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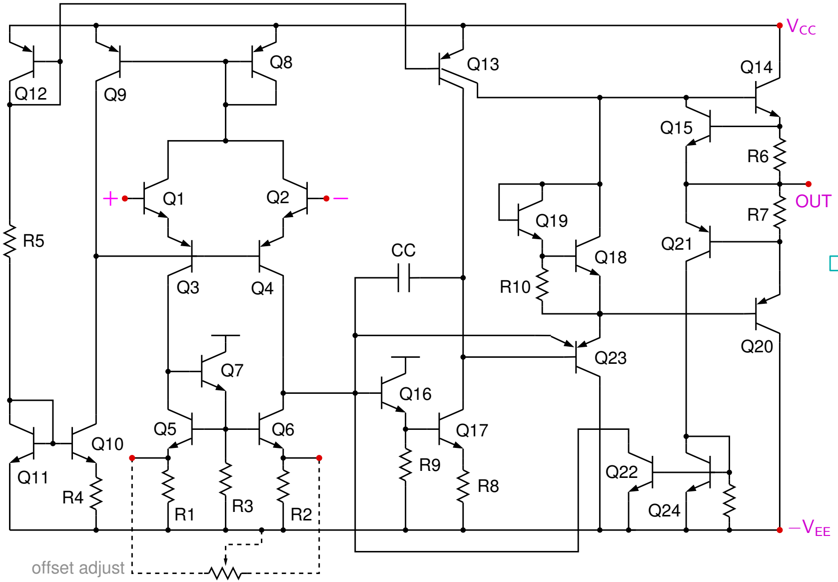
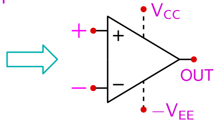
* Amplifiers built with op-amps work with DC input voltages as well $\rightarrow$ useful in sensor applications (e.g., temperature, pressure)
* The user can generally carry out circuit design without a thorough knowledge of the intricate details of an op-amp. This makes the design process simple.

# Op-Amp 741

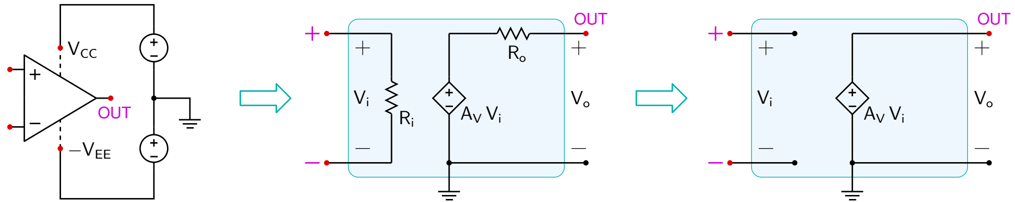


Vcc
Q8
Q9
Q12
Q13
Q14
Q15
R6
OUT
R7
Q21
Q1
Q2
+
−
R5
Q3
Q4
CC
Q19
Q18
R10
Q20
Q23
Q7
Q16
Q5
Q6
Q17
Q10
Q11
R4
R1
R3
R2
R9
R8
Q22
Q24
−VEE
offset adjust

Symbol
VCC
+
−
OUT
−VEE

# Op-amp: equivalent circuit



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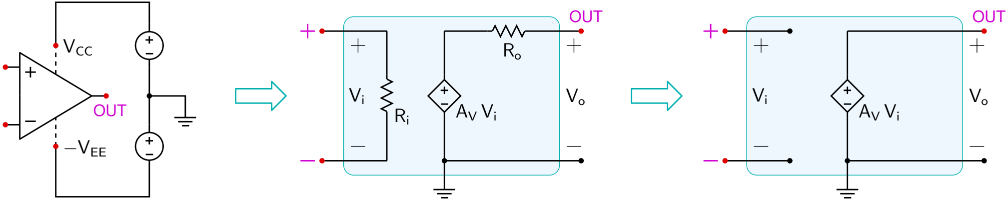


* The external resistances ($\sim$ a few k$\Omega$) are generally much larger than $R_o$ and much smaller than $R_i$ $\rightarrow$ we can assume $R_i \rightarrow \infty$, $R_o \rightarrow 0$ without significantly affecting the analysis.

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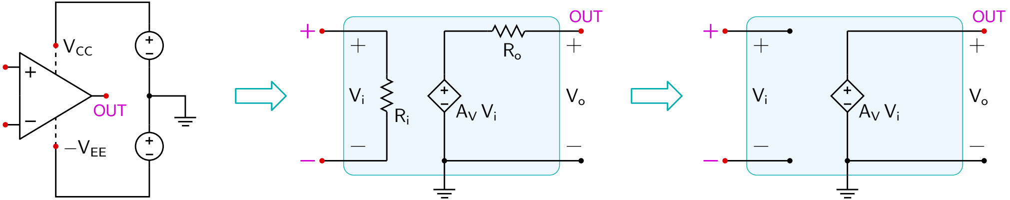


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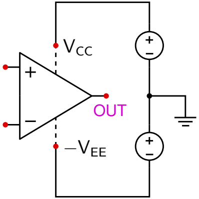


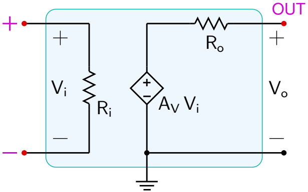


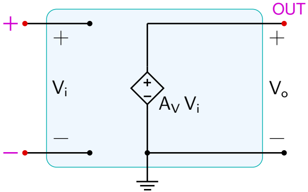


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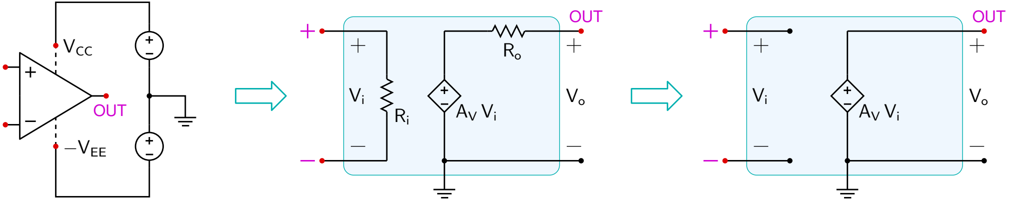


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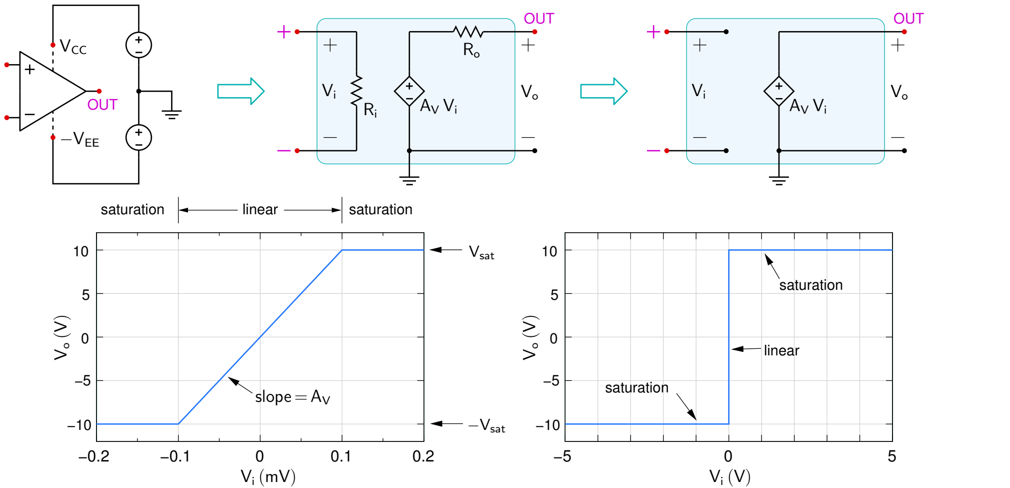
*

| Parameter | Ideal Op-Amp | 741 |
|---|---|---|
| $A_V$ | $\infty$ | $10^5$ (100 dB) |
| $R_i$ | $\infty$ | 2 M$\Omega$ |
| $R_o$ | 0 | 75 $\Omega$ |

# Op-Amp: equivalent circuit



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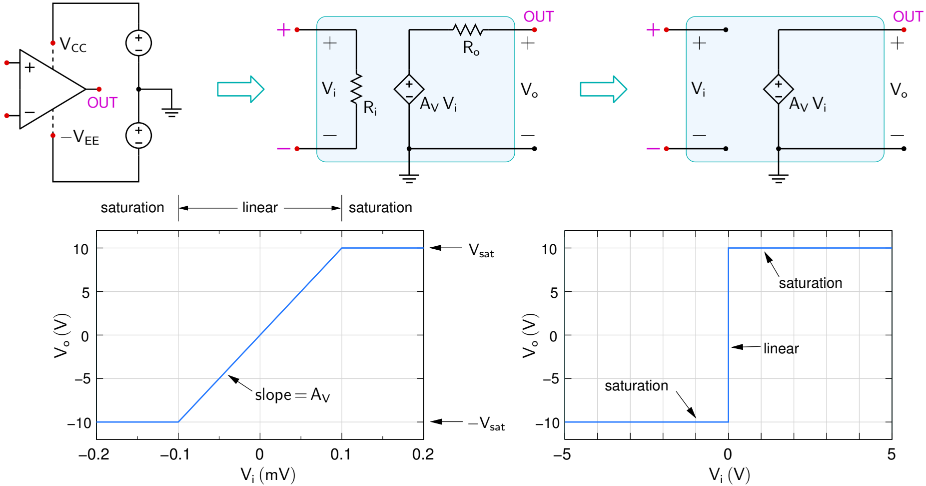
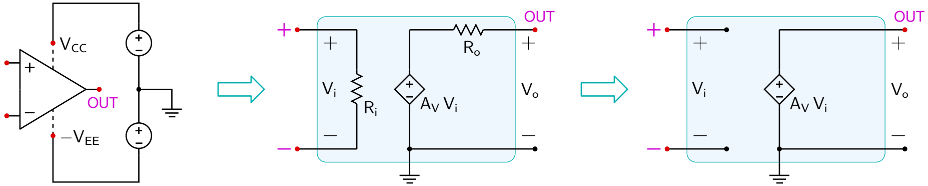
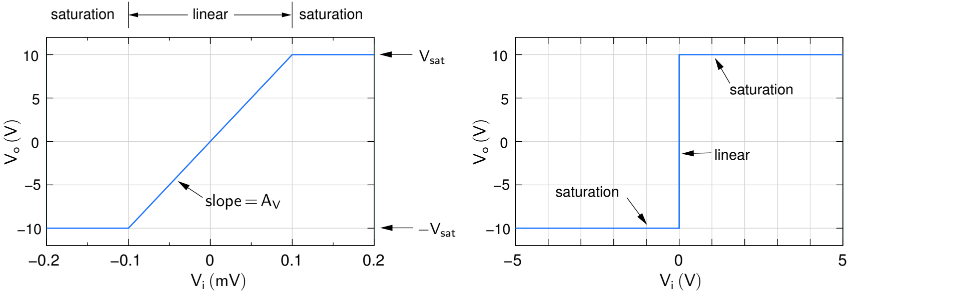


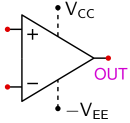

* The output voltage $V_o$ is limited to $\pm V_{\text{sat}}$, where $V_{\text{sat}} \sim 1.5\ V$ less than $V_{CC}$.

