An introduction to coding theory

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Lecture #1A: Introduction to error control coding-I
Books

Textbook:

Books

Textbook:

References:
Books

Textbook:

References:
All communications involves three basic steps

- Encoding a message at its source.
- Transmitting that message through a communication medium.
All communications involves three basic steps

- Encoding a message at its source.
- Transmitting that message through a communication medium.
- Decoding the message at its destination.

The transmission medium in communication is known as channel.

Binary Symmetric Channel

Binary Erasure Channel

The channel capacity is a measure of the amount of information that can be conveyed between the input $X$ and the output $Y$ of a channel.
In his landmark paper in 1948, A Mathematical Theory of Communication, in Bell System Technical Journal, Shannon introduced the concept of channel capacity.

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Shannon in his celebrated noisy channel coding theorem proved the existence of channel coding schemes that can achieve an arbitrarily low error probability as long as the information can be transmitted across the channel at a rate less than the channel capacity, C.

Example: If the channel capacity of a particular communication link is (say) 2 Gbps. We can communicate over this channel at any desired rate less than 2 Gbps, and achieve arbitrary low error rates.
Introduction

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- A channel code is designed by properly adding redundant bits (parity bits) to the source bits. These redundant bits facilitate the detection and correction of transmission (storage) errors.
- Channel coding is used in digital communication systems to control transmission errors caused by channel noise, fading, interference.
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- The goal of coding theory is to reach this limit.
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- Channel coding is used in digital communication systems to control transmission errors caused by channel noise, fading, interference.
- In digital storage systems, channel coding is used to control errors caused by storage medium defects, dust particles, radiation.

Channel Coding

- Example: Repetition codes
Channel Coding

- Example: Repetition codes
  - Rate R=1/2 code
    
    
    $0 \rightarrow 00 \quad 1 \rightarrow 11$

- Rate R=1/3 code
  
  $0 \rightarrow 000 \quad 1 \rightarrow 111$
Example (contd.)

Information bits: 0 0 1 1 0 1
Coded bits using Rate 1/2 Repetition codes: 00 00 11 11 00 11
Received coded bits (Single Error): 10 00 11 11 00 11
Received coded bits (Double Error): 11 00 11 11 00 11

Example (contd.)

Information bits: 0 0 1 1 0 1
Coded bits using Rate 1/3 Repetition codes: 000 000 111 111 000 111
Received coded bits (Single Error): 100 000 111 111 000 111
Received coded bits (Double Error): 110 000 111 111 000 111
Channel Coding

A message of content and clarity has gotten to be quite a rarity. To combat the terror of serious error use bits of appropriate parity.

– Solomon Golomb