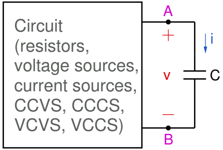
RC circuits with DC sources



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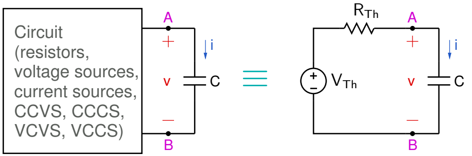

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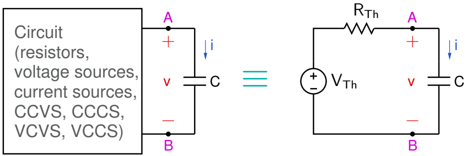
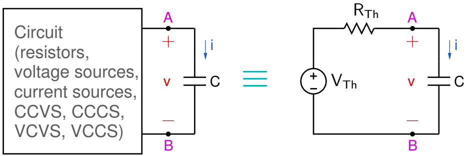


* If all sources are DC (constant), $V_{Th} = \text{constant}$.



* If all sources are DC (constant), $V_{Th} = \text{constant}$.
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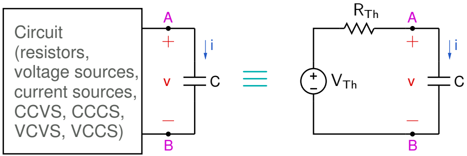

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$\dfrac{dv}{dt} + \dfrac{1}{\tau}\, v = 0$, where $\tau = R_{Th}\, C$ is the "time constant." $\left(\dfrac{\text{V}}{\text{Coul/sec}} \times \dfrac{\text{Coul}}{V}\right)$

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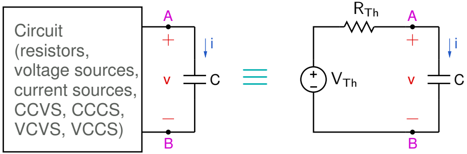

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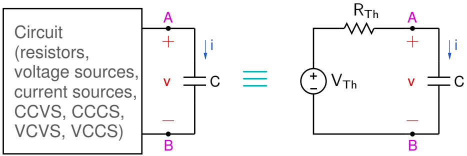

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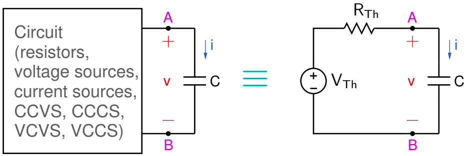

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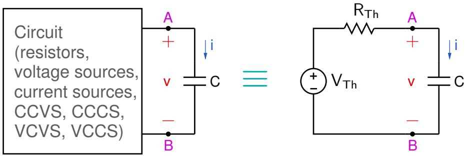

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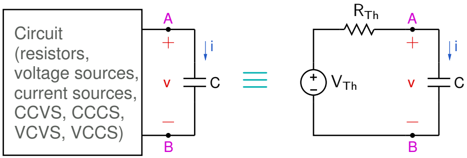

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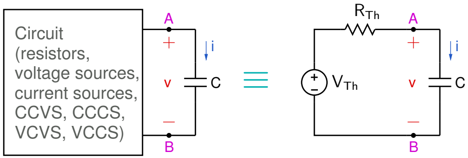
RC circuits with DC sources (continued)



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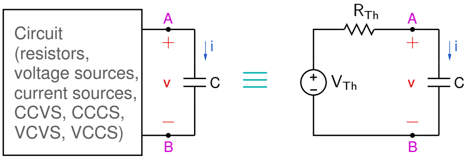

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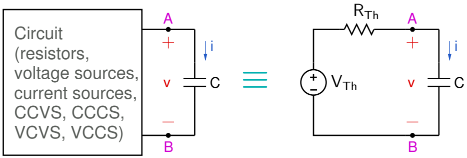

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* Since the circuit in the black box is linear, *any* variable (current or voltage) in the circuit can be expressed as
  $x(t) = K_1\ \exp(-t/\tau) + K_2\,,$
  where K_1 and K_2 can be obtained from suitable conditions on $x(t)$.

Plot of $f(t) = e^{-t/\tau}$

t/τ	$e^{-t/\tau}$	$1 - e^{-t/\tau}$
0.0	1.0	0.0
1.0	0.3679	0.6321
2.0	0.1353	0.8647
3.0	4.9787×10^{-2}	0.9502
4.0	1.8315×10^{-2}	0.9817
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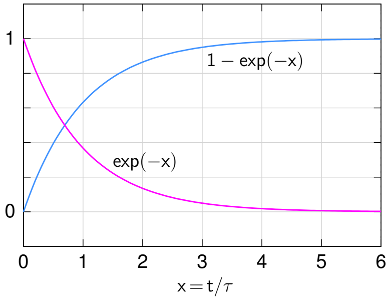

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Plot of $f(t) = A\, e^{-t/\tau} + B$

* At $t = 0$, $f = A + B$.
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* The graph of $f(t)$ lies between $(A + B)$ and B.

  Note: If $A > 0$, $A + B > B$. If $A < 0$, $A + B < B$.

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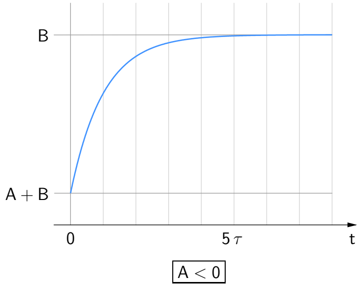

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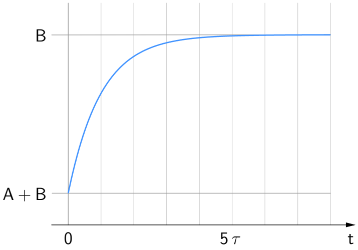

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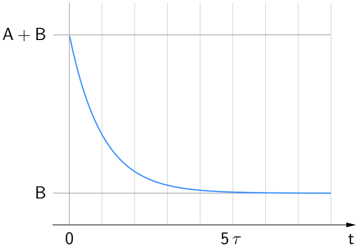

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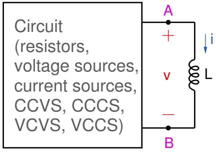
RL circuits with DC sources



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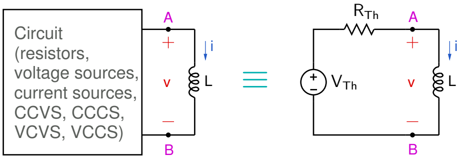