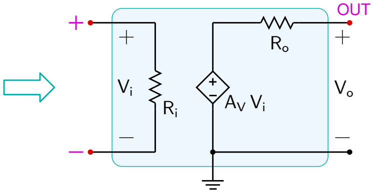


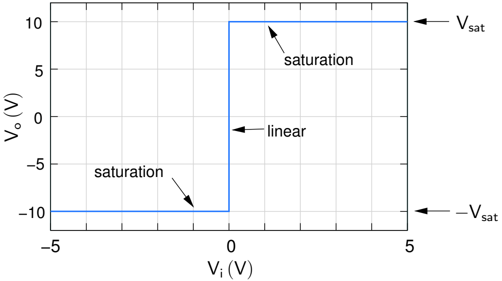



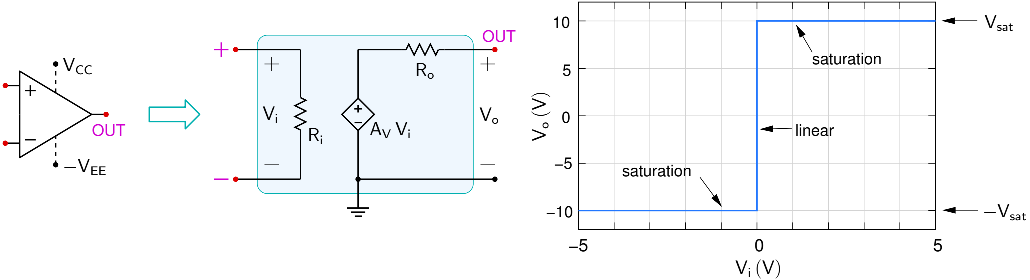

* The output voltage $V_o$ is limited to $\pm V_{\rm sat}$, where $V_{\rm sat} \sim 1.5\ V$ less than $V_{CC}$.
* For $-V_{\rm sat} < V_o < V_{\rm sat}$, $V_i = V_+ - V_- = V_o/A_V$, which is very small
  $\rightarrow$ $V_+$ and $V_-$ are *virtually* the same.

# Op-amp circuits



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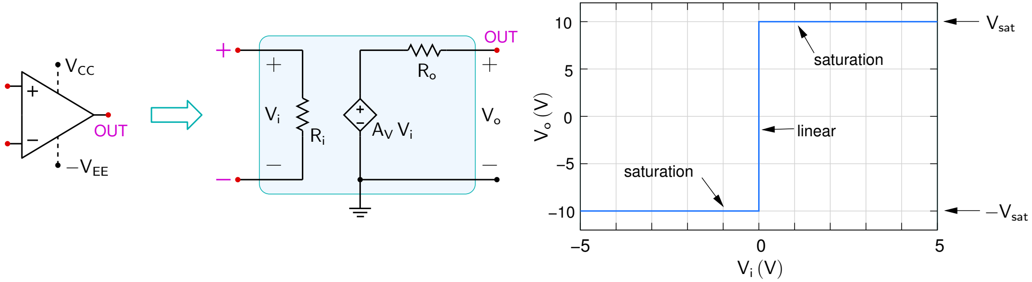


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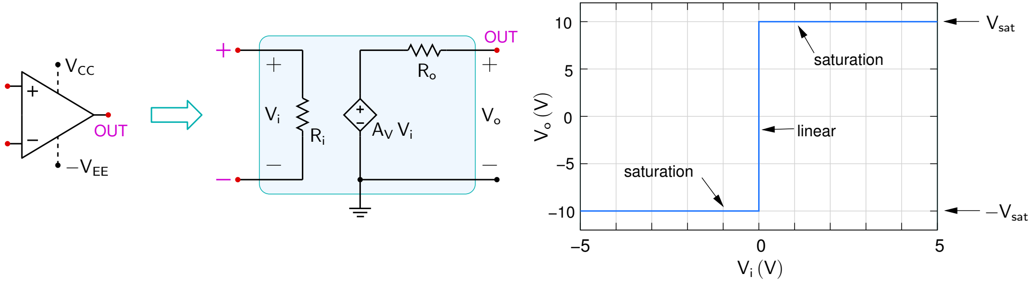


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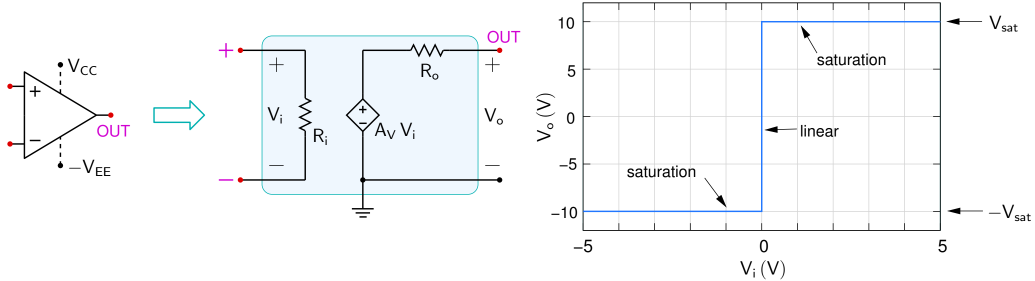


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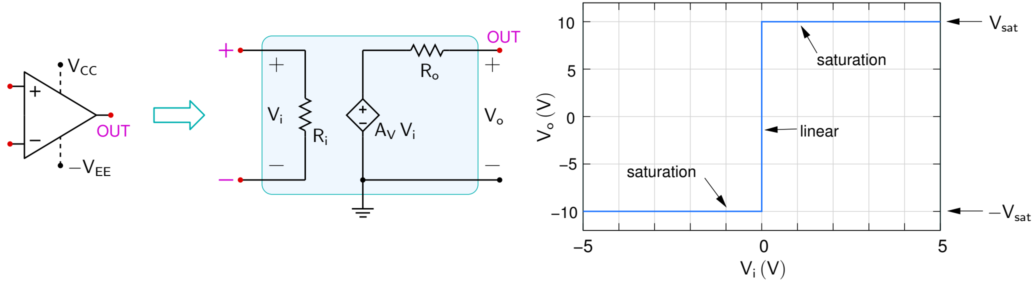


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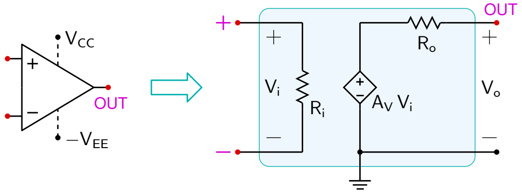


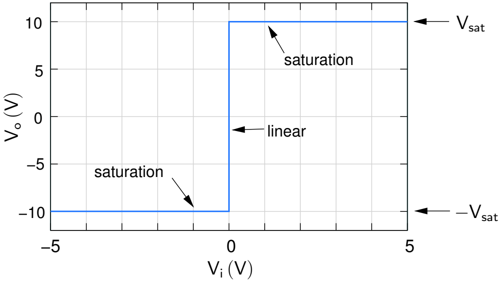


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  - op-amp operating in the linear region
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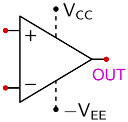
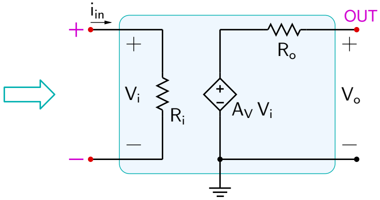
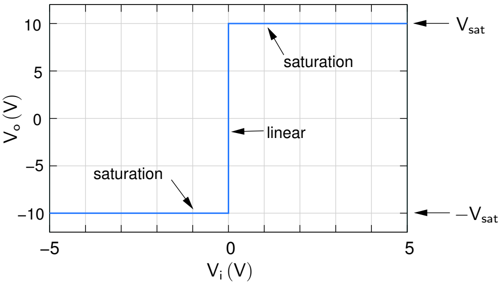
  - type of feedback (negative or positive)
    (We will take a qualitative look at feedback later.)

# Op-amp circuits (linear region)



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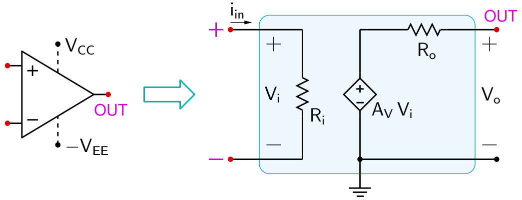
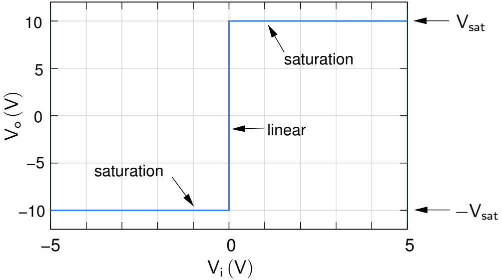
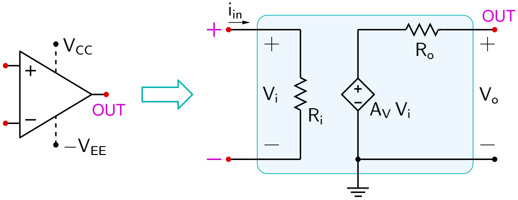


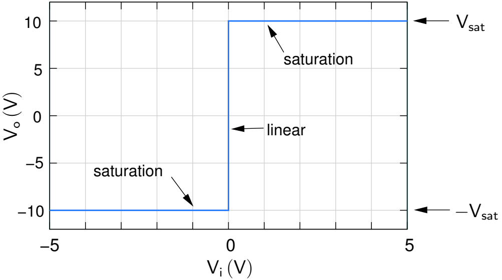




In the linear region,

* $V_o = A_V\,(V_+ - V_-)$, i.e., $V_+ - V_- = V_o/A_V$, which is very small

  $\rightarrow$ $\boxed{V_+ \approx V_-}$

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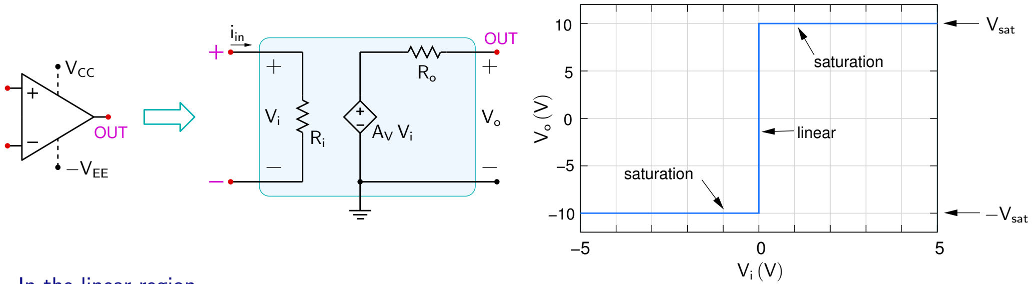


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These two "golden rules" enable us to understand several op-amp circuits.

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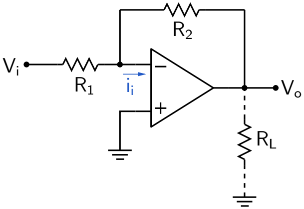

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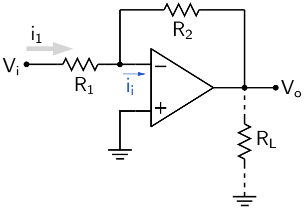


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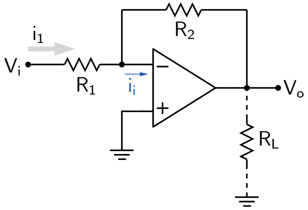


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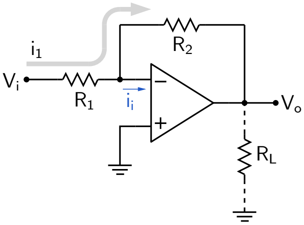


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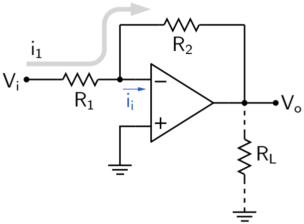


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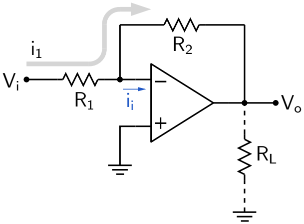


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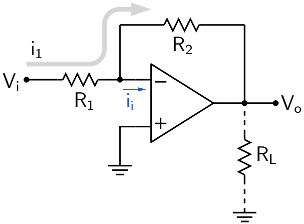


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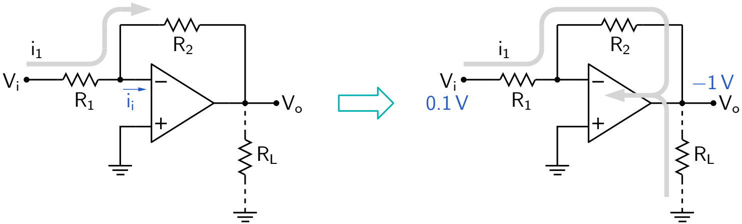
The circuit is called an “inverting amplifier.”

