Switched Mode Power Conversion

Inductors

Devices for Efficient Power Conversion

Switches
Inductors
Transformers
Capacitors
Switched Mode Power Conversion
   Inductors

   Inductors Store Energy
   Inductors Store Energy in a Magnetic Field

   In Power Converters Energy Storage
   Elements Smoothen Power Flow
Switched Mode Power Conversion

Inductors

Inductors are also used as Switch Protection Elements

In Switching Aid Networks Inductors Provide Turn-On di/dt Protection
Switched Mode Power Conversion

Inductors

\[ V = L \frac{dI}{dt} \]

Electrical Circuit Element Equation

\( V, L, I \) are Electrical Circuit Quantities
Switched Mode Power Conversion

Inductors

Electromagnetic Equation – Faraday’s Law

\[ V = N \frac{d\Phi}{dt} \]

\( V \) is Electrical Circuit Quantity
\( N, \Phi \) are Magnetic Circuit Quantities
Switched Mode Power Conversion

Inductors

\[ V = L \frac{dI}{dt} \]

\[ V = N \frac{d\Phi}{dt} \]

\[ LI = N\Phi \]

Relationship Between \( L, I \) & \( N, \Phi \)

\( N\Phi \) is also defined as Flux Linkage in the Inductor
Switched Mode Power Conversion

Inductors

\[ V = L \frac{dl}{dt} \]

\[ LI = N \Phi \]

\[ V = N \frac{d\Phi}{dt} \]

How to Relate the Electrical Circuit \( V, L, I \) and The Magnetic Circuit of the Inductor \( N, \Phi, I \) & \( R \)
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Ohm’s Law

\[ V = IR \]

Bulk Ohm’s Law
Switched Mode Power Conversion

Ohm’s Law at a Point

\[ V = I R = J A \frac{\rho \ell}{A} = J \rho \ell \]

\[ \frac{V}{\ell} = J \rho \]

\[ J = \sigma \varepsilon \]
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Ohm’s Law at a Point

\[ J = \sigma E \]

In a Conducting Material
Electrical Current Density \((J)\) is Proportional to \((\sigma)\), and
Electrical Field Intensity \((E)\)
\(\sigma\) is the conductivity of the material
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Magnetic Ohm’s Law at a Point

\[ B = \mu H \]

In a Magnetic Material
Magnetic Flux Density (B) is Proportional to (\( \mu \)), and
Magnetic Field Intensity (H)

\( \mu \) is the permeability of the material
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Dielectric Ohm’s Law at a Point

\[ D = \varepsilon E \]

In a Dielectric Material
Dielectric Flux Density (D)
is Proportional to (\varepsilon), and
Electrical Field Intensity (E)
\( \varepsilon \) is the permittivity of the material
Switched Mode Power Conversion

Magnetic Ohm’s Law

\[ BA = \mu A \frac{NI}{\ell} = \Phi = \frac{NI}{\ell/A\mu} = \frac{F}{A} \]

\[ \Phi = \frac{F}{A} \]

Bulk Magnetic Ohm’s Law
Switched Mode Power Conversion

Magnetic Circuit Relationships

 Flux in Magnetic Circuit

$$\Phi = \frac{F}{A} = \frac{(NI)}{\ell/A\mu}$$
Inductance of an Electromagnetic Circuit

\[ L = \frac{N \Phi}{I} = \frac{N^2}{\left( \frac{l}{A\mu} \right)} = \frac{N^2 A\mu}{l} \]

Inductance of a Magnetic Circuit
Switched Mode Power Conversion

Conceptual Design of an Inductor

For the desired $L$ and $I$, To select a magnetic core $(A, \mu, l)$, number of turns $N$ and conductor size $a$
Switched Mode Power Conversion
Design of an Inductor

$\mu$ of a magnetic material varies widely. $I$ is not finely controllable.
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Practical Design of an Inductor

Magnetic path is made of:
Large Path of High Permeability and a Small Controlled Path of Low Permeability (Air)
Switched Mode Power Conversion
Practical Design of an Inductor