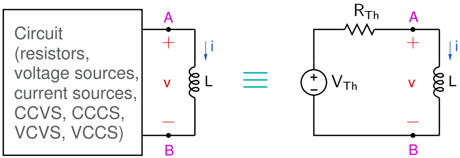

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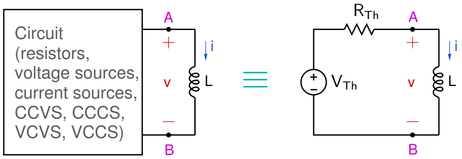

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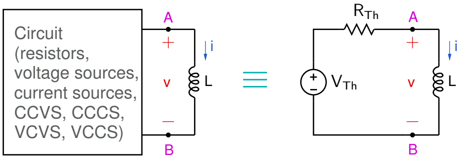

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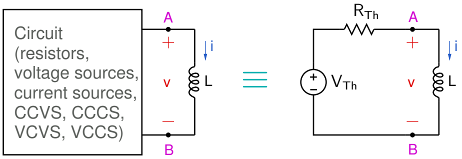

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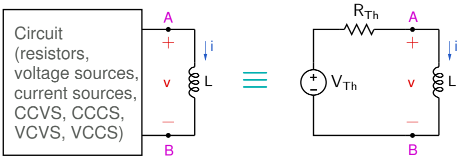

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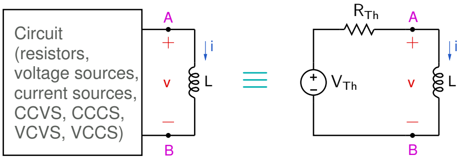

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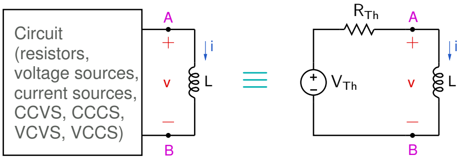

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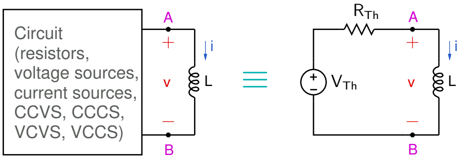

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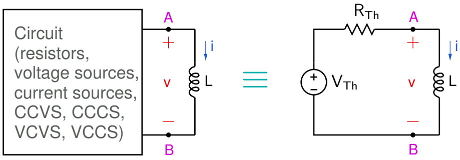

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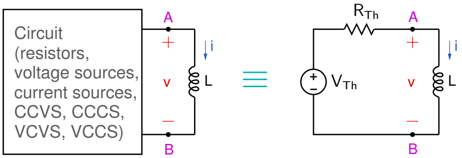

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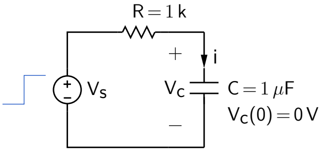
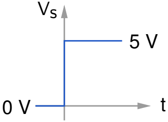
* As $t \rightarrow \infty$, $v \rightarrow 0$, i.e., the inductor behaves like a short circuit since all derivatives vanish.
* Since the circuit in the black box is linear, *any* variable (current or voltage) in the circuit can be expressed as
  $x(t) = K_1\ \exp(-t/\tau) + K_2\,,$
  where K_1 and K_2 can be obtained from suitable conditions on $x(t)$.

RC circuits: Can V_c change "suddenly?"





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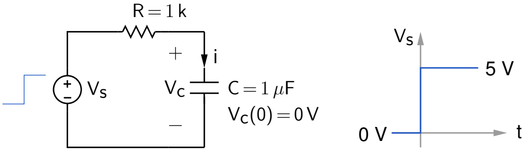

* V_s changes from 0 V (at $t=0^-$), to 5 V (at $t=0^+$). As a result of this change, V_c will rise. How fast can V_c change?

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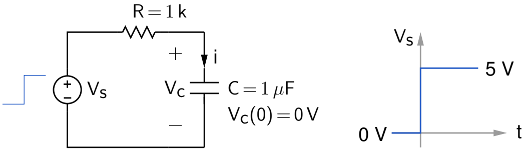
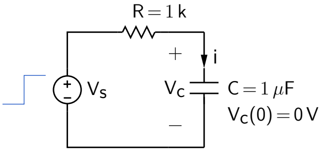
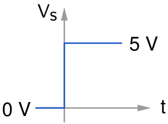


* V_s changes from $0\ V$ (at $t=0^-$), to $5\ V$ (at $t=0^+$). As a result of this change, V_c will rise. How fast can V_c change?
* For example, what would happen if V_c changes by $1\ V$ in $1\,\mu s$ at a constant rate of $1\ V/1\,\mu s = 10^6\ V/s$?

RC circuits: Can V_c change "suddenly?"





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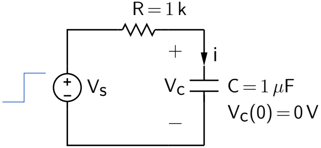

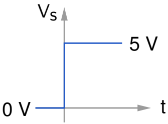

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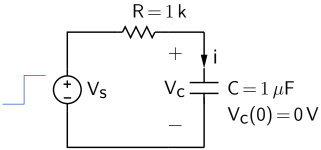
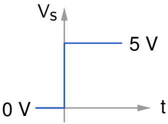
* $i = C\,\dfrac{dV_c}{dt} = 1\,\mu F \times 10^6\,\dfrac{V}{s} = 1\,A\,.$
* With $i=1\,A$, the voltage drop across R would be $1000\,V$! Not allowed by KVL.





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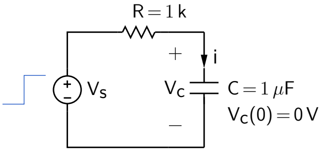
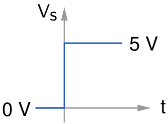
- We conclude that $V_c(0^+) = V_c(0^-) \Rightarrow$ A capacitor does not allow abrupt changes in V_c if there is a finite resistance in the circuit.





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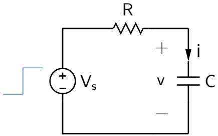
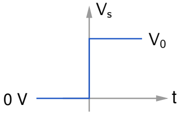
* We conclude that $V_c(0^+) = V_c(0^-) \Rightarrow$ A capacitor does not allow abrupt changes in V_c if there is a finite resistance in the circuit.
* Similarly, an inductor does not allow abrupt changes in i_L.

