Switched Mode Power Conversion

Transformers

Devices for Efficient Power Conversion

Switches
Inductors
Transformers
Capacitors
Transformers Match Voltage Levels. Ideally They Do Not Store Energy.

In Power Converters, Transformers Provide Electrical Isolation as Well.
Switched Mode Power Conversion

Transformers

Transformers Match Voltage Levels.

\[
\frac{V_1}{V_2} = \frac{N_1}{N_2}
\]
Switched Mode Power Conversion

Transformers

Transformers Provide Electrical Isolation

\[
\frac{V_1}{V_2} = \frac{N_1}{N_2}
\]
Switched Mode Power Conversion

Transformers

Efficiency of Power Conversion is Unity

\[ V_1 I_1 = V_2 I_2 \]

\[ \frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} \]
Switched Mode Power Conversion

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Circuit Schematic

\[ V_2 = \frac{N_2}{N_1} V_1 \]

\[ I_1 = \frac{N_2}{N_1} I_2 \]
Switched Mode Power Conversion

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Magnetic Circuit

\[ V_2 = \frac{N_2}{N_1} V_1 \]

\[ I_1 = \frac{N_2}{N_1} I_2 \]
Switched Mode Power Conversion

Transformers
Magnetic Circuit

Transformer Does Not Store Energy
The Core Reluctance is Ideally Zero
Switched Mode Power Conversion

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Magnetic Circuit

Core Shape is Chosen for Convenience
Switched Mode Power Conversion

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Magnetic Circuit

EE Cores
Switched Mode Power Conversion

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Magnetic Circuit

Window Accommodates Both Primary & Secondary
Core Links Both Primary & Secondary
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Voltage Equation

\[ V_1 = N_1 \frac{d\Phi}{dt}; \quad V_2 = N_2 \frac{d\Phi}{dt} \]
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Flux Waveform – Square Wave Primary Voltage

\[ V = N \frac{d\Phi}{dt} \]

Flux Swings Symmetrically Between \(-\Phi_m\) and \(+\Phi_m\)
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Quasi-Square Wave Primary Voltage

\[ V = N \frac{d\Phi}{dt} \]

Flux Swings Symmetrically Between \(-\Phi_m\) and \(+\Phi_m\)

Notice the Dwell Time in Flux
Switched Mode Power Conversion
Transformers
Voltage – Core Area Equation

\[ V = N \frac{2\Phi_m}{T_S / 2} = N \times 4 \times B_m \times A_C \times f_s \]

\[ V_1 = N_1 \times 4 \times B_m \times A_C \times f_s ; V_2 = N_2 \times 4 \times B_m \times A_C \times f_s \]

Flux Swings Symmetrically Between \(-\Phi_m\) and \(+\Phi_m\)
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Voltage – Core Area Equation

\[ V_1 = 4N_1B_mA CF_s \]

\[ V_2 = 4N_2B_mA CF_s \]

\( A_C \) is a Function of Voltage and Frequency
Switched Mode Power Conversion

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Window Space

$A_W$ is a Function of Conductor Size & No. of Turns

$N_1a_1 + N_2a_2 = k_w A_W$

$A_W$ is a Function of Conductor Size & No. of Turns
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Window Space

$A_W$ is a Function of rms Currents

$N_1 \frac{I_1(rms)}{J} + N_2 \frac{I_2(rms)}{J} = k_w A_W$

$A_W$ is a Function of rms Currents
Switched Mode Power Conversion

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Design Equations

\[ V_1 = 4N_1B_m A_C f_S \]

\( V_1/2f_S \) is the Volt-Sec Integral

\( 2N_1B_m A_C \) is the Flux Linkage

\[ 2N_1 \frac{I_{(rms)}}{J} = k_w A_w \]

Primary Ampere-Turns Equal Secondary Ampere-Turns
Switched Mode Power Conversion

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Area-Product Equation

\[
V_1 = 4N_1 B_m A_c f_S
\]

\[
2 N_1 \frac{I_1(rms)}{J} = k_w A_w
\]

\[
A_c A_w = \frac{V_1 I_1(rms)}{2 k_w B_m J f_S}
\]
Switched Mode Power Conversion

Popular Geometry (EE)

Notice the Low Reluctance (No Air Gap in the Core) Geometry
Switched Mode Power Conversion

Popular Geometry (EE)

On Account of Low Reluctance Magnetic Path
Magnetising Inductance is Infinity Ideally

\[ L_{mp} = \frac{N_1^2}{\mathcal{R}} \]

\[ L_{ms} = \frac{N_2^2}{\mathcal{R}} \]
# Switched Mode Power Conversion
## ETD Core Series Data

<table>
<thead>
<tr>
<th>Type Number</th>
<th>AC mm²</th>
<th>AW mm²</th>
<th>AC AW mm⁴</th>
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<tbody>
<tr>
<td>ETD 29/16/10</td>
<td>76</td>
<td>128</td>
<td>9728</td>
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<tr>
<td>ETD 34/17/11</td>
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<td>171</td>
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<td>473</td>
<td>174064</td>
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**Standard Cores**

ETD
Switched Mode Power Conversion

Few Other Geometries (EE)

Standard Cores
ETD

Low Profile Cores
EFD

Pot Cores
P
Switched Mode Power Conversion

Sample Core Selection

\[ V_1 = 48 \text{ V} ; \]
\[ V_2 = 400 \text{ V} ; I_2 = 3 \text{ A} ; \]

High Frequency (50 kHz) Application

\[
A_c A_w = \frac{400 \times 3 \times 10^{12}}{2 \times 0.35 \times 0.2 \times 3 \times 10^6 \times 50000} = 57142 \text{ mm}^4
\]
## Switched Mode Power Conversion

