State Space Representation

State
Power source

Storage

Kinetic energy storage

$E = \frac{1}{2} Li^2$

L (inductor)

Transform

Power sink

Load

Dissipation (Resistor)

Storage

Potential energy storage

$E = \frac{1}{2} CV^2$

Capacitor
Inductor, $L \Rightarrow v_L = L \frac{di_L}{dt}$

or

\[ i_L = \frac{1}{L} \int v_L \, dt \]

\[ \int_{i_L} \]

\[ \int_{v_L} \]

CAUSALITY

Integral Causal
Capacitor, $C = \int v_c \, dt$

- Model 1:
  \[ v_c = \frac{1}{C} \int i_c \, dt \]

- Model 2:
  \[ i_c = C \frac{dv_c}{dt} \]
Dissipator, Resistor, $R$

\[ V_R = i_R \cdot R \] — (1)

\[ i_R = \frac{V_R}{R} \] — (2)
\[ \begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*} \] state space representation

State Equation

Output Equation

STANDARD
\[ \dot{x} = Ax + Bu \]
\[ y = Cx \]
Step 1: Identify energy storing elements

ORDER of system

L = 1 first order system
Step 2: Identify the variables that will be used for modelling.

System's response $\Rightarrow$ 

1. Input - $u$
2. Integrator output values.

State variables $x$
Step 3: Starting with dynamic elements, obtain differential equations.

\[
\frac{dV_c}{dt} = \frac{1}{CR} \left[ V_g - V_c \right] - \frac{1}{CR_0} \cdot V_c - \frac{V_c}{R_0}
\]

\[
V_c = \frac{1}{C} \int i_C \, dt
\]

\[
i_R - i_0
\]
Step 4: Segregate into form

\[ \dot{x} = \Delta A \, x + B \, u \]

State space representation

\[ v_c = \left( \begin{array}{r} -\frac{1}{RC} \\ -\frac{1}{R_0C} \end{array} \right) v_c + \left( \frac{1}{RC} \right) v_g \]  
- State equation

\[ y = C \, x + Du \]

Output equation

\[ v_0 = \begin{bmatrix} 1 \end{bmatrix} [v_c] + \begin{bmatrix} 0 \end{bmatrix} v_g \]
Steps

1. Identify the energy storage elements
2. Identify variables - $u, x, [A]$
3. Starting from Dynamic components
4. Put all equations in the form
   \[
   \dot{x} = Ax + Bu
   \]
   \[
   y = Cx + Du
   \]
Practice