A. Consider a Hawk, Dove game in which each player can be aggressive and choose to be a Hawk (H) or a Dove (D). If both choose H, both receive a payoff of 0 each and if both choose D, both receive a payoff of 3 each. However, if one chooses H and other chooses D, player choosing H receives a payoff of 6 while the other receives 1. Formulate the game table for this game and answer the questions below.

1. Which one is a pure strategy Nash Equilibrium of the game above

   a) H, D
   b) D, H
   c) Both of the above
   d) None of the above

   Ans c)

2. If the row player i.e. player 1 is mixing H, D with probabilities 1/2, 1/2, what are the payoffs to the column player for choosing the pure strategies H, D respectively

   a) 2, 3
   b) 3, 2
   c) 2, 2
   d) 3, 3

   Ans b)

3. If the column player i.e. player 2 is mixing (H, D) with probabilities (1/3, 2/3), what are the payoffs to the row player for choosing the pure strategies H, D respectively

   a) 4, 7/3
   b) 3, 3
   c) 7/3, 3
   d) 2, 1/3

   Ans a)

4. If the row player i.e. player 1 is mixing (H, D) with probabilities (p, 1 – p), what is the value of p such that the payoffs to the row player for choosing the pure strategies H, D are equal

   a) ¼
   b) ½
c) \( \frac{3}{4} \)
d) 1

Ans c)

5. At the mixed strategy Nash equilibrium of this game, what are the mixtures employed by the row and column players

a) \( (1/2,1/2), (1/2,1/2) \)
b) \( (1/2,1/2), (1/4, \frac{3}{4}) \)
c) \( (1/3,2/3), (1/3, 2/3) \)
d) \( (3/4,1/4), (3/4,1/4) \)

Ans d)

6. How many Nash equilibria does this game have?

a) 4
b) 3
c) 2
d) 1

Ans b)

B. Consider a modified battle of sexes game in which boy and girl who are players 1, 2 respectively can choose C,H. The various payoffs are as follows. \((C,C) = (4,2), (H,H)=(2,4), (C,H)=(1,0), (H,C) = (0,1)\). Answer the questions below

1. A pure strategy Nash equilibrium of the game above is

a) \((C,C)\)
b) \((C,H)\)
c) Neither of above
d) Both of above

Ans a)

2. If boy is mixing C,H with probabilities 1/3, 2/3, what are the payoffs to the girl for the pure strategies C,H respectively

a) 4/3, 2/3
b) 4/3, 4/3  
c) 4/3, 8/3  
d) 2/3, 4/3  

Ans c)

3. What are the mixed strategies employed by the boy and the girl at the Nash equilibrium

a) (1/2, 1/2), (1/2, 1/2)  
b) (1/3, 2/3), (2/3, 1/3)  
c) (1/4, 3/4), (3/4, 1/4)  
d) (3/5, 2/5), (1/5, 4/5)  

Ans d)

4. What are the payoffs to the boy and girl at the mixed strategy Nash equilibrium

a) 8/5, 8/5  
b) 4/5, 4/5  
c) 2/5, 2/5  
d) 1/5, 1/5  

Ans a)

C. Consider a 3 player version of the ‘Tragedy of commons’ described in the lectures, in which the payoff to each player \( i = 1, 2, 3 \) for his action \( e_i \) is given as \( e_i(1 - (e_1 + e_2 + e_3)) \), where each \( e_i \) lies between 0,1.

1. What is the Nash equilibrium action \( e_i^* \) for each player \( i \) in this game

a) 1/6  
b) 1/3  
c) 1/4  
d) 1/5  

Ans c)

2. What is the Nash payoff to each player

a) 1/16  
b) 0
c) 2/25

d) 1/12

Ans a)

3. What is the ‘collaborative payoff’ to each player when they are maximizing the sum payoff and sharing the outcome

a) 1/6
b) 0
c) 2/25
d) 1/12

Ans d)

D. Two people are engaged in a joint project. If each person $i$ puts in the effort $x_i$, a non-negative number equal to at most 1, which costs her $c(x_i) = (x_i)^2$, the outcome of the project is worth $f(x_1, x_2) = 3x_1x_2$. The worth of the project is split equally between the two people, regardless of their effort levels.

1. The Nash equilibrium of the game above is at $(x_1, x_2)$ equal to

a) (1,1)
b) (3/4, 3/4)
c) (0,0)
d) (1/2, 1/2)

Ans c)

2. Which is the outcome $(x_1, x_2)$ which yields the highest collaborative payoff for both of them

a) (1,1)
b) (1/2, 1/2)
c) (1/3, 1/3)
d) (3/4, 3/4)

Ans a)