RC circuits: charging and discharging transients





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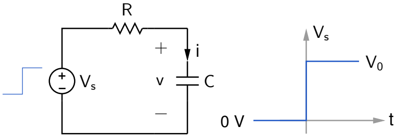
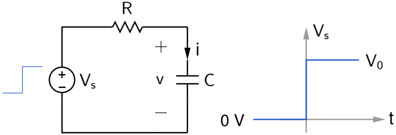


Let $\mathsf{v}(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

RC circuits: charging and discharging transients



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Conditions on $v(t)$:

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Note that we need the condition at 0^+ (and not at 0^-)

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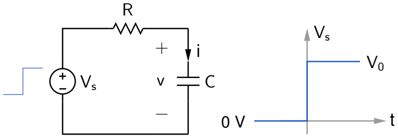

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$t = 0^+$: $0 = A + B\,,$

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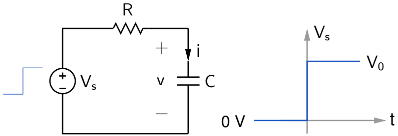
RC circuits: charging and discharging transients



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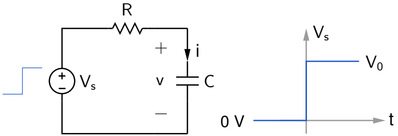
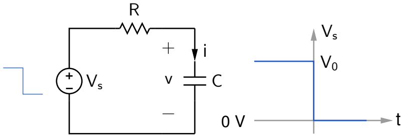
$$\boxed{v(t) = V_0\left[1 - \exp(-t/\tau)\right]}$$

RC circuits: charging and discharging transients





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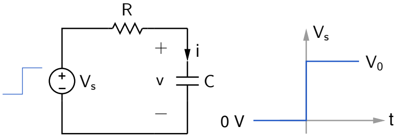

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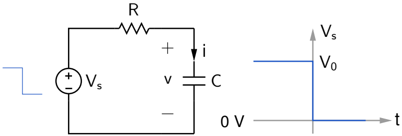

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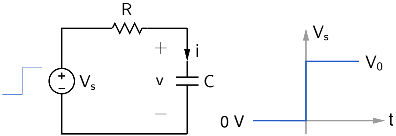

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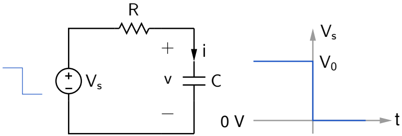

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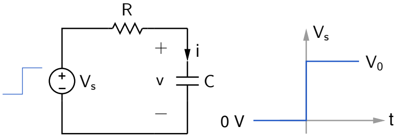
RC circuits: charging and discharging transients



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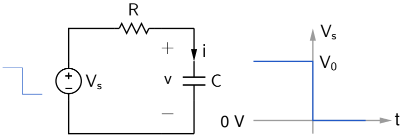

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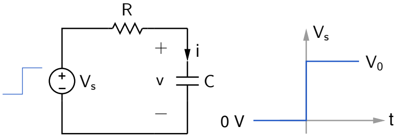
RC circuits: charging and discharging transients



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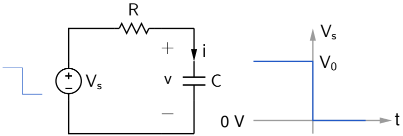

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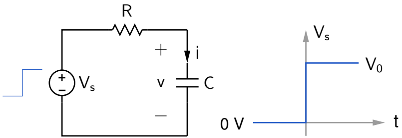
RC circuits: charging and discharging transients



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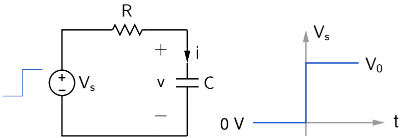

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(A) $$i(t) = C\frac{d}{dt}V_0\left[1 - \exp(-t/\tau)\right]$$
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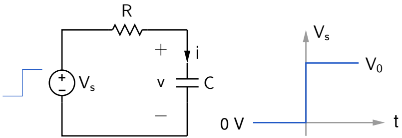

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(B) Let $i(t) = A' \exp(-t/\tau) + B'$, $\quad t > 0$.

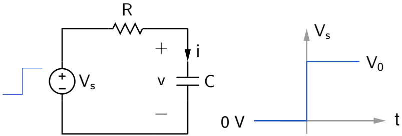
$t = 0^+$: $v = 0$, $V_S = V_0 \Rightarrow i(0^+) = V_0/R$.

$t \to \infty$: $i(t) = 0$.

Using these conditions, we obtain

$$A' = \frac{V_0}{R},\ B' = 0 \Rightarrow i(t) = \frac{V_0}{R} \exp(-t/\tau)$$

RC circuits: charging and discharging transients



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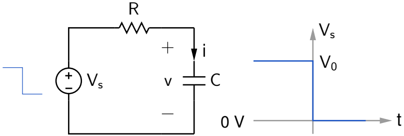

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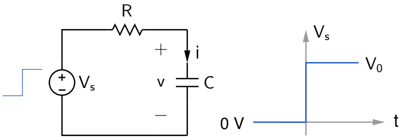
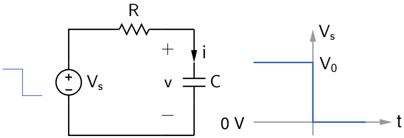
RC circuits: charging and discharging transients





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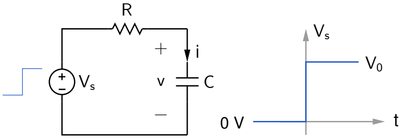
$$A' = \frac{V_0}{R},\ B' = 0 \Rightarrow i(t) = \frac{V_0}{R} \exp(-t/\tau)$$

Compute $i(t)$, $t > 0$.

(A) $i(t) = C \dfrac{d}{dt} V_0 \left[\exp(-t/\tau)\right]$

$$= -\frac{CV_0}{\tau} \exp(-t/\tau) = -\frac{V_0}{R} \exp(-t/\tau)$$

RC circuits: charging and discharging transients



Compute $i(t)$, $t > 0$.

(A) $i(t) = C\,\dfrac{d}{dt}\,V_0\,[1 - \exp(-t/\tau)]$

$$= \frac{CV_0}{\tau}\exp(-t/\tau) = \frac{V_0}{R}\exp(-t/\tau)$$

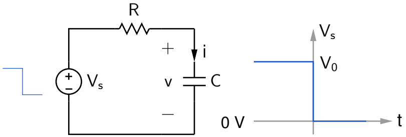
(B) Let $i(t) = A'\exp(-t/\tau) + B'$, $t > 0$.

$t = 0^+$: $v = 0$, $V_S = V_0 \Rightarrow i(0^+) = V_0/R$.

$t \rightarrow \infty$: $i(t) = 0$.

Using these conditions, we obtain

$$A' = \frac{V_0}{R},\ B' = 0 \Rightarrow i(t) = \frac{V_0}{R}\exp(-t/\tau)$$



Compute $i(t)$, $t > 0$.