Inductance is Independent of Core Material ($\mu$)
Inductance is Independent of Core Shape ($l_m$)

$$L = \frac{N\Phi}{I} = \frac{N^2}{\left(\frac{l_m}{A\mu_o\mu_m} + \frac{l_g}{A\mu_o}\right)} \approx \frac{N^2 A\mu_o}{l_g}$$
Switched Mode Power Conversion

Popular Geometry of Inductor (EE)

EE Cores
Switched Mode Power Conversion

Popular Geometry of Inductor (EE)

$A_C$ is the area of Magnetic Path supporting the Flux

$A_W$ is the area of Window Supporting the Electric Current
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Definition of Inductance

Inductance is defined as Flux Linkage per Ampere. The slope of \( N\Phi \) vs \( I \) curve is the Inductance.
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Saturation Effect

Flux Linkage Saturates in Most Magnetic Materials Beyond a Certain Level
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Saturation Flux Density

Peak Flux Density in Magnetic Materials is Limited by $B_m$
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Saturation Limit

\[ L I_m = N \Phi_m = N B_m A_C \]

Magnetic Path Constraint
Switched Mode Power Conversion

Heat Produced in the Conductors

\[ Q = I^2Rt \]

\( I^2R \) Heating in the Conductors
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Thermal Limit

\[ N I_{\text{rms}} = k_w A_w J \]

Current Carrying Capacity Constraint
Switched Mode Power Conversion

Energy Equation

\[ NI_{\text{rms}} = k_w A_w J \]

\[ LI_m = N\Phi_m = N B_m A_C \]

\[ \frac{LI_m I_{\text{rms}}}{k_w B_m J} = A_C A_w \]

Energy (Area Product) Equation for Inductor Design
Switched Mode Power Conversion

Size of Magnetic Core

\[
\frac{L I_m I_{\text{rms}}}{k_w B_m J} = A_c A_w
\]

The Magnetic Core has to be Big Enough to Handle the Stored Energy in the Core

The Core Size is Inversely Proportional to Saturation Flux Density \(B_m\), Peak Current Carrying Capacity \(J\), Window Space factor \(k_w\)
Switched Mode Power Conversion

Typical Design Constraints

The Core Size is Inversely Proportional to Saturation Flux Density $B_m$

$B_m = 0.2 \ T$ for Ferrite Cores

Peak Current Carrying Capacity $J$

$J = 3 \ A/mm^2$ for Copper, Natural Cooling

Window Space factor $k_w$

$k_w = 0.35$ for round Conductors
Switched Mode Power Conversion

Sample Core Selection

\[ L = 20 \, \mu H; \, \text{DC Current of} \, 5 \, A; \]

\[ \text{High Frequency (20 kHz) Application} \]

\[ I_m = 5 \, A; \, I_{\text{rms}} = 5 \, A; \, L = 20 \, \mu H \]

\[ A_c A_w = \frac{20 \mu \ast 5 \ast 5}{0.35 \ast 0.2 \ast 3 \ast 10^6} = 2381 \, mm^4 \]
Switched Mode Power Conversion

Sample Core Selection

<table>
<thead>
<tr>
<th>Type Number</th>
<th>$A_C$</th>
<th>$A_W$</th>
<th>$A_{C,A_W}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mm²</td>
<td>mm²</td>
<td>mm⁴</td>
</tr>
<tr>
<td>E16/8/5</td>
<td>20.1</td>
<td>37.6</td>
<td>755.8</td>
</tr>
<tr>
<td>E21/9/5</td>
<td>21.6</td>
<td>66.0</td>
<td>1425.6</td>
</tr>
<tr>
<td>E20/10/6</td>
<td>32.1</td>
<td>57.4</td>
<td>1842.5</td>
</tr>
<tr>
<td><strong>E25.4/10/7</strong></td>
<td><strong>38.2</strong></td>
<td><strong>80.0</strong></td>
<td><strong>3056.0</strong></td>
</tr>
<tr>
<td>E25/13/7</td>
<td>52.5</td>
<td>87.0</td>
<td>4567.5</td>
</tr>
</tbody>
</table>