Where does the current go?

# Op-amp circuits (linear region)

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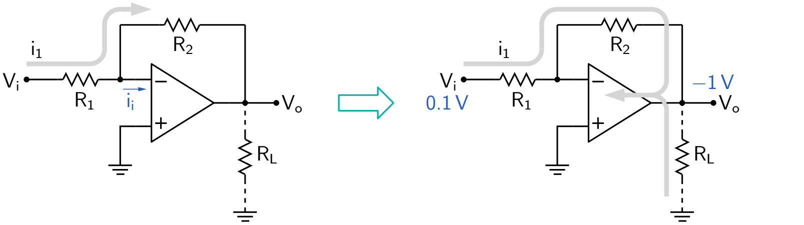


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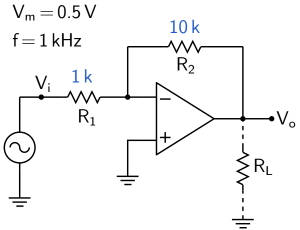
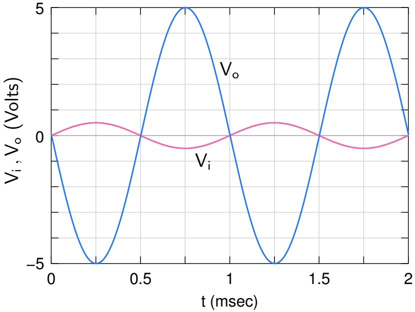
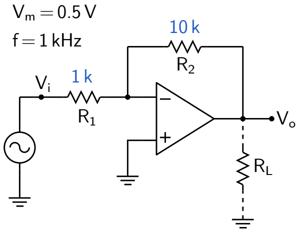
Where does the current go?

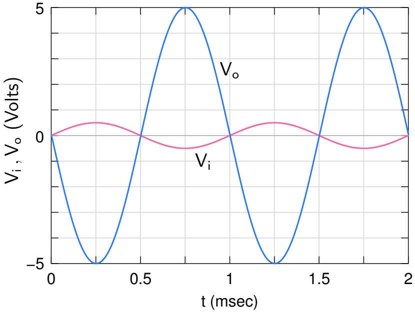
(Op-amp 741 can source or sink about 25 mA.)

# Op-amp circuits: inverting amplifier



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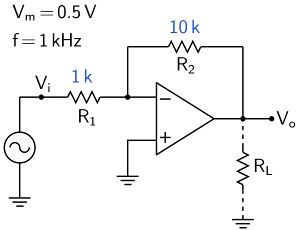


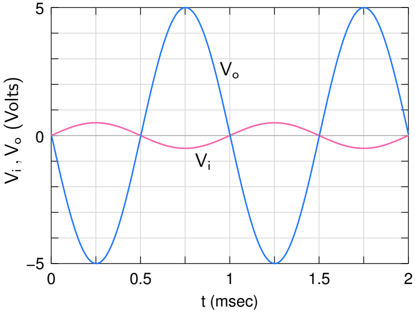


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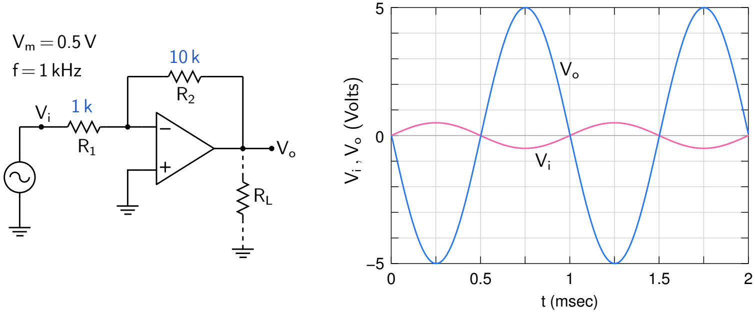
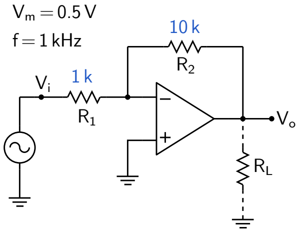
* The gain can be adjusted simply by changing $R_1$ or $R_2$ !

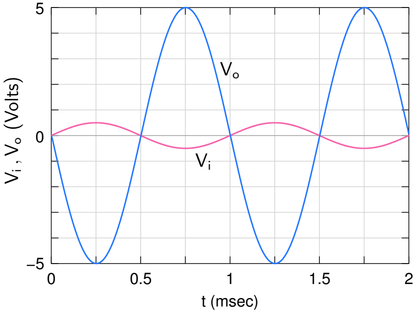
# Op-amp circuits: inverting amplifier




* The gain of the inverting amplifier is $-R_2/R_1$. It is called the "closed-loop gain" (to distinguish it from the "open-loop gain" of the op-amp which is $\sim 10^5$).
* The gain can be adjusted simply by changing $R_1$ or $R_2$ !
* For the common-emitter amplifier, on the other hand, the gain $-g_m\,(R_C \parallel R_L)$ depends on how the BJT is biased (since $g_m$ depends on $I_C$).

# Op-amp circuits: inverting amplifier

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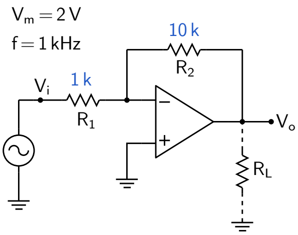
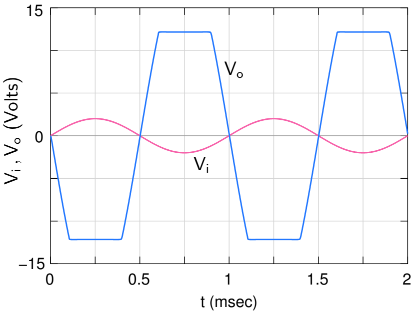
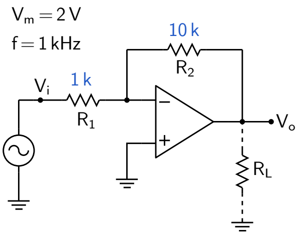
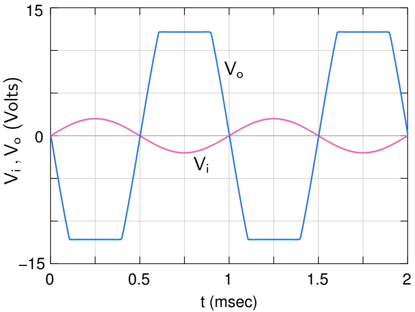
(SEQUEL file: `ee101_inv_amp_1.sqproj`)

# Op-amp circuits: inverting amplifier

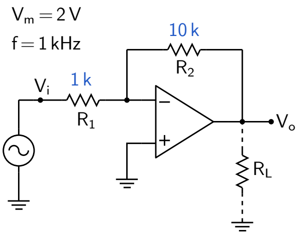


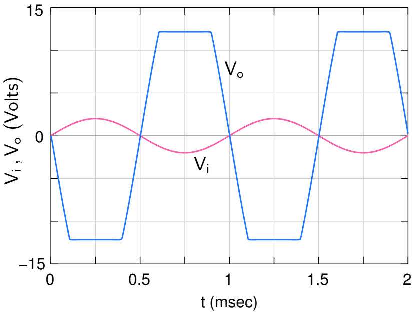





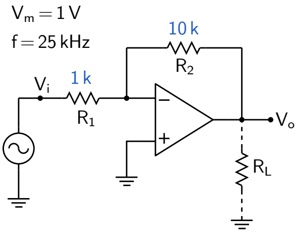




* The output voltage is limited to $\pm V_{sat}$.

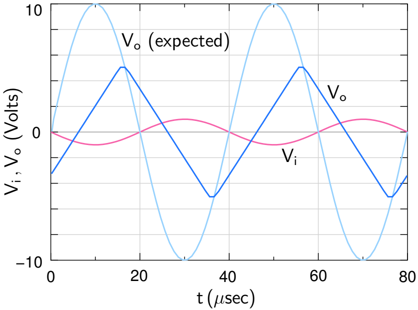


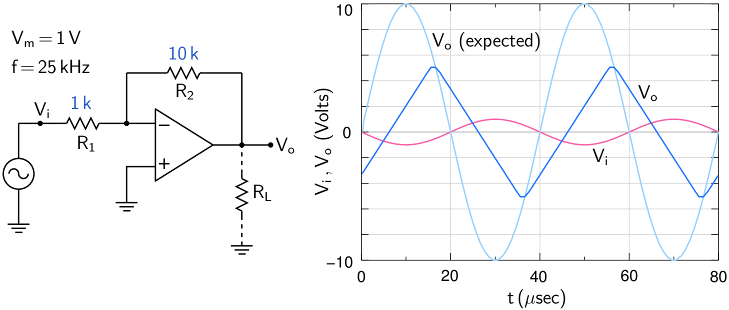

* The output voltage is limited to $\pm V_{sat}$.
* $V_{sat}$ is $\sim 1.5\,V$ less than the supply voltage $V_{CC}$.



$V_m = 1\,V$
$f = 25\,kHz$
10 k
$R_2$
$V_i$
1 k
$R_1$
$V_o$
$R_L$

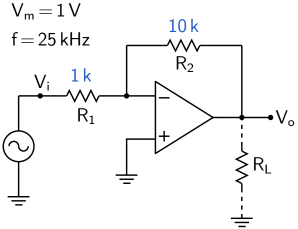



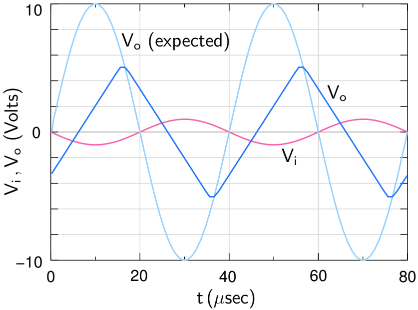
10
0
−10
0
20
40
60
80
$V_o$ (expected)
$V_o$
$V_i$
$V_i$, $V_o$ (Volts)
t ($\mu$sec)



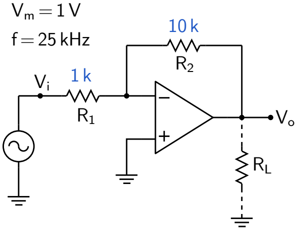

* If the signal frequency is too high, a practical op-amp cannot keep up with the input due to its "slew rate" limitation.

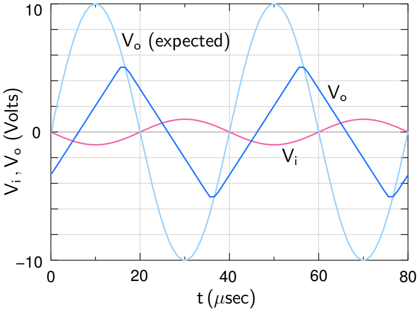
# Op-amp circuits: inverting amplifier




* If the signal frequency is too high, a practical op-amp cannot keep up with the input due to its "slew rate" limitation.
* The slew rate of an op-amp is the maximum rate at which the op-amp output can rise (or fall).