(A) $i(t) = C\,\dfrac{d}{dt}\,V_0\,[\exp(-t/\tau)]$

$$= -\frac{CV_0}{\tau}\exp(-t/\tau) = -\frac{V_0}{R}\exp(-t/\tau)$$

(B) Let $i(t) = A'\exp(-t/\tau) + B'$, $t > 0$.

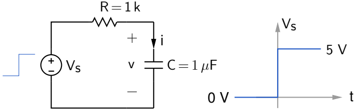
$t = 0^+$: $v = V_0$, $V_S = 0 \Rightarrow i(0^+) = -V_0/R$.

$t \rightarrow \infty$: $i(t) = 0$.

Using these conditions, we obtain

$$A' = -\frac{V_0}{R},\ B' = 0 \Rightarrow i(t) = -\frac{V_0}{R}\exp(-t/\tau)$$

RC circuits: charging and discharging transients



$$v(t) = V_0\,[1 - \exp(-t/\tau)]$$

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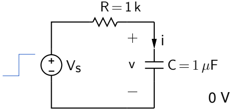
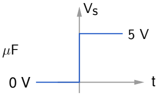
RC circuits: charging and discharging transients





$$v(t) = V_0 \left[1 - \exp(-t/\tau)\right]$$

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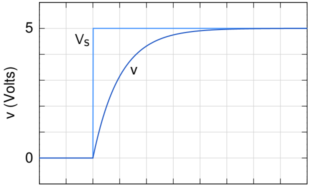
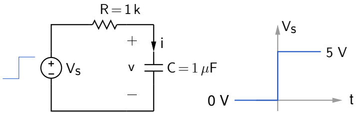


RC circuits: charging and discharging transients



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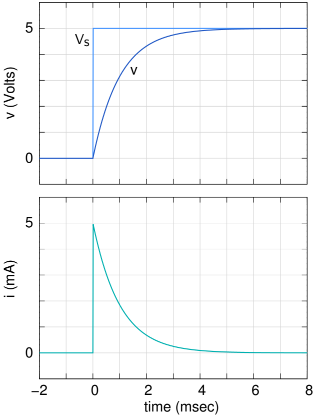
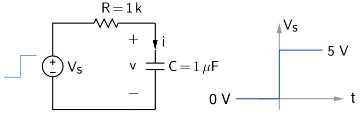


RC circuits: charging and discharging transients



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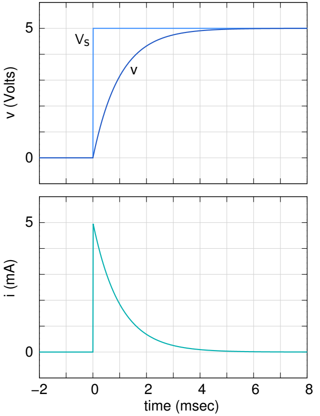

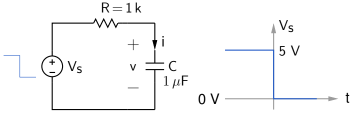

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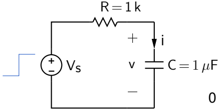
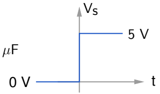
RC circuits: charging and discharging transients





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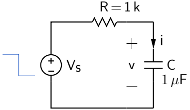

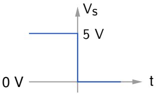

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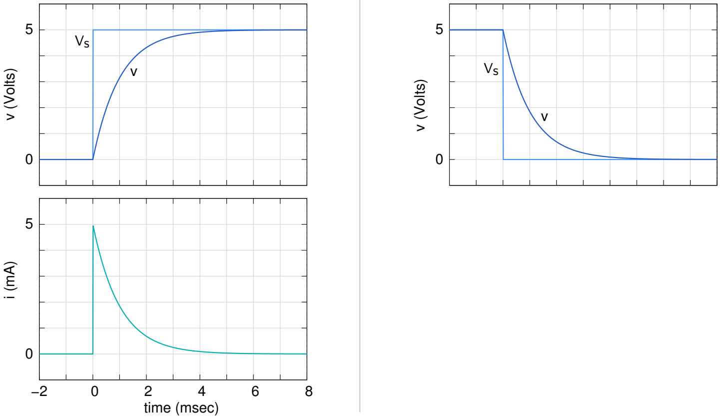
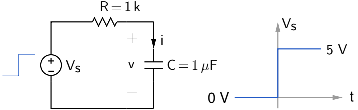


RC circuits: charging and discharging transients



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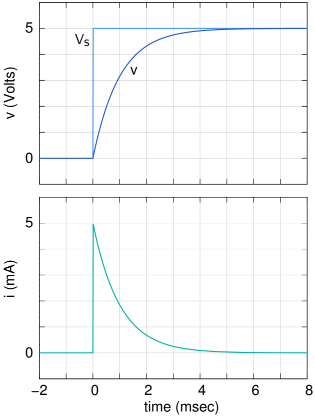

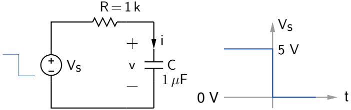

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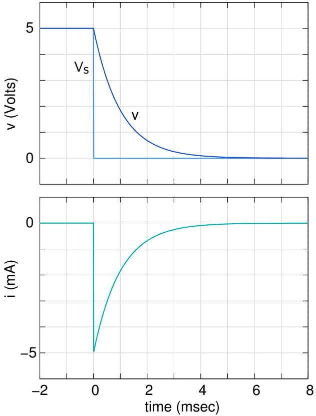
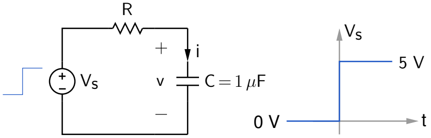


RC circuits: charging and discharging transients



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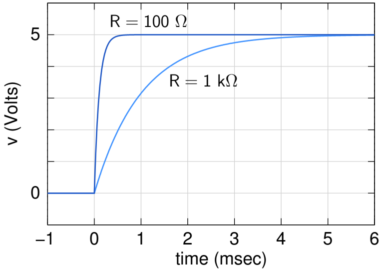

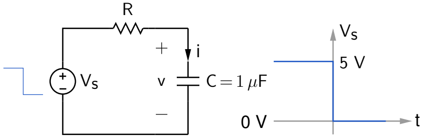

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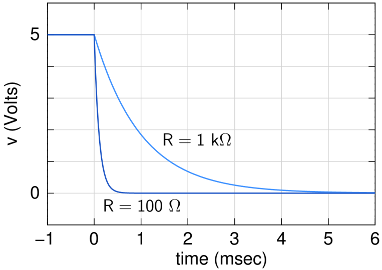
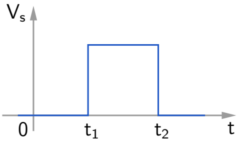


