Selected Problems:
Devices for Efficient Power Conversion
Switched Mode Power Conversion

Problem Set 02

Problem No. 1

\[ C = 10 \, \mu F; \quad R_C = 0.02 \, \Omega; \quad V_C(0) = 0 \, V \]
Problem No. 1

Switched Mode Power Conversion

Problem Set 02

Problem No. 1

\[
A = 10\, \mu s \times \frac{50 \times 2}{11} \\
= 150\, \mu C + 318.3\, \mu C = 468.3\, \mu C \\
I^* \times 10\, \mu C = 468.3\, \mu C \\
I^* = 46.83\, A
\]
Switched Mode Power Conversion

Problem Set 02

Problem No. 1

\[ \theta = \frac{46.83 \times 10 \mu F}{16 \mu F} = 46.83V \]
Problem No. 1

Switched Mode Power Conversion

Problem Set 02

Problem No. 1

\[ \left( 2 \times 15 \times \frac{50 \times 2}{11} \times \frac{10 \mu}{30 \mu} \right) \]

\[ 15^2 \frac{10 \mu}{30 \mu} \]

\[ \frac{50^2}{2} \frac{10 \mu}{30 \mu} \]

\[ \int (A+B) \, dt \]

\[ 2 \times 15 \int 50 \sin \theta \, d\theta \]
Switched Mode Power Conversion
Problem Set 02
Problem No. 1

\[ 15^2 \frac{1}{3} + \frac{50^2}{2} \times \frac{1}{3} + 30 \times \frac{100}{\pi} \frac{1}{3} \]

\[ + \frac{46.83^2}{3} \frac{1}{3} \]

\[ = I_{rms}^2 = 1533.7 \text{ A}^2 \]

\[ \text{Loss} = I_{rms}^2 \times 0.02 = 30.67 \text{ W} \]

\[ \text{VA} = 65 \times 46 \approx 3000 \text{ VA} \]
Switched Mode Power Conversion

Problem Set 02

Problem No. 2

\[ Z_C = \frac{1}{\omega C} \]
\[ Z_L = \omega L \]
\[ Z_R = 30 \text{ m\Omega} \]

\( C = 10 \mu F \); \( R_C = 30 \text{ m\Omega} \); \( L = 75 \text{ nH} \)
Switched Mode Power Conversion

Problem Set 02

Problem No. 2

\[ C = 10 \, \mu F \]
Switched Mode Power Conversion
Problem Set 02
Problem No. 2

$L = 75 \text{ nH}$
Switched Mode Power Conversion

Problem Set 02

Problem No. 2

\[ R_C = 30 \, \text{m}\Omega \]

\[ 20 \log_{10} (0.03) \]

\[ -30.5 \, \text{dBn} \]
Switched Mode Power Conversion

Problem Set 02

Problem No. 2

$C = 10 \, \mu F; \quad R_C = 30 \, m\Omega; \quad L = 75 \, nH$
Switched Mode Power Conversion

Problem Set 02

Problem No. 3

Transformer with Sinusoidal Excitation

\[ V = N \frac{d\phi}{dt} \]

\[ \phi = \int_{0}^{t} \frac{1}{N} V \, dt \]
Switched Mode Power Conversion

Problem Set 02

Problem No. 3

\[ \phi(t) = -\Phi_m \cos \omega t \]

\[ V(t) = N \frac{d\phi}{dt} = N \Phi_m \omega \sin \omega t \]

\[ V(t) = 2\pi f \Phi_m N \sin \omega t \]

\[ V_{\text{rms}} = \sqrt{\frac{2\pi}{\sqrt{2}}} \int B_m A c \; N \]

\[ V_{\text{rms}} = 4.44 \int B_m A c \; N \]

Transformer with Sinusoidal Excitation
Switched Mode Power Conversion

Problem Set 02

Problem No. 3

Transformer with Sinusoidal Excitation

\[
K_{WAW} = \left(N_1 a_1 + N_2 a_2 \right) y = 2 N_1 a_1 = 2 N_1 \frac{I_{1rms}}{J}
\]
Switched Mode Power Conversion