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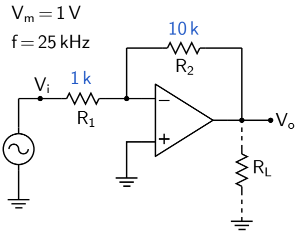
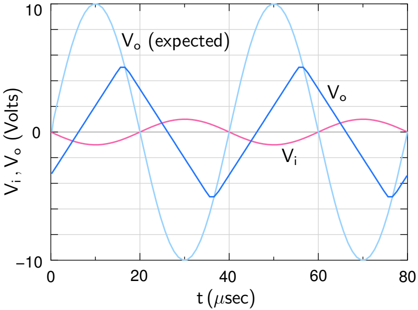
* For the 741, the slew rate is 0.5 $V/\mu$sec.

# Op-amp circuits: inverting amplifier

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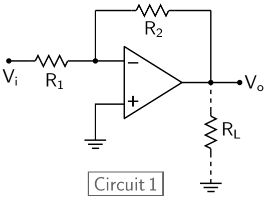
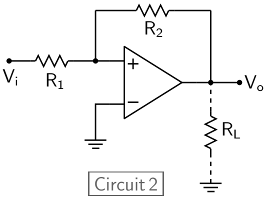
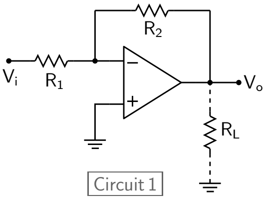
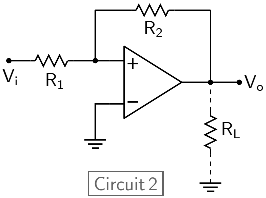
(SEQUEL file: `ee101_inv_amp_2.sqproj`)




What if the $+$ (non-inverting) and $-$ (inverting) inputs of the op-amp are interchanged?

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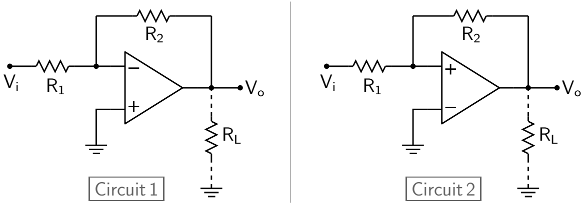
Our previous analysis would once again give us $V_o = -\dfrac{R_2}{R_1}\,V_i$ .

# Op-amp circuits: inverting amplifier

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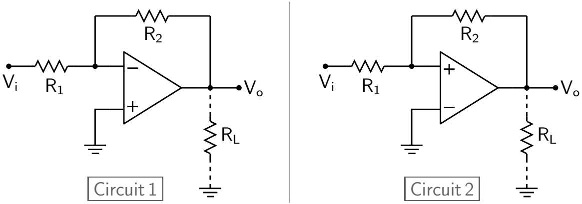
However, from Circuit 1 to Circuit 2, the nature of the feedback changes from negative to positive.

$\rightarrow$ Our assumption that the op-amp is working in the linear region does not hold for Circuit 2, and $V_o = -\dfrac{R_2}{R_1}\, V_i$ does not apply any more.

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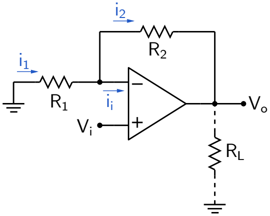
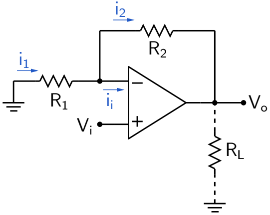
(Circuit 2 is also useful, and we will discuss it later.)

# Op-amp circuits (linear region)



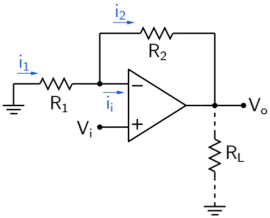

* $V_+ \approx V_- = V_i$

# Op-amp circuits (linear region)




* $V_+ \approx V_- = V_i$

  $\rightarrow i_1 = (0 - V_i)/R_1 = -V_i/R_1$ .




* $V_+ \approx V_- = V_i$

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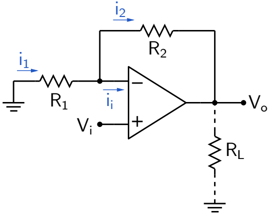
* Since $i_i = 0$, $i_2 = i_1 \rightarrow V_o = V_- - i_2\, R_2 = V_+ - i_1\, R_2 = V_i - \left(-\dfrac{V_i}{R_1}\right) R_2 = V_i \left(1 + \dfrac{R_2}{R_1}\right)$ .

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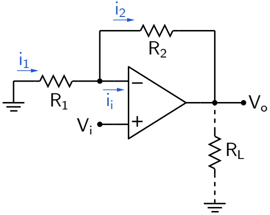
* This circuit is known as the "non-inverting amplifier."

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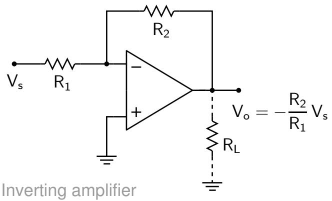
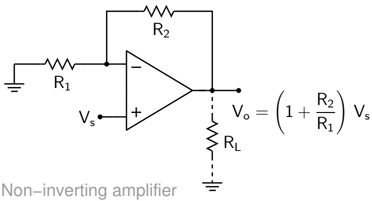
* This circuit is known as the "non-inverting amplifier."
* Again, interchanging + and − changes the nature of the feedback from negative to positive, and the circuit operation becomes completely different.



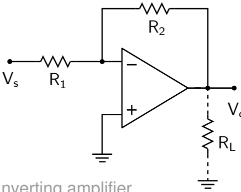

Inverting amplifier




Non–inverting amplifier

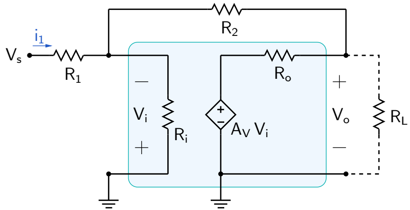
* If the sign of the output voltage is not a concern, which configuration should be preferred?

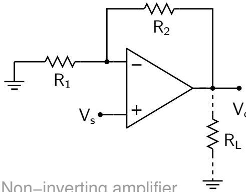
# Inverting or non-inverting?




$$V_o = -\frac{R_2}{R_1} V_s$$

Inverting amplifier




$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_s$$

Non–inverting amplifier

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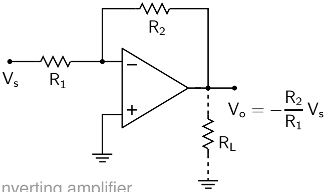
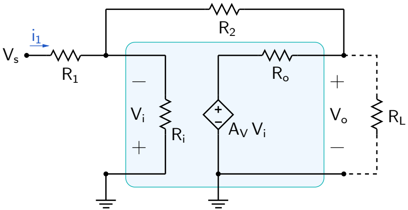
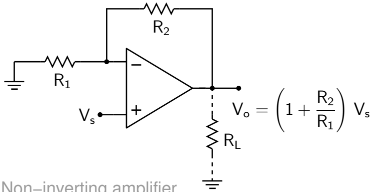


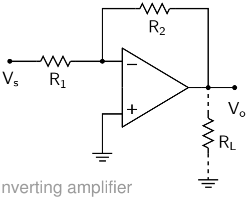

Inverting amplifier



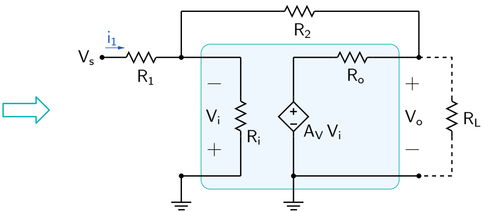

Non–inverting amplifier

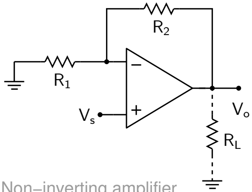
* If the sign of the output voltage is not a concern, which configuration should be preferred?
* For the inverting amplifier, since $V_- \approx 0\ V$, $i_1 = V_s/R_1 \rightarrow R_{in} = V_s/i_1 = R_1$ .




$V_o = -\dfrac{R_2}{R_1}\, V_s$

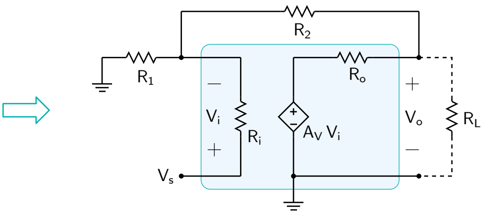
Inverting amplifier




$V_o = \left(1 + \dfrac{R_2}{R_1}\right) V_s$

Non–inverting amplifier

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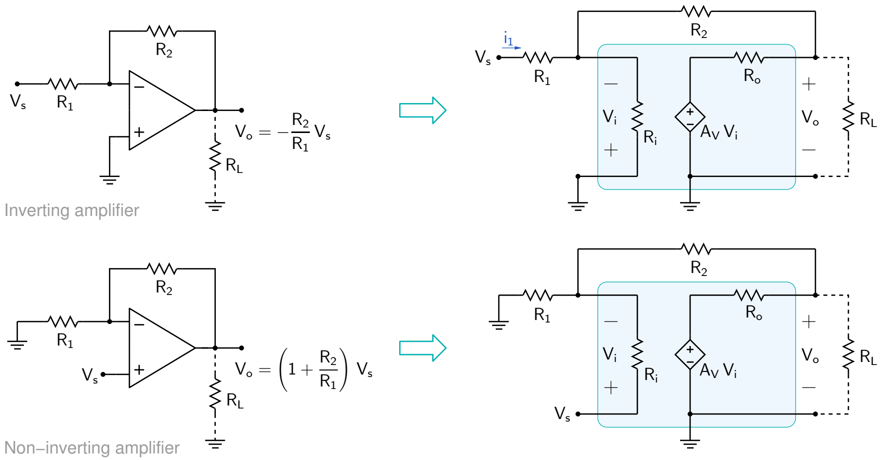
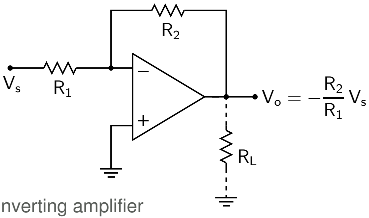
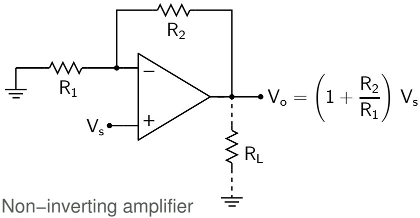
# Inverting or non-inverting?




* If the sign of the output voltage is not a concern, which configuration should be preferred?
* For the inverting amplifier, since $V_- \approx 0\ V$, $i_1 = V_s/R_1 \rightarrow R_{in} = V_s/i_1 = R_1$ .
* For the non-inverting amplifier, $R_{in} \sim R_i\, A_V \dfrac{R_1}{R_1 + R_2}$. Huge!