* Identify intervals in which the source voltages/currents are constant.
  For example,



(1) $t < t_1$

(2) $t_1 < t < t_2$

(3) $t > t_2$

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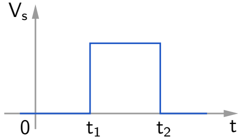

(1) $t < t_1$

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* For *any* current or voltage $x(t)$, write general expressions such as,
  $x(t) = A_1 \exp(-t/\tau) + B_1\,, \quad t < t_1\,,$
  $x(t) = A_2 \exp(-t/\tau) + B_2\,, \quad t_1 < t < t_2\,,$
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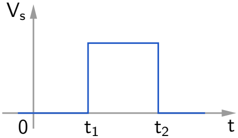

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* Work out suitable conditions on $x(t)$ at specific time points using

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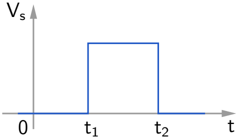

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* Work out suitable conditions on $x(t)$ at specific time points using

  (a) If the source voltage/current has not changed for a "long" time (long compared to τ), all derivatives are zero.

  $$\Rightarrow i_C = C\,\frac{dV_c}{dt} = 0\,, \text{ and } V_L = L\,\frac{di_L}{dt} = 0\,.$$

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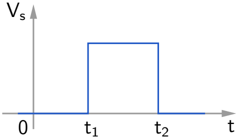

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  (b) When a source voltage (or current) changes, say, at $t = t_0$,
  $V_c(t)$ or $i_L(t)$ cannot change abruptly, i.e.,
  $V_c(t_0^+) = V_c(t_0^-)$, and $i_L(t_0^+) = i_L(t_0^-)$.

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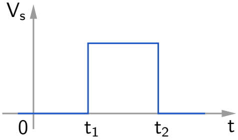

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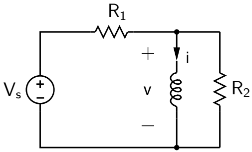
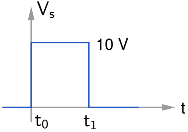
* Compute A_1, B_1, $\cdots$ using the conditions on $x(t)$.

RL circuit: example



Find i(t).



$R_1 = 10\,\Omega$

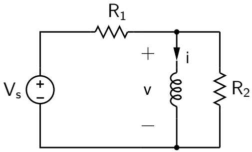
$R_2 = 40\,\Omega$

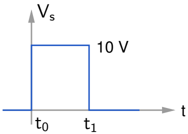
$L = 0.8\,H$

$t_0 = 0$

$t_1 = 0.1\,s$

RL circuit: example





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There are three intervals of constant V_s:

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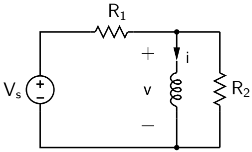

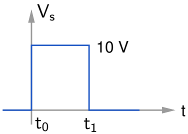

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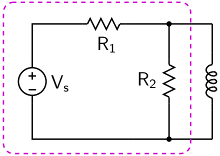
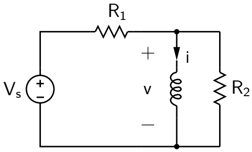
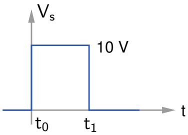
R_{Th} seen by L is the same in all intervals:



$$R_{Th} = R_1 \parallel R_2 = 8\,\Omega$$

$$\begin{aligned}\tau &= L/R_{Th} \\ &= 0.8\ H/8\,\Omega \\ &= 0.1\ s\end{aligned}$$





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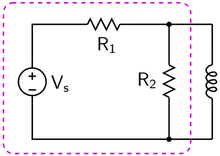


$R_{Th} = R_1 \parallel R_2 = 8\,\Omega$

$\tau = L/R_{Th}$

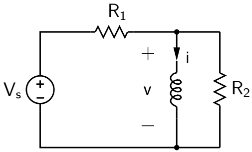
$= 0.8\ \mathrm{H}/8\,\Omega$

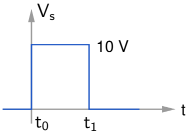
$= 0.1\ \mathrm{s}$

At $t = t_0^-$, $v = 0$ V, $V_s = 0$ V.

$\Rightarrow i(t_0^-) = 0$ A $\Rightarrow i(t_0^+) = 0$ A.

RL circuit: example





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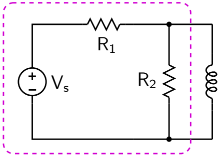

$R_{Th} = R_1 \parallel R_2 = 8\,\Omega$

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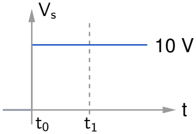
At $t = t_0^-$, $v = 0$ V, $V_s = 0$ V.

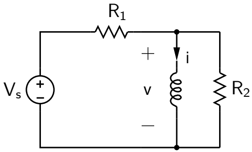
$\Rightarrow i(t_0^-) = 0\ \mathrm{A} \Rightarrow i(t_0^+) = 0\ \mathrm{A}$.

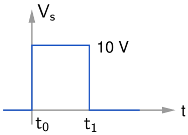
If V_s did not change at $t = t_1$,

we would have



$v(\infty) = 0$ V, $i(\infty) = 10\ \mathrm{V}/10\ \Omega = 1$ A.





$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,H$

$t_0 = 0$

$t_1 = 0.1\,s$

Find $i(t)$.

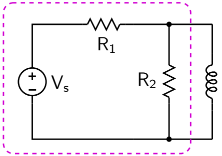
There are three intervals of constant V_s:

(1) $t < t_0$

(2) $t_0 < t < t_1$

(3) $t > t_1$

R_{Th} seen by L is the same in all intervals:



$R_{Th} = R_1 \parallel R_2 = 8\,\Omega$

$\tau = L/R_{Th}$

$= 0.8\text{ H}/8\,\Omega$

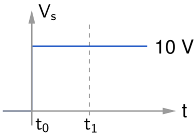
$= 0.1\text{ s}$

At $t = t_0^-$, $v = 0$ V, $V_s = 0$ V .

$\Rightarrow i(t_0^-) = 0\text{ A} \Rightarrow i(t_0^+) = 0\text{ A}$.