### ETD Core Series Data

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<th>$A_c \text{ mm}^2$</th>
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<th>$A_cA_w \text{ mm}^4$</th>
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Select ETD 49/25/16

N67 Material
Switched Mode Power Conversion

No. of Turns

<table>
<thead>
<tr>
<th>Type Number</th>
<th>$A_C \text{ mm}^2$</th>
<th>$A_W \text{ mm}^2$</th>
<th>$A_{CAW} \text{ mm}^4$</th>
</tr>
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<tr>
<td>$N_1$</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_2$</td>
<td>47</td>
<td></td>
<td></td>
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</table>

$$N = \frac{V}{4B_mCf_S} = \frac{48}{4 \times 0.2 \times 211 \times 10^{-6} \times 50000}$$

Select $N_1 = 6$ Turns ; $N_2 = 47$ Turns
Switched Mode Power Conversion

Wire Size

<table>
<thead>
<tr>
<th>Type Number</th>
<th>$A_C \text{ mm}^2$</th>
<th>$A_W \text{ mm}^2$</th>
<th>$A_C A_W \text{ mm}^4$</th>
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<td>$N_2$</td>
<td>47</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_1$</td>
<td>8.33 mm$^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_2$</td>
<td>1 mm$^2$</td>
<td></td>
<td></td>
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$$a_{w2} = \frac{3}{3} = 1 \text{ mm}^2$$  $$a_{w1} = 8.33 \text{ mm}^2$$
## Switched Mode Power Conversion

### Wire Size – Skin Effect

<table>
<thead>
<tr>
<th>Type Number</th>
<th>( A_C \text{ mm}^2 )</th>
<th>( A_W \text{ mm}^2 )</th>
<th>( A_C A_W \text{ mm}^4 )</th>
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<td></td>
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<td>( N_2 )</td>
<td>47</td>
<td></td>
</tr>
<tr>
<td>( a_1 )</td>
<td>8.33 mm²</td>
<td></td>
</tr>
<tr>
<td>( a_2 )</td>
<td>1 mm²</td>
<td></td>
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</tbody>
</table>

\[
\text{Skin Depth} = \frac{1}{\sqrt{\pi \mu \sigma f}}
\]

\[
\mu = 4\pi 10^{-7} \text{H/m} \quad \sigma = 59.6 \times 10^6 / \Omega \cdot \text{m}
\]

For Copper @50 kHz, Skin Depth = 0.29 mm : 0.26 mm² (24 SWG)
Switched Mode Power Conversion
Primary & Secondary Conductors

<table>
<thead>
<tr>
<th>Type Number</th>
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- $N_1$ = 6
- $N_2$ = 47
- $a_1 = 8.33 \text{ mm}^2$
- $a_2 = 1 \text{ mm}^2$

For Secondary 4 wires of 24 SWG in Parallel

For Primary 32 wires of 24 SWG in Parallel
Switched Mode Power Conversion

Transformers

Parasitic Resistance

Mean Length of a Turn: 83 mm

Primary Resistance: 10 $\mu\Omega$
Secondary Resistance: 629 $\mu\Omega$
Switched Mode Power Conversion

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Core Loss

Loss at 0.2 T at 100 kHz: 15.5 W
Loss at 0.2 T at 50 kHz: 7.75 W

Core Loss Resistance: 297 Ω

ETD 49/25/16
N67 Material
Switched Mode Power Conversion
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Magnetising Inductance

Primary Inductance: 0.133 mH
Secondary Inductance: 8.17 mH

le = 114 mm; A_C = 211 mm^2; \mu_r = 1590
Reluctance = 270405 /H
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Primary Magnetising Current Peak

\[ I_m(\text{peak}) = \frac{48}{2 \times 0.133 \mu} \times 10 = 1.8 \text{A} \]

Magnetising Current Peak << Primary Current
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Stored Energy in the Core

\[ E_m = \frac{L_m I_m^2}{2} = 0.5 \times 0.133 \times 1.8^2 = 0.22 \, \text{mJ} \]

Stored Energy in the Core = 0.22 mJ
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Leakage Energy in the Window

Leakage is Another Non-ideality in the Magnetic Circuit
Switched Mode Power Conversion

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Leakage Energy in the Window

Leakage is Another Non-ideality in the Magnetic Circuit
Switched Mode Power Conversion

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Leakage Energy in the Window

\[ H(x) = \frac{N_1 I_1 x}{h b} \]

\[ B(x) = \frac{\mu_0 N_1 I_1 x}{h b} \]

\[ L_\ell = \frac{\mu_0}{I_1^2} \iiint H^2(x) \, dv \]

\[ dv = 2hw \, dx \]
Switched Mode Power Conversion

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Leakage Energy in the Window

\[ L = \frac{\mu_0 N_1^2}{b^2} \int_0^b x^2 \, dx \]

\[ L = \frac{2\mu_0 N_1^2 b w}{3 \cdot h} \]

0.07 \, \mu H \ll 133 \, \mu H

\[ L_{pu} = 6.1 \, \mu H \]
Switched Mode Power Conversion
Transformers
Circuit Model with Dominant Non-idealities
Switched Mode Power Conversion

Transformers

Measurement of Circuit Parameters

Open Circuit & Short Circuit Test
Switched Mode Power Conversion

Transformers

Devices for Efficient Power Conversion

Switches
Inductors
Transformers
Capacitors
Switched Mode Power Conversion

Capacitors

Devices for Efficient Power Conversion

Switches
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