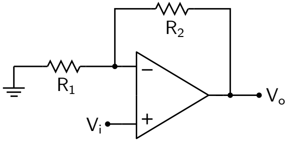



Inverting amplifier

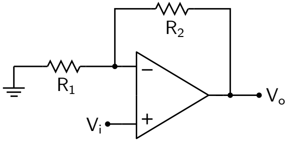


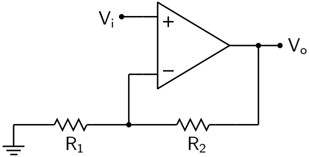

Non–inverting amplifier

# Non-inverting amplifier

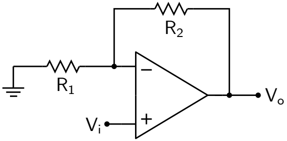


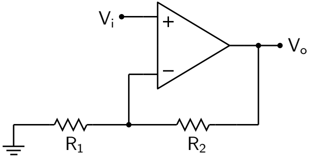
# Non-inverting amplifier

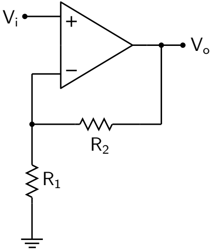


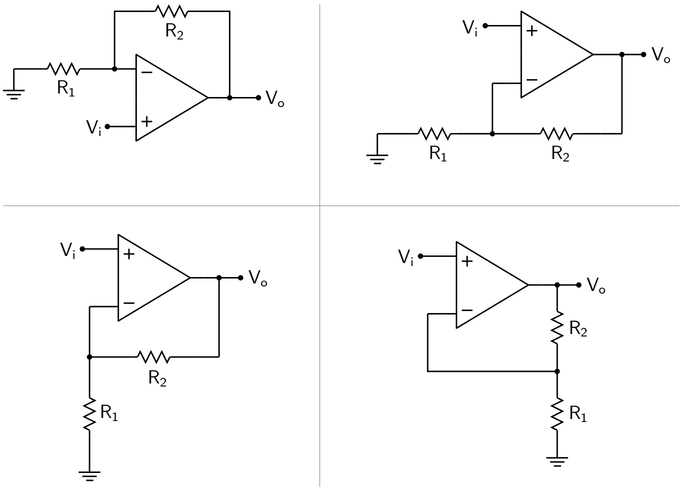



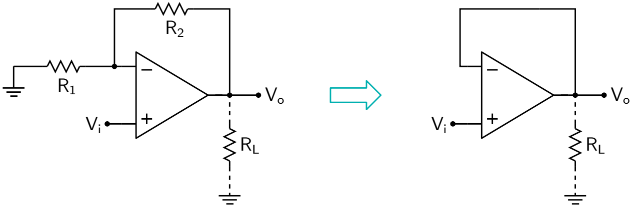
# Non-inverting amplifier



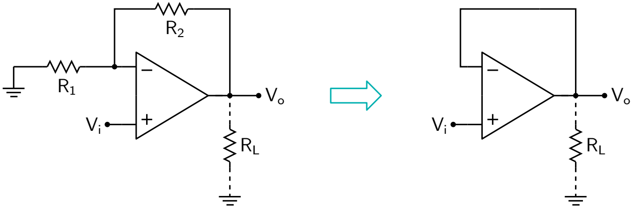




R2
R1
Vi
Vo
Vi
Vo
R1
R2
Vi
Vo
R2
R1
Vi
Vo
R2
R1



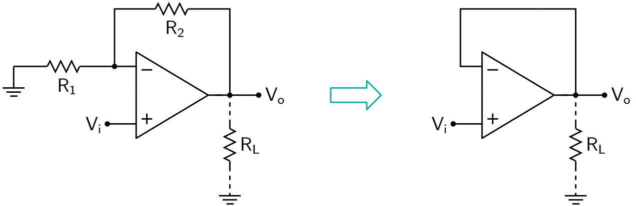

Consider $R_1 \rightarrow \infty\,,\;\; R_2 \rightarrow 0\,.$

# Non-inverting amplifier




Consider $R_1 \to \infty\,,\ \ R_2 \to 0\,.$

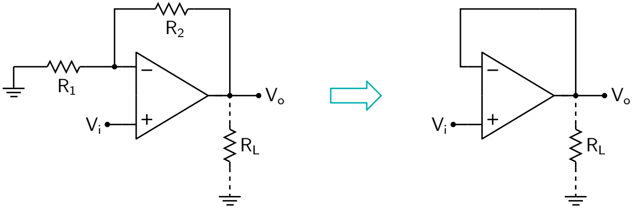
$$\frac{V_o}{V_i} \to 1 + \frac{R_2}{R_1} \to 1\,, \text{ i.e., } V_o = V_i\,.$$




Consider $R_1 \to \infty$ , $R_2 \to 0$ .

$\dfrac{V_o}{V_i} \to 1 + \dfrac{R_2}{R_1} \to 1$ , i.e., $V_o = V_i$ .

This circuit is known as unity-gain amplifier/voltage follower/buffer.

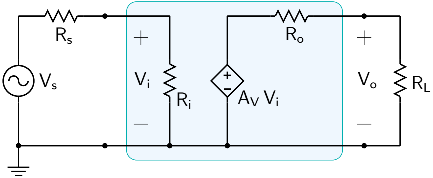



Consider $R_1 \to \infty$, $R_2 \to 0$.

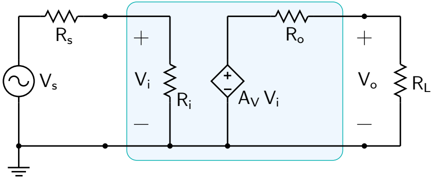
$\dfrac{V_o}{V_i} \to 1 + \dfrac{R_2}{R_1} \to 1$, i.e., $V_o = V_i$.

This circuit is known as unity-gain amplifier/voltage follower/buffer.

What has been achieved?



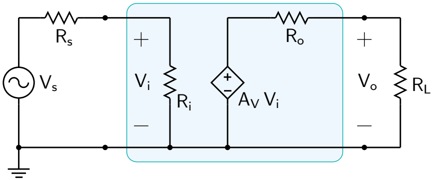

Consider an amplifier of gain $A_V$. We would like to have $V_o = A_V\, V_s$ .




Consider an amplifier of gain $A_V$. We would like to have $V_o = A_V\, V_s$ .

However, the actual output voltage is,

$$V_o = \frac{R_L}{R_o + R_L}\, A_V\, V_i = A_V\, \frac{R_L}{R_o + R_L}\, \frac{R_i}{R_i + R_s}\, V_s\,.$$

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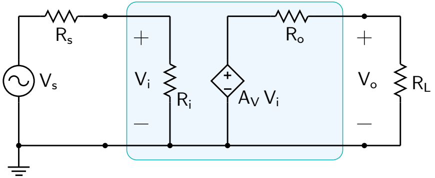
To obtain the desired $V_o$, we need $R_i \rightarrow \infty$ and $R_o \rightarrow 0$ .

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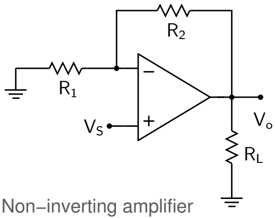
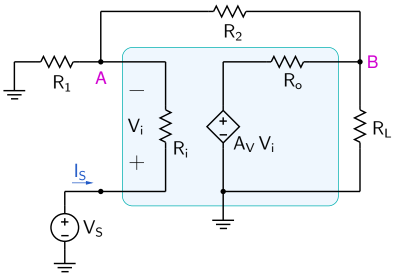
The buffer (voltage follower) provides these features.

# Op-amp buffer: input resistance

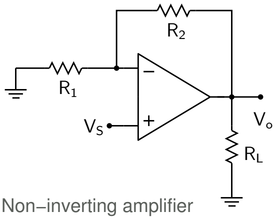



$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_s$$

Non−inverting amplifier



# Op-amp buffer: input resistance

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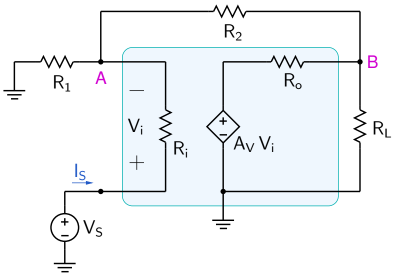
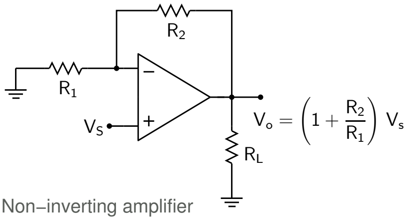
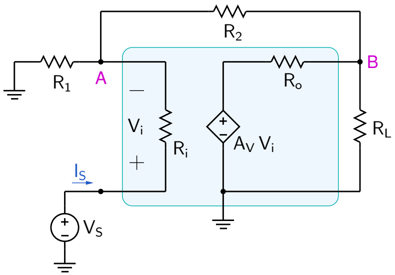



KCL at B: $$\frac{V_B}{R_L} + \frac{V_B - A_V V_i}{R_o} + \frac{V_B - V_A}{R_2} = 0.$$

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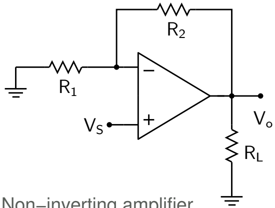
# Op-amp buffer: input resistance




Non–inverting amplifier

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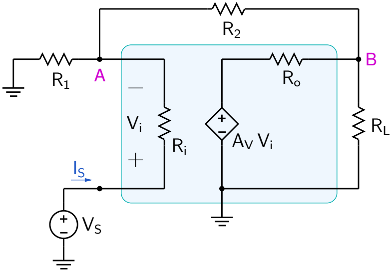


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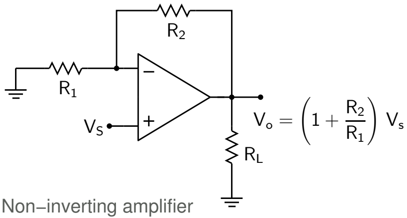
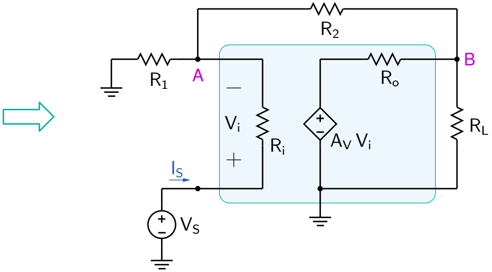
Using $V_i = I_S R_i$, $V_A = V_S - V_i$, and after some algebra, we get

$$R_{\text{in}} = \frac{V_S}{I_S} = \frac{\left(1 + \dfrac{R_o}{R_L} + \dfrac{R_o}{R_2}\right) + R_i\left[\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)\left(1 + \dfrac{R_o}{R_L} + \dfrac{R_o}{R_2}\right) - \dfrac{R_o}{R_2^2} + \dfrac{A_V}{R_2}\right]}{\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)\left(1 + \dfrac{R_o}{R_L} + \dfrac{R_o}{R_2}\right) - \dfrac{R_o}{R_2^2}}.$$

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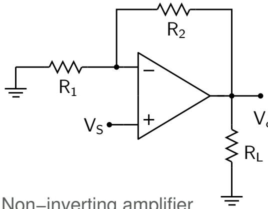


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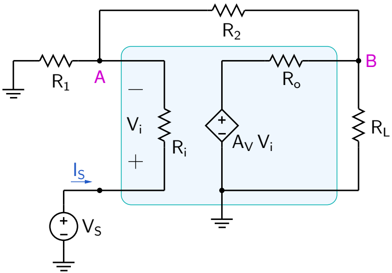


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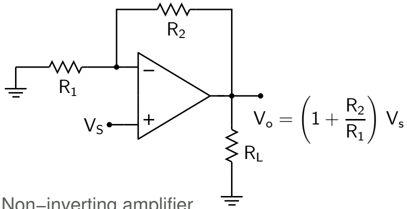
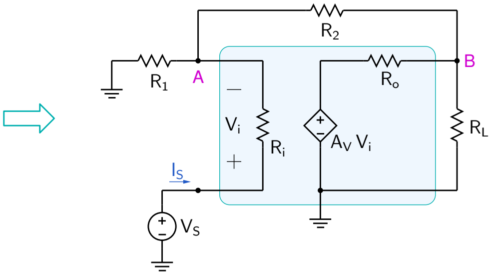
# Non-inverting amplifier: input resistance (continued)