If V_s did not change at $t = t_1$,

we would have

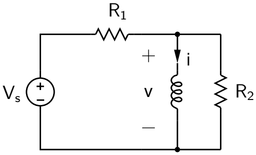


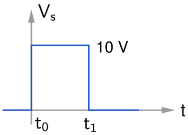
$v(\infty) = 0$ V, $i(\infty) = 10\text{ V}/10\,\Omega = 1$ A .

Using $i(t_0^+)$ and $i(\infty)$, we can obtain

$i(t)$, $t > 0$ (See next slide).

RL circuit: example





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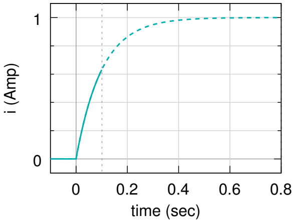
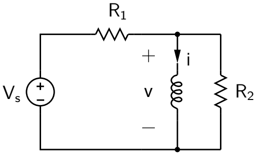
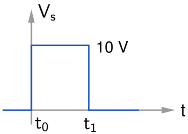


RL circuit: example





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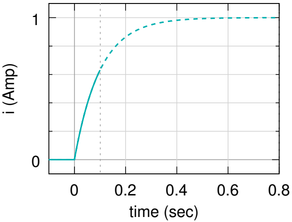
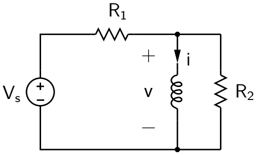
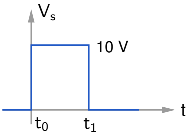


In reality, V_s changes at $t = t_1$, and we need to work out the solution for $t > t_1$ separately.





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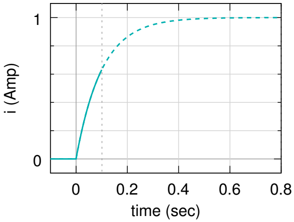

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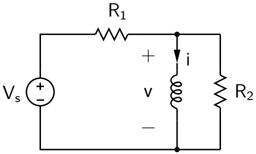
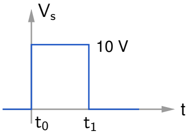
For $t_0 < t < t_1$, $i(t) = 1 - \exp(-t/\tau)$ Amp.

Consider $t > t_1$.

RL circuit: example





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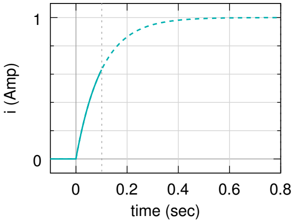

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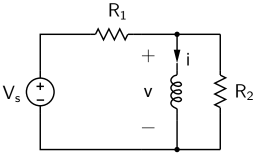
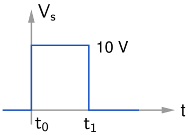
Consider $t > t_1$.

$i(t_1^+) = i(t_1^-) = 1 - e^{-1} = 0.632$ A (Note: $t_1/\tau = 1$).

$i(\infty) = 0$ A.

RL circuit: example





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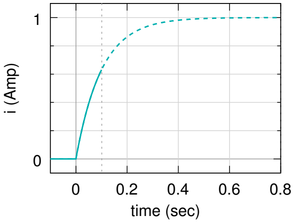

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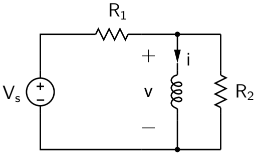
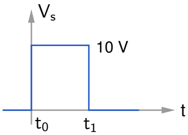
RL circuit: example





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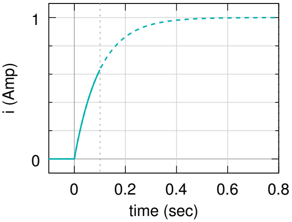

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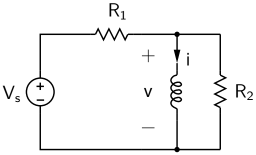
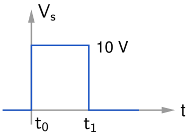
$i(t) = A' \exp[-(t - t_1)/\tau] + B$.

RL circuit: example





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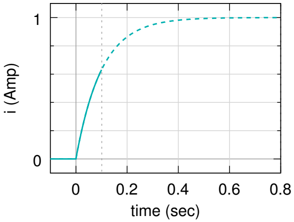

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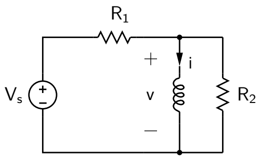
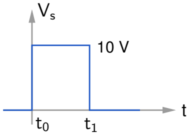
RL circuit: example





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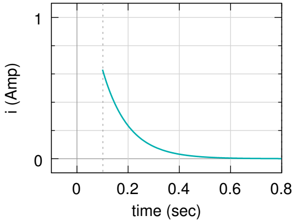

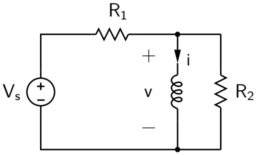

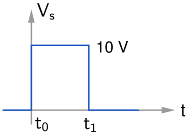

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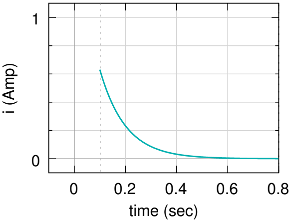
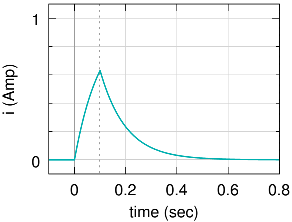
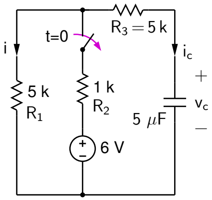


Combining the solutions for $t_0 < t < t_1$ and $t > t_1$, we get



(SEQUEL file: ee101_rl1.sqproj)

RC circuit: The switch has been closed for a long time and opens at $t = 0$.