Problem Set 02

Problem No. 3

\[ V_{\text{rms}} = 4.44 f B_m A_c N_1 \]

\[ 2 N_1 I_{\text{rms}} = \frac{K_w A_w}{J} \]

\[ V_{\text{rms}} I_{\text{rms}} = 2.22 J B_m f K_w \]

\[ A_c A_w = \frac{V_{\text{rms}} I_{\text{rms}}}{2.22 J B_m f K_w} \]

Transformer with Sinusoidal Excitation
Switched Mode Power Conversion

Problem Set 02

Problem No. 4

150 VA, 230 V, 1:1, 50 Hz, Isolation Transformer
Switched Mode Power Conversion

Problem Set 02

Problem No. 4

\[ A_c A_w = \frac{V_{rms} I_{rms}}{2.22 f J B_m K W} = \frac{150}{2.22} \]

\[ = \frac{5^0}{2.5 \times 10^{-6} \text{ A/m}^2 \cdot 1.2 \text{ T}} \]

\[ = 12,870.01 \text{ mm}^4 \]

150 VA, 230 V, 1:1, 50 Hz, Isolation Transformer
Switched Mode Power Conversion
Problem Set 02
Problem No. 4

$$A_{c}A_{w} \Rightarrow \sqrt{16}$$

$$\sqrt{A_{c}} = 1452 \text{ mm}^2$$

$$\sqrt{A_{w}} = 1093 \text{ mm}^2$$

$$230 = 4.44 \times 10^{-6} \times \frac{1452 N}{4.44 \times 10^{-6}}$$

$$N = \frac{230}{1452}$$

$$N_1 = N_2 = 5.95 \text{ turns}$$

Wire Data & Core Data
Switched Mode Power Conversion

Problem Set 02

Problem No. 4

\[ I_{\text{rms}} = \frac{150 \text{VA}}{230 \text{V}} = 0.65 \text{A} \]

\[ J = 2.5 \text{ A/mm}^2 \implies A_n = \frac{0.65}{2.5} \]

\[ = 0.26 \text{ mm}^2 \]

\[ \underline{23 \text{ SWG}} \]

150 VA, 230 V, 1:1, 50 Hz, Isolation Transformer
Switched Mode Power Conversion

Problem Set 02

Problem No. 4

\[ T \quad 16 \quad \text{core} \quad EI \]
\[ A_c = 1452 \text{ mm}^2 \quad A_w = 1092 \text{ mm}^2 \]
\[ N_1 = 595T \quad N_2 = 595T \]
\[ a_{w1} = 23 \text{ SWG} \quad a_{w2} = 23 \text{ SWG} \]

150 VA, 230 V, 1:1, 50 Hz, Isolation Transformer
Switched Mode Power Conversion

Problem Set 02

Problem No. 4

\[
\begin{align*}
A_cA_w \\
\text{Select a Core} \\
A_c \\
N_1 & \quad N_2 \\
I_1 & \quad I_2 \\
aw_1 & \quad aw_2 \\
sw & \quad sw_g
\end{align*}
\]

150 VA, 230 V, 1:1, 50 Hz, Isolation Transformer
Switched Mode Power Conversion

Problem Set 02

Problem No. 5

Coupled Inductor
Problem No. 5
Switched Mode Power Conversion
Problem Set 02
Problem No. 5

$N_1 = 100 \, \text{T;} \quad N_2 = 200 \, \text{T;} \quad Ag_1 = Ag_2 = 40 \, \text{mm}^2; 
Ag = 80 \, \text{mm}^2; \quad l_{g1} = 1 \, \text{mm;} \quad l_{g2} = 2 \, \text{mm;} \quad l_g = 1.5 \, \text{mm}$
Switched Mode Power Conversion