Non–inverting amplifier

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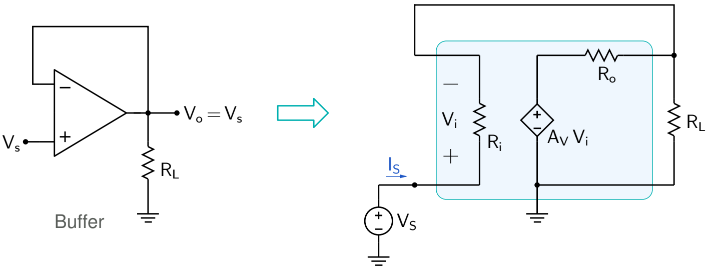
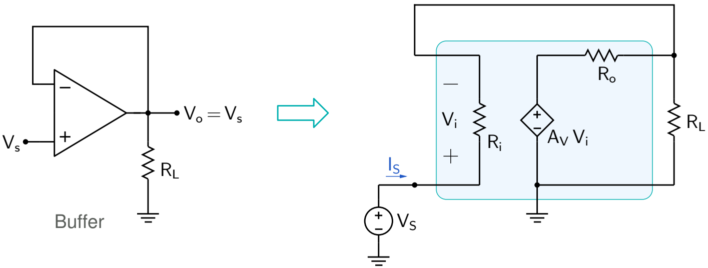
Since $R_o$ is much smaller than $R_1$, $R_2$, $R_L$, or $R_i$,

$$R_{\text{in}} \approx \frac{1 + R_i\left[\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{A_V}{R_2}\right]}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \approx \frac{R_i\left[\frac{R_1 + R_2}{R_1 R_2} + \frac{A_V}{R_2}\right]}{\frac{R_1 + R_2}{R_1 R_2}} \approx A_V R_i \frac{R_1}{R_1 + R_2}.$$



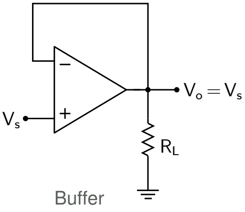
Vs
Vo = Vs
RL
Buffer
Ro
Vi
Ri
AV Vi
RL
IS
VS

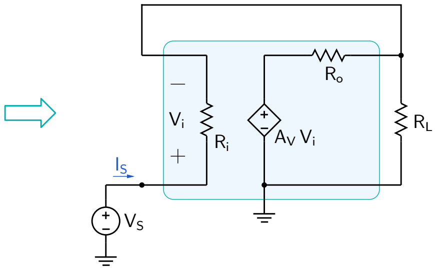
# Op-amp buffer: input resistance




Let $R_o \to 0$.

# Op-amp buffer: input resistance

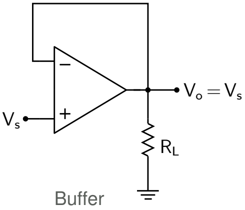


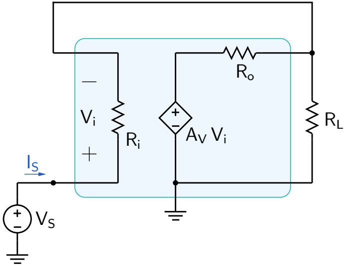




Let $R_o \to 0$.

$V_S = V_i + A_V V_i = V_i(1 + A_V)$.

# Op-amp buffer: input resistance



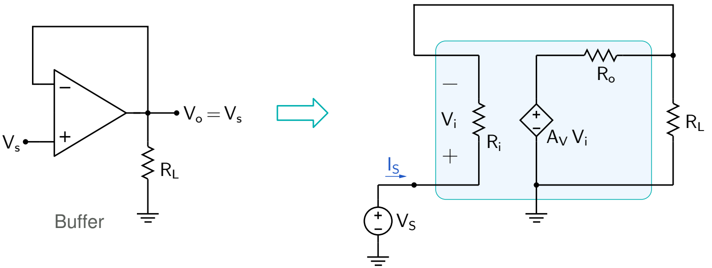




Let $R_o \rightarrow 0$.

$V_S = V_i + A_V V_i = V_i(1 + A_V)$.

$I_S = \dfrac{V_i}{R_i}$.

# Op-amp buffer: input resistance



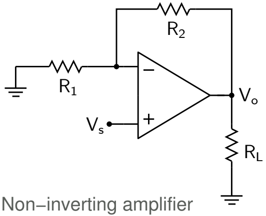

Let $R_o \to 0$.

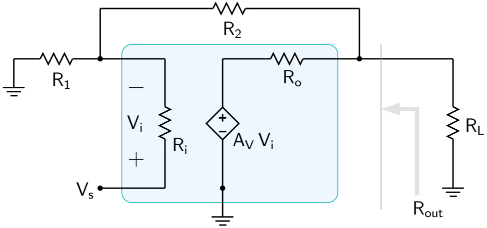
$V_S = V_i + A_V V_i = V_i(1 + A_V)$.

$I_S = \dfrac{V_i}{R_i}$.

$\to R_{\text{in}} = \dfrac{V_S}{I_S} = R_i(A_V + 1)$

# Op-amp buffer: output resistance

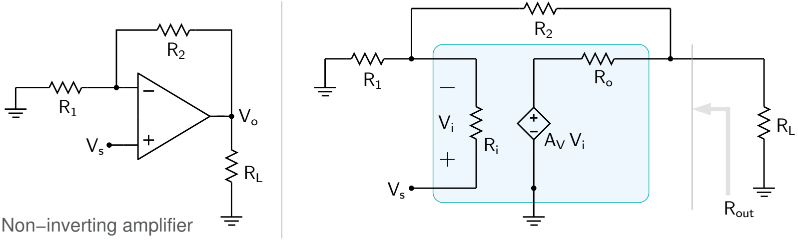



To find $R_{out}$,

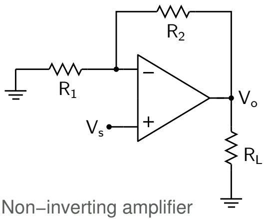
∗ Deactivate the input source.

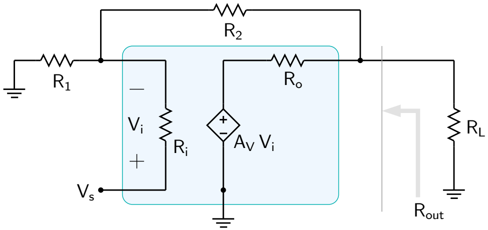
# Op-amp buffer: output resistance




To find $R_{\mathrm{out}}$,

* Deactivate the input source.
* Replace $R_L$ with a test source $V'$.

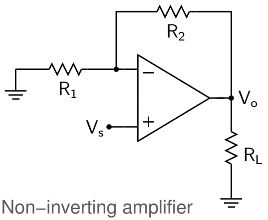


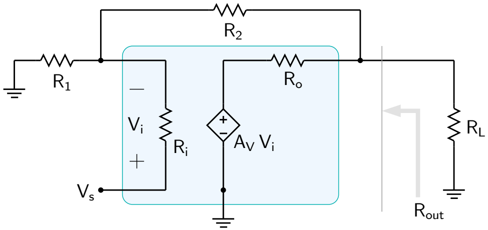




To find $R_{\text{out}}$,

* Deactivate the input source.
* Replace $R_L$ with a test source $V'$.
* Find the current ($I'$) through $V'$.

# Op-amp buffer: output resistance

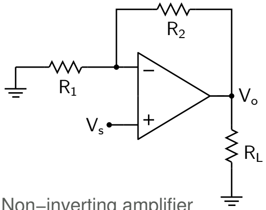


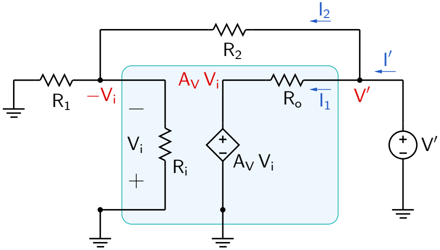

To find $R_{\text{out}}$,

* Deactivate the input source.
* Replace $R_L$ with a test source $V'$.
* Find the current ($I'$) through $V'$.
* $R_{\text{out}} = \dfrac{V'}{I'}$.

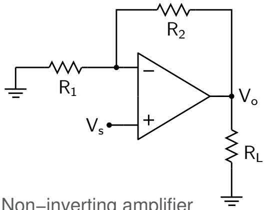
# Op-amp buffer: output resistance (continued)



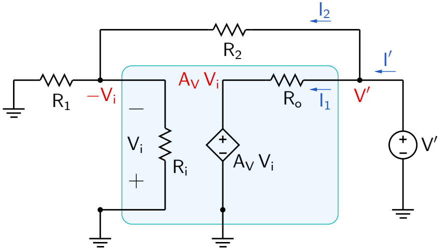

Non–inverting amplifier



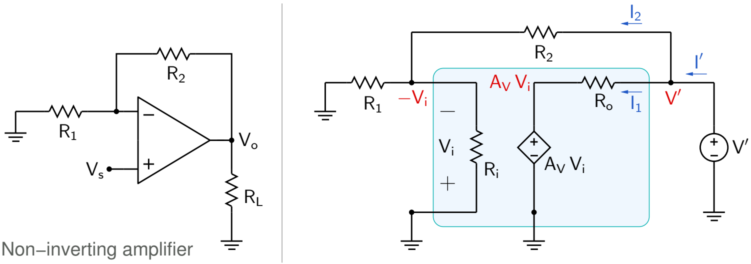
# Op-amp buffer: output resistance (continued)




Non–inverting amplifier

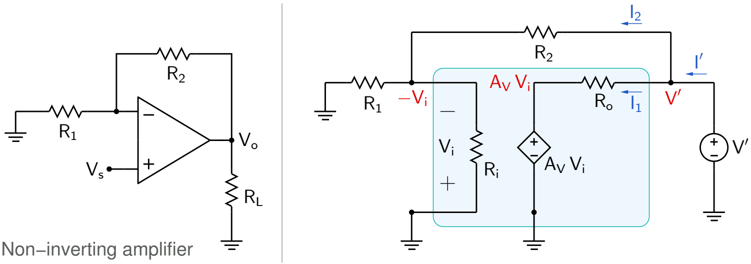



$$V_i = -\frac{(R_i \parallel R_1)}{R_2 + (R_i \parallel R_1)}\, V' \equiv -kV'.$$




$$V_i = -\frac{(R_i \parallel R_1)}{R_2 + (R_i \parallel R_1)}\, V' \equiv -kV'.$$

$$I' = I_1 + I_2 = \frac{V' - A_V V_i}{R_o} + \frac{V' - (-V_i)}{R_2} = \frac{1}{R_o}\,(V' + kA_V V') + \frac{1}{R_2}\,(V' - kV').$$

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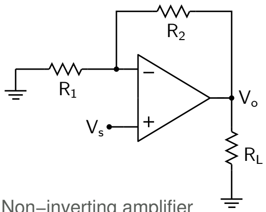
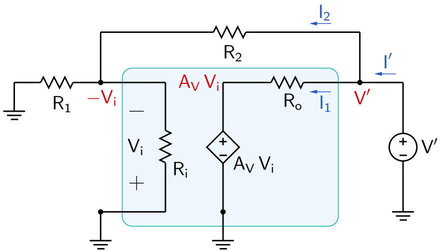
$$\frac{I'}{V'} = \frac{1}{R_o}\,(1 + kA_V) + \frac{1}{R_2}\,(1 - k) \rightarrow R_{out} = \frac{V'}{I'} = \frac{R_o}{(1 + kA_V)} \parallel \frac{R_2}{(1 - k)} \approx \frac{R_o}{(1 + kA_V)}$$



Non–inverting amplifier



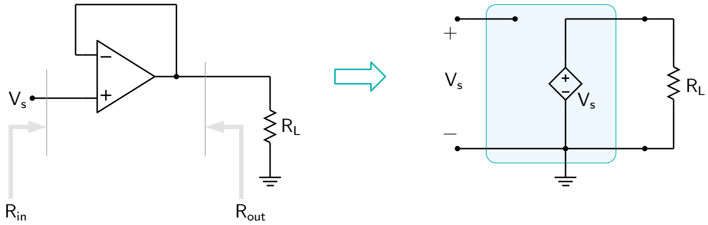

$$V_i = -\frac{(R_i \parallel R_1)}{R_2 + (R_i \parallel R_1)}\, V' \equiv -kV'.$$

$$I' = I_1 + I_2 = \frac{V' - A_V V_i}{R_o} + \frac{V' - (-V_i)}{R_2} = \frac{1}{R_o}\,(V' + kA_V V') + \frac{1}{R_2}\,(V' - kV').$$