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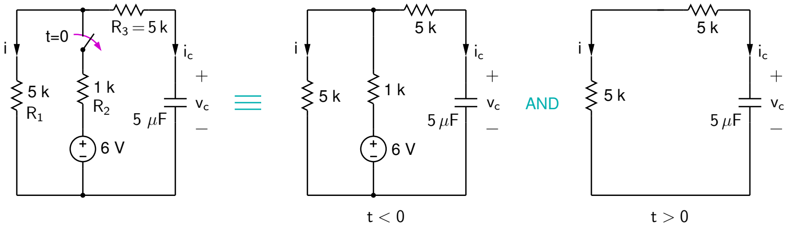

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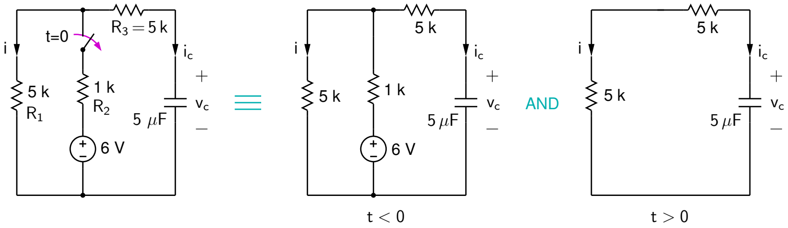
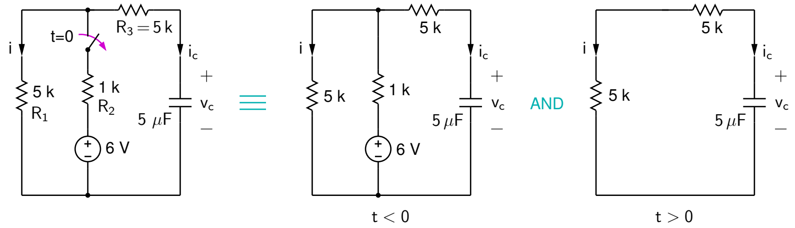


$t = 0^-$: capacitor is an open circuit $\Rightarrow i(0^-) = 6\ \text{V}/(5\ \text{k} + 1\ \text{k}) = 1$ mA.

RC circuit: The switch has been closed for a long time and opens at $t = 0$.



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$v_c(0^-) = i(0^-)\, R_1 = 5\,\text{V} \Rightarrow v_c(0^+) = 5\,\text{V}$

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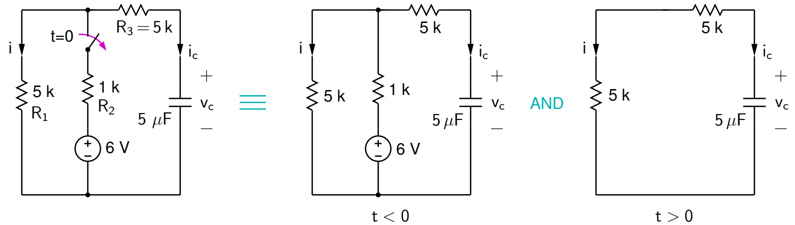

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$\Rightarrow i(0^+) = 5\text{ V}/(5\text{ k} + 5\text{ k}) = 0.5\text{ mA}$.

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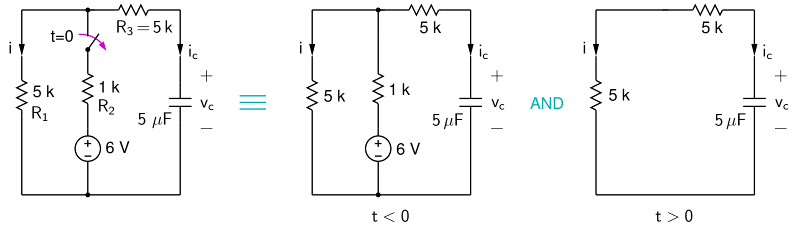

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Let $i(t) = A\exp(-t/\tau) + B$ for $t > 0$, with $\tau = 10\text{ k} \times 5\,\mu\text{F} = 50\text{ ms}$.

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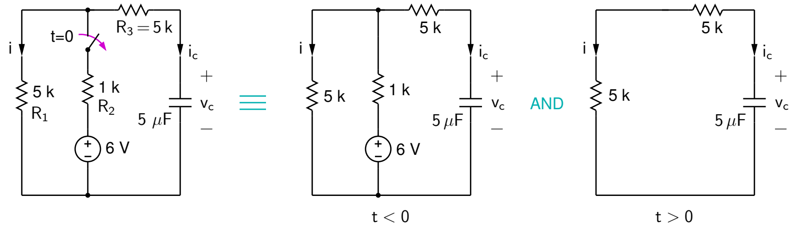

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Using $i(0^+)$ and $i(\infty) = 0$ A, we get

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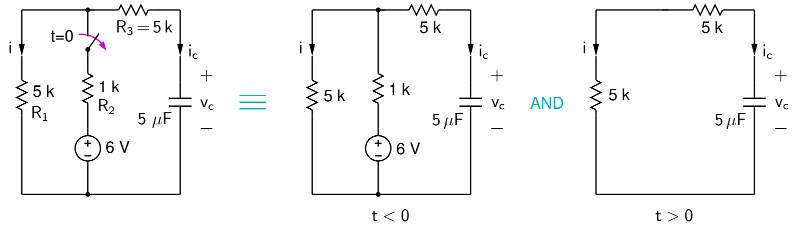

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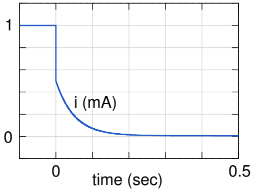

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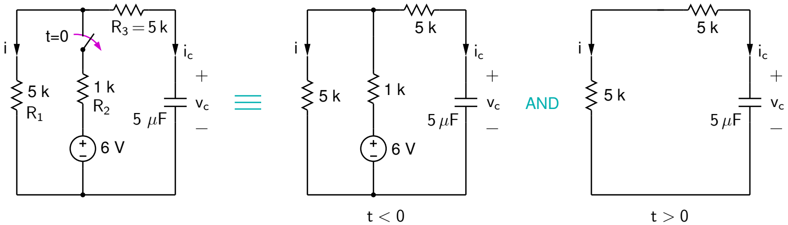

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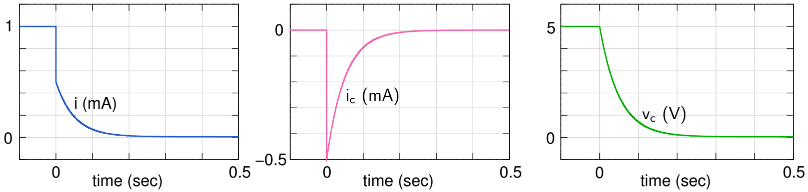

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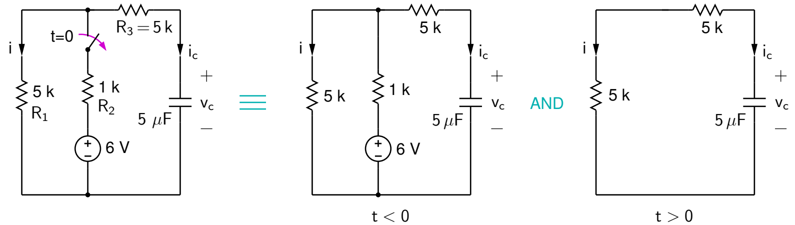

