Basic Tools of Economic Analysis

4: Basic Tools of Economic Analysis and Optimization Techniques

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Lecture No -
Session Outline

Basic Tools of Economic Analysis and Optimization Techniques

Functional relationship between the economic variables
Some important economic functions
Slope and its use in economic analysis
Derivatives of various functions
Optimization techniques
Constrained optimization
Some Important Economic Functions

- **Demand function**: It represents the relationship between the price and quantity of the product.

- **Production function**: It represents the relationship between input like labour, capital, etc. and output.

- **Cost function**: It represents the relationship between the output and the cost of production.

- **Total revenue function**: It represents the combined function of quantity produced and price function derived from the demand function.

- **Profit function**: It represents the PROFIT obtained by subtracting the total cost function from the total revenue function.

Source: Managerial Economics; D N Dwivedi, 7th Edition
General Forms of Function used in Economic Analysis

Polynomial function: It represents those functions that have various terms of measure for the same independent variables.

Linear function: It is used when the relationship between the dependent and independent variables remains constant.

Nonlinear function: Used where relationship between the dependent & the independent variables does not remain constant. It shows a curvilinear relationship between the two variables.

Source: Managerial Economics; D N Dwivedi, 7th Edition
Linear Function

The relationship is linear – change in the dependent variable remains constant throughout for a one unit change in the independent variable, irrespective of the level of the dependent variable.

\[ Qx = 20 - 2Px \]

Each one rupee change in price, the demand for commodity changes by 2 units.

Linear demand function is a straight line.
Managerial Economics

![Demand Curve](image)

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Non Linear Demand Function

The quantitative relation between the dependent and independent variables does not remain constant.

It changes with the change in the level of independent variable.
Non Linear Demand Function

\[ D_x = aP_x^{-b} \]

\( -b \) is the exponent of variables \( P_x \)

Constant \( a \) is the coefficient of variables \( P_x \).

\[ D_x = 32 P_x^{-2} = 32/P_x^2 \]

This demand function produces a nonlinear or a curvilinear demand curve.
Non Linear Demand Schedule

<table>
<thead>
<tr>
<th>Px</th>
<th>Dx</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>32.00</td>
</tr>
<tr>
<td>2</td>
<td>8.00</td>
</tr>
<tr>
<td>3</td>
<td>3.50</td>
</tr>
<tr>
<td>4</td>
<td>2.00</td>
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<td>5</td>
<td>1.33</td>
</tr>
<tr>
<td>6</td>
<td>0.90</td>
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</table>
Non Linear Demand Curve
Polynomial Function

Functions containing many terms of the same independent variable are called polynomial function.

Consider a short run production function: \( Q = f(L) \)
Polynomial Function

A Quadratic Function: $Q = a + bL - cL^2$

A Cubic Function: $Q = a + bL + cL^2 - dL^3$

A Power Function: $Q = aL^b$

Where $Q$ = Output, $L$ = labour, $a, b, c$ and $d$ are constant.
The Degree of Polynomial Function

\[ Q = a + bL - cL^2 \]

The highest power is 2, so a polynomial function of degree 2.

A polynomial function of Power 2 is also called a quadratic function.
The Degree of Polynomial Function

$Q = a + bL + cL^2 - dL^3$

Here the highest power is 3, so a polynomial function of degree 3.

A function of power 3 is also called a cubic function.
The Degree of Polynomial Function

\[ Q = aL^b \]

The range of power is between

\[ b < 1, \ b = 1 \text{ and } b > 1 \]

Except zero, it can take any power.
Solving Polynomial Function

• Factoring methods
• Quadratic Formula

• Property of quadratic and cubic equation: more than one solution
Solving Quadratic and Cubic Equations

**Factoring method:** It involves the following two steps:
- The quadratic equation is set equal to zero.
- The equation is factored for obtaining two values of the variables x and y.

**Quadratic formula:** It involves determining the values of the variables using the formula:
- The quadratic equation is set equal to zero.
- The equation is factored for obtaining two values of the variables x and y.

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Source: Managerial Economics; D N Dwivedi, 7th Edition

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Concept of Slope

It is used for measuring relationship between marginal changes in two related variables.

It can be defined as the rate of change in the dependent variables as a result of change in the independent variable.
Concept of Slope

Representing relationship between two variables in a line or curve, the slope of the line or curve shows how strongly or weakly are the two variables related.

The steeper the curve or line, the weaker the relationship

The flatter the curve or line, the stronger the relationship.
Concept of Slope

With respect to demand curve, slope is the ratio of change in the dependent variable (D) to the change in the independent variable (P).

The movement down the demand line/curve gives the decrease in price ($-\Delta P$) and the consequent increase in the demand ($\Delta D$).

The ratio $-\Delta P / \Delta D$ gives the slope of the demand curve.
Concept of Slope

Measuring slope at a point on a curve: Limitation

- This method is not very reliable if changes in the independent variable is large because slope is different between any set of two points within the chosen two points of the curve.

- This method is not much of help in case of optimum solution to the business problem has to be found because an optimization problem may involve polynomial function.
Technique of Differential Calculus

It provides a technique of measuring the marginal change in the dependent variable (Y) due to change in independent variable (X), when the change in X approaches to zero.

The measure of such marginal change is known as **Derivative**.

The derivative of a dependent variable (Y) is the limit of change in Y when the change in the independent variable (X) approaches zero.
Differential Calculus

It is used for finding an optimum solution to a problem

- Derivative of constant function: The derivative of a constant function is always equal to zero.
  \[ \frac{\partial Y}{\partial X} = 0 \]

- Derivative of a power function: \( Y = f(X) = aX^b \) where, \( a \) and \( b \) are constants.

- Derivatives of functions of sum and difference of functions:
  \( Y = f(X) + g(X) \) and \( Y = f(X) - g(X) \) where, \( f(X) \) and \( g(X) \) are two different functions.

- Derivative of a function as a product of two functions:
  \[ \frac{\delta Y}{\delta X} = f(X) \times \frac{\delta g(X)}{\delta X} + g(X) \times \frac{\delta f(X)}{\delta X} \]

- Derivative of a quotient:
  \[ \frac{\delta Y}{\delta X} = \frac{[g(X) \frac{\delta f(X)}{\delta X} - f(X) \frac{\delta g(X)}{\delta X}]}{[g(X)]^2} \]

- Derivative of a function of a function:
  \[ \frac{\delta Y}{\delta X} = \frac{\delta Y}{\delta U} \times \frac{\delta U}{\delta X} \]

Source: Managerial Economics; D N Dwivedi, 7th Edition
Session References

Managerial Economics; D N Dwivedi, 7th Edition