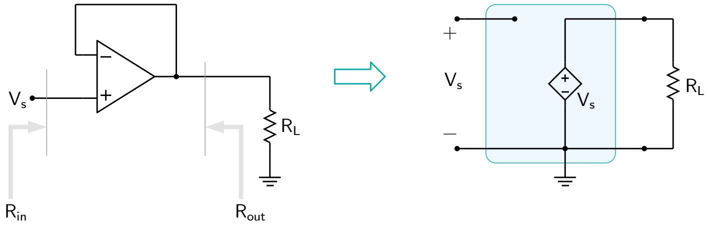
$$\frac{I'}{V'} = \frac{1}{R_o}\,(1 + kA_V) + \frac{1}{R_2}\,(1 - k) \rightarrow R_{\mathrm{out}} = \frac{V'}{I'} = \frac{R_o}{(1 + kA_V)} \parallel \frac{R_2}{(1 - k)} \approx \frac{R_o}{(1 + kA_V)}$$

Special case: Op-amp buffer

$$k = \frac{(R_i \parallel R_1)}{R_2 + (R_i \parallel R_1)} \rightarrow 1 \quad \Rightarrow \quad \boxed{R_{\mathrm{out}} \approx \frac{R_o}{1 + A_V}}$$

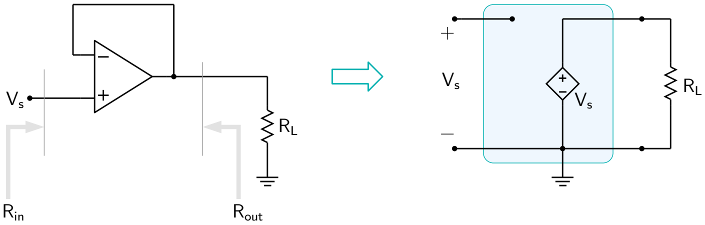



In summary, the buffer (voltage follower) provides



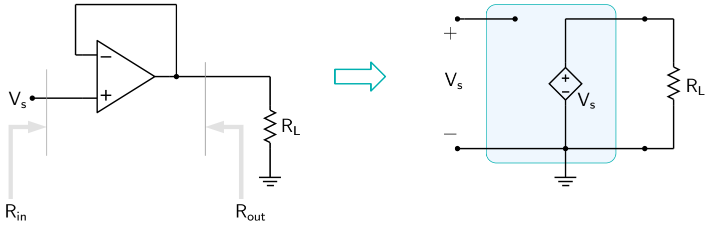

In summary, the buffer (voltage follower) provides

* a large input resistance $R_{\text{in}}$ as seen from the source.




In summary, the buffer (voltage follower) provides

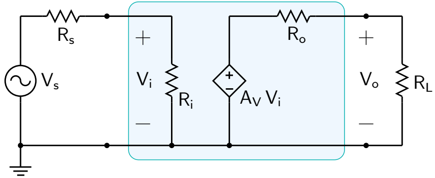
* a large input resistance $R_{\text{in}}$ as seen from the source.
* a small output resistance $R_{\text{out}}$ as seen from the load.




In summary, the buffer (voltage follower) provides

* a large input resistance $R_{\text{in}}$ as seen from the source.
* a small output resistance $R_{\text{out}}$ as seen from the load.
* a gain of 1, i.e., the output voltage simply follows the input voltage.

# Loading effects (revisited)




Problem: We would like to have $V_o = A_V \, V_s$ .

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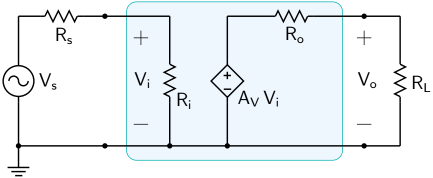
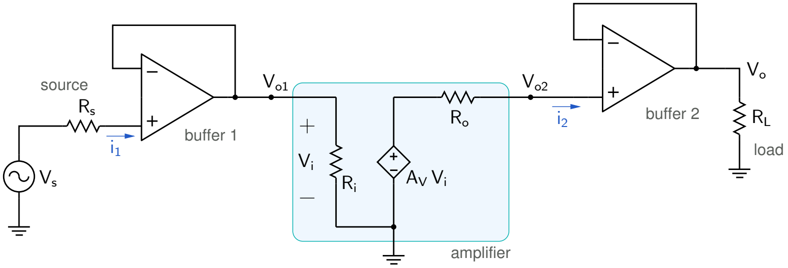


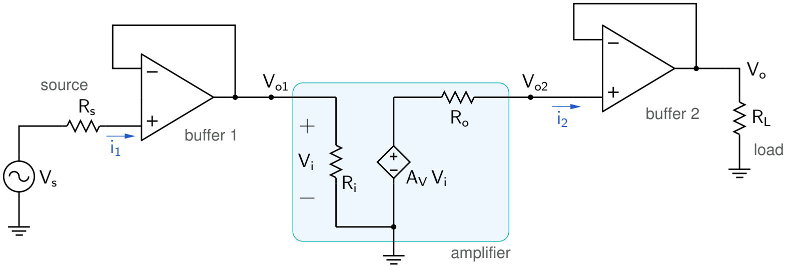

Problem: We would like to have $V_o = A_V\, V_s$ .

But the actual output voltage is,

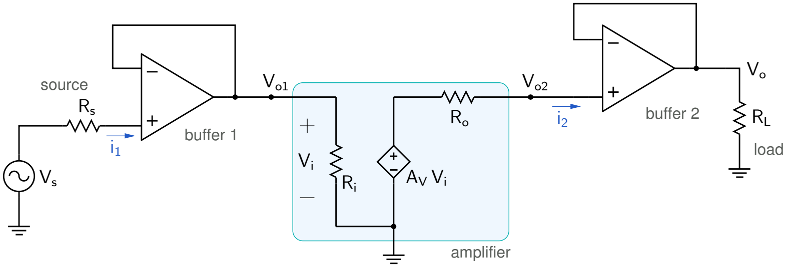
$$V_o = \frac{R_L}{R_o + R_L}\, A_V\, V_i = A_V\, \frac{R_L}{R_o + R_L}\, \frac{R_i}{R_i + R_s}\, V_s .$$


source
$R_s$
$i_1$
$V_s$
buffer 1
$V_{o1}$
$V_i$
$R_i$
$A_V V_i$
$R_o$
amplifier
$V_{o2}$
$i_2$
buffer 2
$V_o$
$R_L$
load

# Op-amp buffer




Since the buffer has a large input resistance, $i_1 \approx 0\,A$,
and $V_+$ (on the source side) $= V_s \rightarrow V_{o1} = V_s$ .

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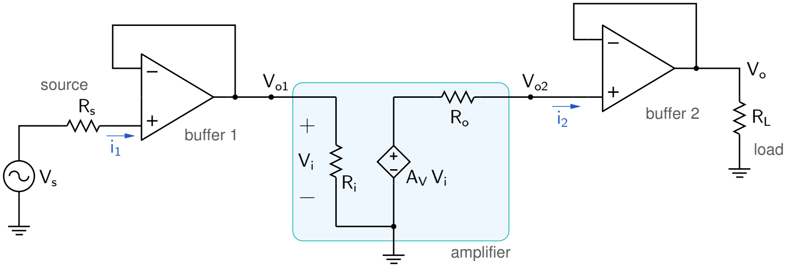


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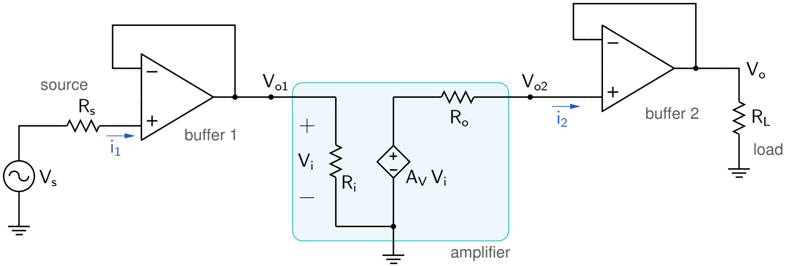
Finally, $V_o = V_{o2} = A_V\,V_s$ , as desired, *irrespective* of $R_S$ and $R_L$.

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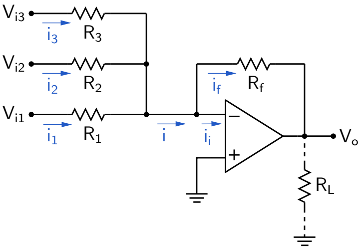
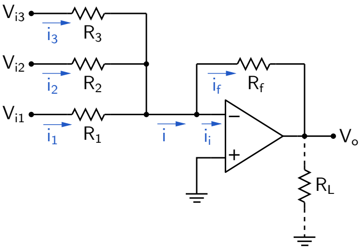
Note that the load current is supplied by the second buffer which acts as a voltage source ($= A_V\,V_s$) with zero source resistance.

# Op-amp circuits (linear region)



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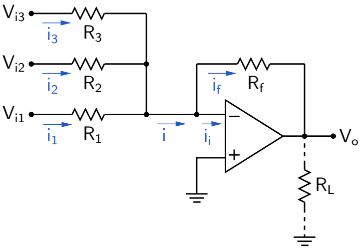


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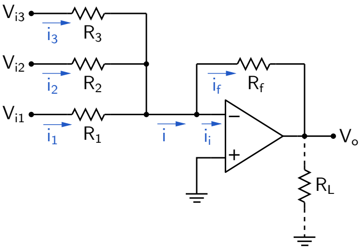


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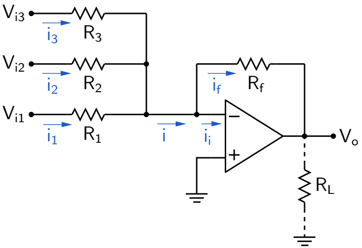


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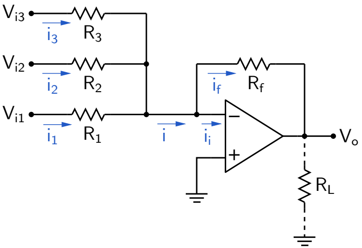
$$V_o = V_- - i_f\,R_f = 0 - \left(\frac{V_{i1}}{R_1} + \frac{V_{i2}}{R_2} + \frac{V_{i3}}{R_3}\right) R_f = -\left(\frac{R_f}{R_1}\,V_{i1} + \frac{R_f}{R_2}\,V_{i2} + \frac{R_f}{R_3}\,V_{i3}\right),$$

i.e., $V_o$ is a *weighted sum* of $V_{i1}$, $V_{i2}$, $V_{i3}$.

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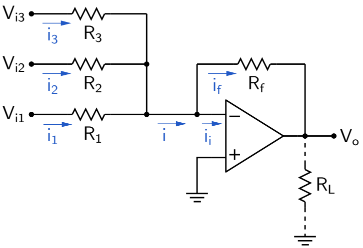
i.e., $V_o$ is a *weighted sum* of $V_{i1}$, $V_{i2}$, $V_{i3}$.

If $R_1 = R_2 = R_3 = R$ , the circuit acts as a summer, giving

$\boxed{V_o = -K\,(V_{i1} + V_{i2} + V_{i3})}$ with $K = R_f/R$ .

