, 4/5))\)

Ans (d)

10. Payoff to boy at Mixed NE is \(p \times \text{payoff from C} + (1-p) \times \text{payoff from H}\).

However, payoff from C = payoff from H = \(1+3q = 2-2q = 1+3/5 = 8/5\)

Payoff to boy = \(p \times 8/5 + (1-p) \times 8/5 = 8/5\)

Similarly, payoff to girl is \(1+p = 1+3/5 = 8/5\)

Answer (a)

11. 3 Player tragedy of commons. Payoff to each player \(i\) is \(e_i(1-(e_1+e_2+e_3))\)

Consider payoff to player 1 given as, \(e_1(1-(e_1+e_2+e_3))\)

\[ = e_1 - e_1^2 - e_1 \times e_2 - e_1 \times e_3 \]

To find best response \(e_1\), differentiate with respect to \(e_1\) and set equal to zero. Hence, we have,

\[1-2e_1-e_2-e_3 = 0\]
\[\Rightarrow e_1 = (1-e_2-e_3)/2\]

For Nash equilibrium, we have, \(e_1^* = (1-e_2^*-e_3^*)/2\) since each player is playing his best response. In addition, considering the symmetry in the game, at Nash equilibrium, we can assume \(e_1^* = e_2^* = e_3^* = x\)

Therefore, we have, \(x = (1-x-x)/2 \Rightarrow x = 1/4\)

Therefore, \(e_1 = e_2 = e_3 = 1/4\) is the Nash equilibrium. Ans (c)
12. Nash payoff to each player is \( \frac{1}{4} X (1 - \frac{3}{4}) = \frac{1}{16} \). Ans (a)

13. Sum payoff = \((e_1 + e_2 + e_3) (1 - (e_1 + e_2 + e_3))\) Let \((e_1 + e_2 + e_3) = t\). Sum payoff to \( t(1-t) \). This is maximum for \( t = \frac{1}{2} \). Which means, \( e_1 = e_2 = e_3 = \frac{1}{3} \cdot t = \frac{1}{6} \) Payoff to each player is \( \frac{1}{6} X (1 - \frac{3}{6}) = \frac{1}{12} \) Ans (d)

14. Payoff to each player = \(3x_1 x_2 / 2 = 3/2 x_1 x_2\). Further, cost to players 1, 2 are \(x_1^2\) and \(x_2^2\) respectively. Therefore, payoff to each is,
Payoff to player 1 is \(3/2 x_1 x_2 - x_1^2\)
Payoff to player 2 is \(3/2 x_1 x_2 - x_2^2\)
Find best response by differentiating payoff of player 1 with respect to \(x_1\) and setting equal to zero
\[\frac{3}{2} x_2 = 2x_1\]
Similarly, differentiate payoff to player 2 with respect to \(x_2\) and set equal to zero. This yields \(\frac{3}{2} x_1 = 2x_2\)
From above two equations we have \(x_1 = x_2 = 0\) is Nash equilibrium.
Therefore, it can be seen that \(x_1 = x_2 = 0\) ie \((0,0)\) is the Nash equilibrium.
Ans (c)

15. Collaborative payoff i.e. sum payoff = \(3x_1 x_2 - x_1^2 - x_2^2 = x_1 x_2 - (x_1 - x_2)^2\). It can be clearly seen that second term is minimum for any \(x_1 = x_2\). Further, first term is maximum for \(x_1 = x_2 = 1\). Therefore, maximum overall payoff occurs for \(x_1 = x_2 = 1\), which is equal to 1. Hence, Ans (a).