Problem Set 02

Problem No. 5

\[ R_1 = \frac{Lg_1}{Ag_1 \mu_0} \quad R = \frac{Lg}{Ag \mu_0} \quad R_2 = \frac{Lg_2}{Ag_2 \mu_0} \]

Reluctance Model
Switched Mode Power Conversion

Problem Set 02

Problem No. 5

\[ R_1 = \frac{l g_1}{A g_1 \mu_0} = \frac{1 \times 10^{-3}}{40 \times 10^{-6} \frac{1}{4\pi 10^{-7}}} = 19.9 \times 10^6 \frac{1}{H} \]

\[ R_2 = \frac{l g_2}{A g_2 \mu_0} = \frac{39.8 \times 10^6 \frac{1}{H}}{1.5 \times 10^{-3}} \]

\[ R = \frac{l g}{A g \mu_0} = \frac{14.92 \times 10^6 \frac{1}{H}}{80 \times 10^{-6} \frac{1}{4\pi 10^{-7}}} \]

Reluctance Calculation
Switched Mode Power Conversion

Problem Set 02

Problem No. 5

\[ R_{\text{net}} = R_1 + \left( R_1 \parallel R_2 \right) \]

\[ \Rightarrow R_2 + \left( R_1 \parallel R_1 \right) \]

Reluctance Calculation
Switched Mode Power Conversion

Problem Set 02

Problem No. 5

\[
\frac{\psi_1}{i_1} = \frac{N_1}{i_1} \frac{N_1 I_1}{R_1 + (R_{11} R_2)}
\]

\[
L_1 = \frac{N_1^2}{R_1 + (R_{11} R_2)}
\]

\[
L_1 = \frac{100 \times 100}{19.9 + 10.85} \times 10^6 = 325 \mu H
\]
Switched Mode Power Conversion

Problem Set 02

Problem No. 5

\[
L_2 = \frac{N_2^2}{R_2 + (R_{11} R_1)}
\]

\[
L_2 = \frac{\Psi_2}{i_2} \bigg|_{i_1=0} \Rightarrow \frac{N_2}{\frac{1}{2} \left( R_2 + (R_{11} R_1) \right)}
\]

\[
\frac{200 \times 200}{(39.8 + 8.52)} \times 10^{-6} \Rightarrow 8.28 \mu H
\]
Switched Mode Power Conversion

Problem Set 02

Problem No. 5

\[
\frac{\psi_1}{i_2} \bigg|_{i_1 = 0} = L_{12}
\]

\[
\frac{N_1 \phi_1}{i_2} \bigg|_{i_1 = 0} = L_{12} \cdot \frac{N_2 x_2}{R_2 + (R_1 R_4)} \cdot \frac{R}{R + R_1}
\]

L_{12}
Switched Mode Power Conversion

Problem Set 02

Problem No. 5

\[ L_{21} = \frac{N_1 N_2 R}{R_2 + (R_{11} R_1)} \frac{R}{R + R_1} \]

\[ = \frac{100}{14.92} \frac{200}{14.92 + 39.79} -6 \]

\[ = 177 \; \mu H \]

\[ L_{12} = 177 \; \mu H \]

\[ L_{21} \]
Switched Mode Power Conversion

Problem Set 02

Problem No. 5

\[ L_1 = 356 \, \mu H \]
\[ L_2 = 828 \, \mu H \]
\[ L_{12} = L_{21} = 177 \, \mu H \]

Self and Mutual Inductances
Switched Mode Power Conversion

Problem Set 02

Problem No. 6

Lifting Magnet

\[ L = \frac{N^2}{R} \]

\[ R = \frac{2x}{A_c \mu_0} \]
Switched Mode Power Conversion

Problem Set 02

Problem No. 6

\[
R = \frac{1_g}{A_C \mu_O \mu_r} = \frac{2x}{A_C \mu_O}
\]