# Summer example




$R_1 = R_2 = R_3 = 1\ \text{k}\Omega$

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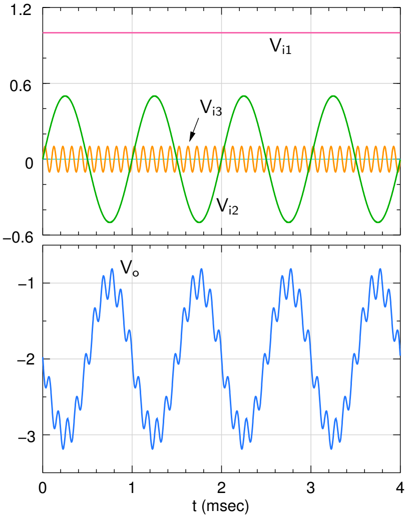
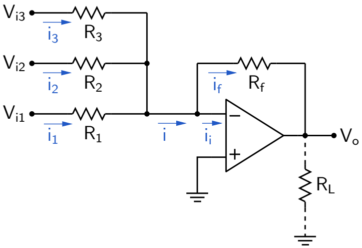
$\rightarrow V_o = -2\,(V_{i1} + V_{i2} + V_{i3})$

SEQUEL file: `ee101_summer.sqproj`



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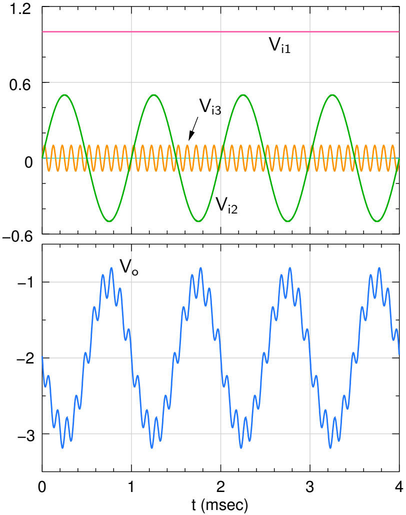
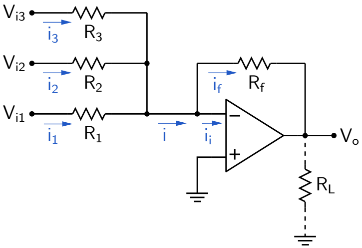



* Note that the summer also works with DC inputs (so do inverting and non-inverting amplifiers).

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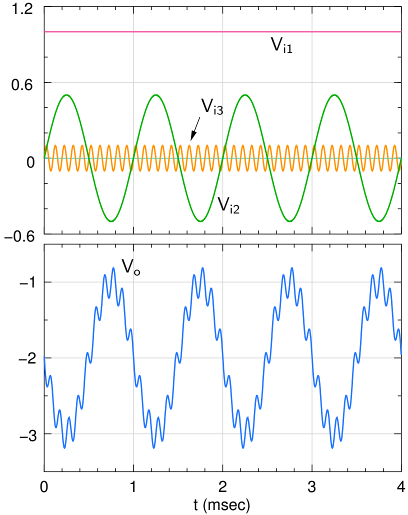


* Note that the summer also works with DC inputs (so do inverting and non-inverting amplifiers).
* Op-amps make life simpler! Think of adding voltages in any other way.

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* Typical resistance values: 0.1 k to 100 k.

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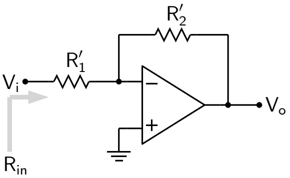

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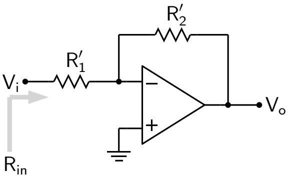


$$R_{\text{in}} = R_1' = 10\,\text{k}.$$

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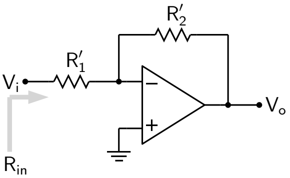


$$R_{\text{in}} = R_1' = 10\,\text{k}.$$

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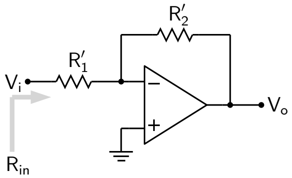


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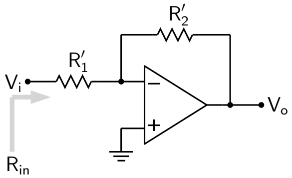


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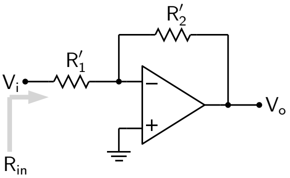


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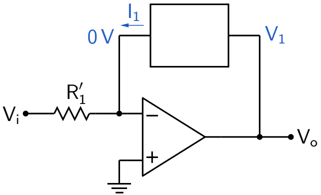

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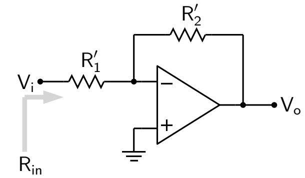


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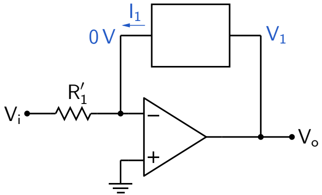
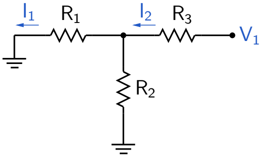
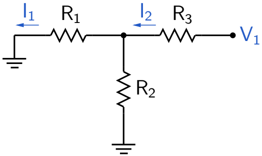
If we ensure $\dfrac{V_1}{I_1} = R'_2$, we will satisfy the gain condition.

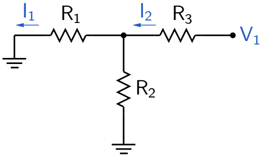




I1
R1
I2
R3
V1
R2




$$I_2 = \frac{V_1}{R_3 + (R_1 \parallel R_2)}$$

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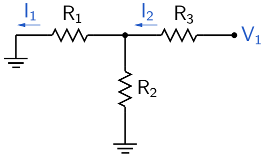


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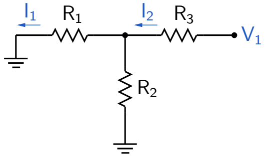


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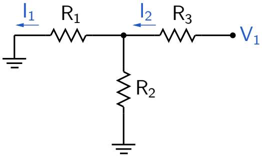


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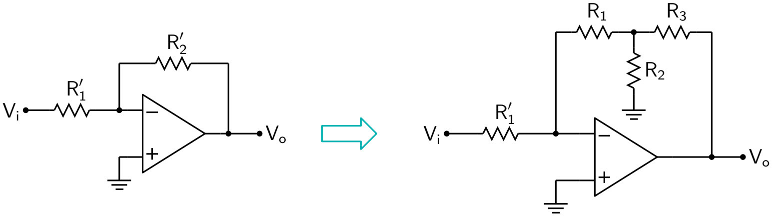

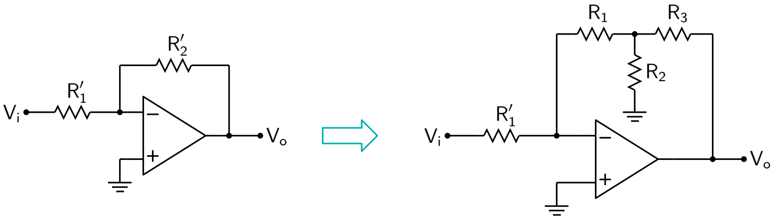


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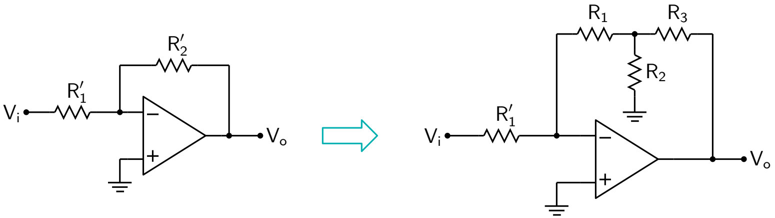


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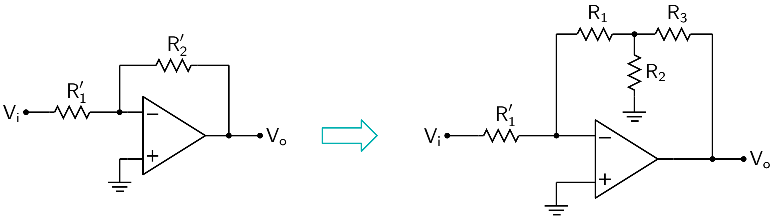
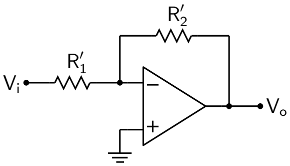
Let $R_1 = R_3 \equiv R$

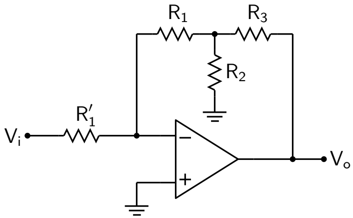



$$R_{\text{eff}} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

We want $R_{\text{eff}} = R_2' = 1\,\text{M}\Omega$.

Let $R_1 = R_3 \equiv R \rightarrow R_{\text{eff}} = \dfrac{R^2 + 2\,R\,R_2}{R_2} = R\left(\dfrac{R}{R_2} + 2\right)$




$$R_{\text{eff}} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

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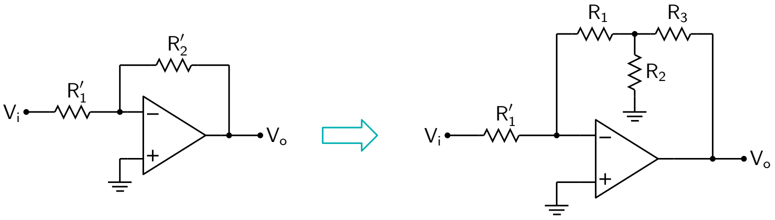
Let $R_1 = R_3 \equiv R \rightarrow R_{\text{eff}} = \dfrac{R^2 + 2\,R\,R_2}{R_2} = R\left(\dfrac{R}{R_2} + 2\right) \rightarrow R_2 = \dfrac{R}{\dfrac{R_{\text{eff}}}{R} - 2}$

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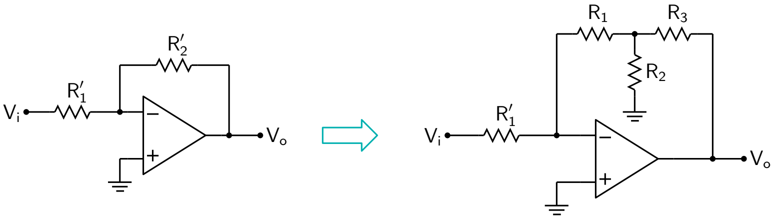
For $R = 10\,\text{k}$, $R_2 = \dfrac{10\,\text{k}}{100 - 2} \approx 102\,\Omega$.

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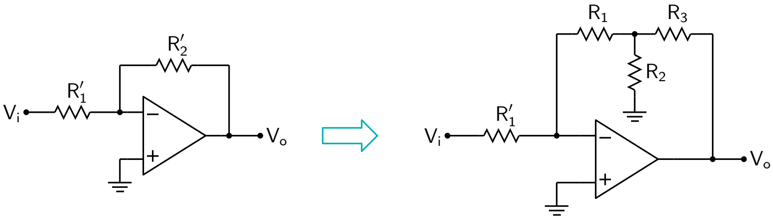
Ref: Wait et al, *Introduction to op-amp theory and applications*, McGraw-Hill, 1992.

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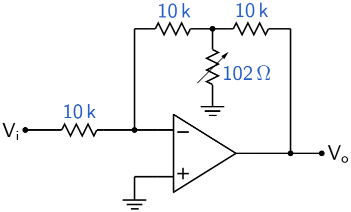


Ref: Wait et al, *Introduction to op-amp theory and applications*, McGraw-Hill, 1992.