Core Data Sheet

Select Core E25.4/10/7
Switched Mode Power Conversion

No. of Turns

<table>
<thead>
<tr>
<th>Type Number</th>
<th>$A_C \text{ mm}^2$</th>
<th>$A_W \text{ mm}^2$</th>
<th>$A_C A_W \text{ mm}^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>E25.4/10/7</td>
<td>38.2</td>
<td>80.0</td>
<td>3056.0</td>
</tr>
<tr>
<td>L</td>
<td>2.00E-05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>13</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
N = \frac{LI}{B_mA_C} = \frac{20 \times 10^{-6} \times 5}{0.2 \times 38.2 \times 10^{-6}}
\]

Select 13 Turns
### Switched Mode Power Conversion

#### Wire Size

<table>
<thead>
<tr>
<th>Type Number</th>
<th>$A_C \text{ mm}^2$</th>
<th>$A_W \text{ mm}^2$</th>
<th>$A_C A_W \text{ mm}^4$</th>
</tr>
</thead>
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<tr>
<td>I</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_W \text{ mm}^2$</td>
<td>1.67</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
a_W = \frac{I}{J} = \frac{5}{3} = 1.67 \text{ mm}^2
\]

Select $a_W$ 1.67 mm$^2$ which is 16 SWG
### Switched Mode Power Conversion

#### Air Gap

<table>
<thead>
<tr>
<th>Type Number</th>
<th>$A_C \text{ mm}^2$</th>
<th>$A_W \text{ mm}^2$</th>
<th>$A_C A_W \text{ mm}^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>E25.4/10/7</td>
<td>38.2</td>
<td>80.0</td>
<td>3056.0</td>
</tr>
<tr>
<td>$L$</td>
<td>2.00E-05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I$</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_W \text{ mm}^2$</td>
<td>1.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$l_g \text{ mm}$</td>
<td>0.41</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$$l_g = \frac{N^2 A \mu}{L}$$

Select Airgap of 0.205 mm on each limb
Switched Mode Power Conversion

A Few Other Geometries (EE)

- Low Profile Cores
  - ELP

- Low Profile Cores
  - EFD

- Pot Cores
  - P
Switched Mode Power Conversion

Back to the Design

$L = 20 \, \mu H; \, DC \, Current \, of \, 5 \, A; \newline 13 \, Turns \, of \, 16 \, SWG \newline Core \, Type \, E25.4/10/7 \newline Airgap \, 0.205 \, mm \, x \, 2$
Switched Mode Power Conversion
Practical Design of an Inductor

Inductance is Independent of Core Material ($\mu$)
Inductance is Independent of Core Shape ($l_m$)
Switched Mode Power Conversion

Back to the Design

Assumptions:
Core Reluctance << Gap Reluctance
Fringing Field at Gap is Negligible
Switched Mode Power Conversion

Core Reluctance : Gap Reluctance

\[
\left( \frac{l_m}{\mu_m} \ll l_g \right)
\]

\[
\frac{47.2}{1510} = 0.031 \ll 0.205
\]
Switched Mode Power Conversion

Fringing Effect

\[
\ell_g \ll \sqrt{A_C}
\]

0.205 \ll 6.2
Switched Mode Power Conversion

Parasitic Resistance of Inductor

\[ R_\ell = \frac{\rho L_w}{a_w} \]

\[ R_\ell = \frac{1.76 \times 10^{-6} \times 13 \times 40 \times 10^{-1}}{1.67 \times 10^{-2}} \Omega = 5.5 \text{ m} \Omega \]
Switched Mode Power Conversion

Losses in the Inductor

If the DC Output Voltage is 7.5 V, this loss is 0.1%.

0.36 W per set
Switched Mode Power Conversion

Measurement of L

\[ V = L \frac{dI}{dt} \]

Pulsed Voltage and Current Rise
Switched Mode Power Conversion

Measurement of L with LCR Meter

\[ \vec{I} = \frac{\vec{V}}{j\omega L} \]

\[ |\vec{V}| = \omega L \]

Sinusoidal Voltage and Current
Switched Mode Power Conversion

Impedance as a Function of $\omega$

$\vec{Z} = \frac{\vec{V}}{\vec{I}} = j\omega L$

Impedance Plot [$dB\Omega$ vs $log (\omega)$]

$\omega = \frac{1}{L}$

$20 dB\Omega/\text{dec}$
Switched Mode Power Conversion

Inductors

Devices for Efficient Power Conversion

Switches

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