Reluctance
Switched Mode Power Conversion

Problem Set 02

Problem No. 6

\[ L = \frac{N^2}{R} = \frac{N^2 A_c \mu_o}{2x} \]

Inductance
Switched Mode Power Conversion

Problem Set 02

Problem No. 6

\[
E = \frac{1}{2} LI^2 = \frac{N^2 A_c \mu_o I^2}{4x}
\]

Stored Energy

\[
E = f(x)
\]
Switched Mode Power Conversion

Problem Set 02

Problem No. 6

\[ F = - \frac{dE}{dx} = \frac{N^2 A_c \mu_0 I^2}{4x^2} \]

Lifting Force
Switched Mode Power Conversion

Problem Set 02

Problem No. 6

Lifting Force: \( N = 200 \), \( A_C = 0.04 \text{ m}^2 \), \( I = 500 \text{ A} \), \( x = 100 \text{ mm} = 0.1 \text{ m} \)

\[ F = \frac{N^2 A C \mu_0 I^2}{4 x^2} \]

\[ F = \frac{200 \cdot 200 \cdot 0.04 \cdot 4 \pi \cdot 10^{-7} \cdot 500 \cdot 500}{4 \cdot 0.1} \]

\[ F = 12566 \text{ N} \]

\[ \Rightarrow 1281 \text{ Kgf} \]
Lifting Force: \( N = 200, \ A_C = 0.04 \ m^2, \ I = 500 \ A, \ x = 100 \ mm = 0.1 \ m \)
Switched Mode Power Conversion

Problem Set 02

Problem No. 7

Non-linear Inductor

a

Gap = 1 mm
Area = 25 mm²
N = 100 T
Bₘ = 0.2 T

b

Gap = 5 mm
Area = 25 mm²
N = 100 T
Bₘ = 0.2 T

Composite
Core

c

φ = φₐ + φₜ
Switched Mode Power Conversion

Problem Set 02

Problem No. 7

\[ L = \frac{N^2}{R} = \frac{100^2}{1 \times 10^{-3}} \times 2 \times 10^{-6} \times 4 \pi \times 10^{-7} \]

\[ = 314 \ \mu \text{T} \quad 0 \leq i \leq 1.59 \text{A} \]

\[ I_m \Rightarrow B_m \]

\[ \frac{L}{L} \frac{I_m}{I_m} = \frac{N}{N} \frac{B_m}{B_m} \frac{A_c}{A_c} \]

\[ I_m = \frac{100 \times 0.2 \times 25 \times 10^{-6}}{314 \times 10^{-6}} \]

\[ = 1.59 \text{A} \]
Switched Mode Power Conversion

Problem Set 02
Problem No. 7

Gap = 1 mm
Area = 25 mm²
N = 100 T
B_m = 0.2 T

Flux – Current Characteristics
Switched Mode Power Conversion
Problem Set 02
Problem No. 7

Gap = 5 mm
Area = 25 mm²
N = 100 T
Bₘ = 0.2 T

\[ L = \frac{100 \times 100 \times 25 \times 10^{-6}}{5 \times 10^{-3}} \]
\[ = 62.8 \ \mu H \]

\[ L I_m = N B_m A_c \]

\[ 62.8 \ I_m = 100 \times 2 \times 25 \times 10^{-6} \]
\[ I_m = \frac{100 \times 2 \times 25 \times 10^{-6}}{62.8 \times 10^{-6}} \]
\[ = 7.96 A \]
Switched Mode Power Conversion

Problem Set 02

Problem No. 7

Gap = 5 mm
Area = 25 mm²
N = 100 T
B_m = 0.2 T

Flux – Current Characteristics
Switched Mode Power Conversion

Problem Set 02

Problem No. 7

Flux – Current Characteristics (Total)
Switched Mode Power Conversion

Problem Set 02

Problem No. 7

Idealised Flux – Current Characteristics
Switched Mode Power Conversion
Problem Set 02
Problem No. 7

Real Flux – Current Characteristics