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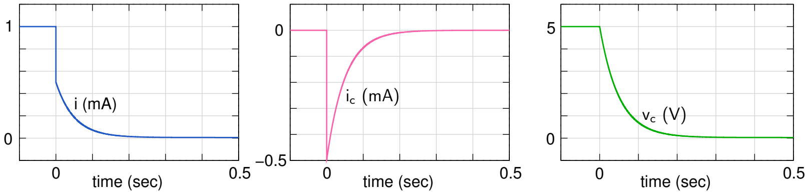
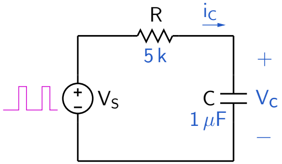
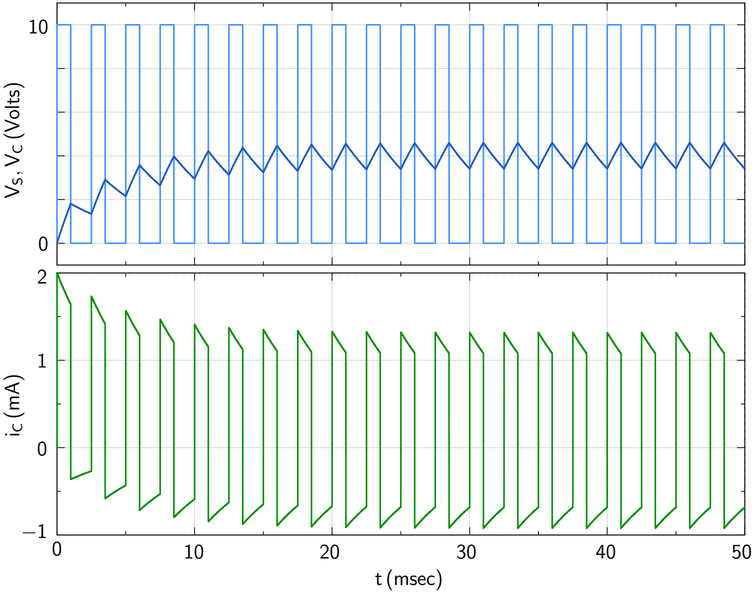
(SEQUEL file: ee101_rc2.sqproj)

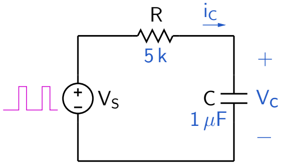


M. B. Patil, IIT Bombay

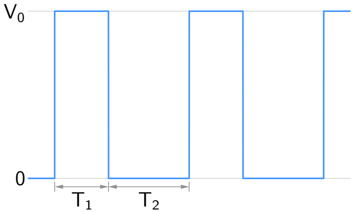
RC circuit: example

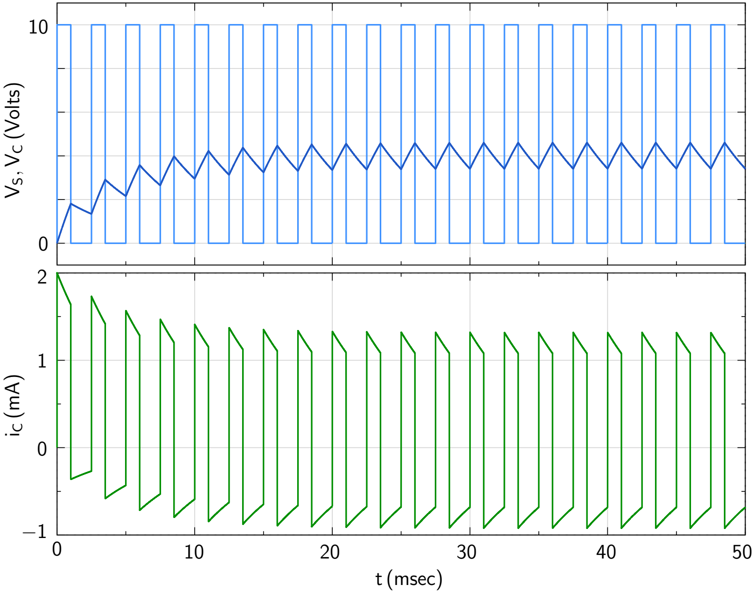






Find expressions for $V_C(t)$ and $i_C(t)$ in steady state (in terms of R, C, V_0, T_1, T_2).





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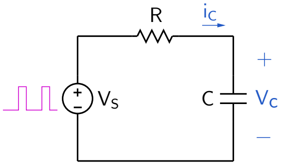
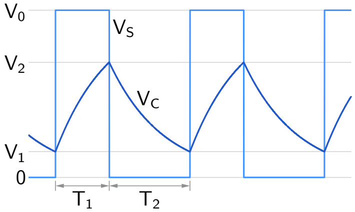
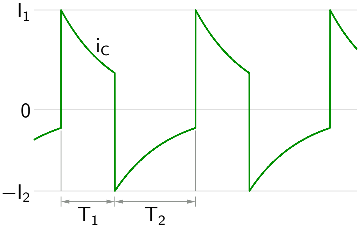
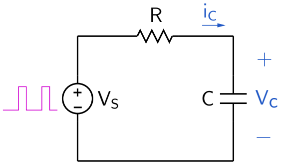
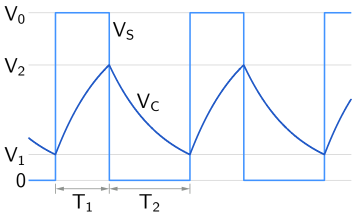
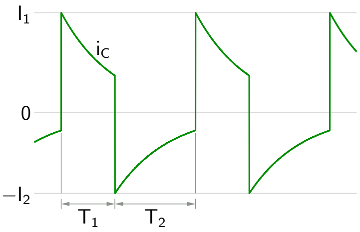






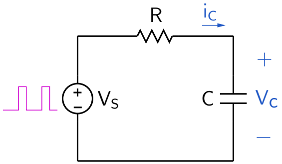


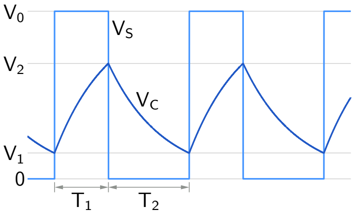


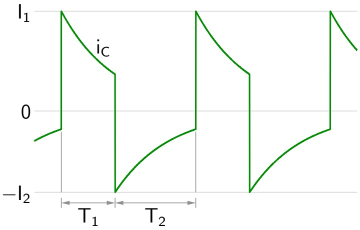


$\boxed{0 < t < T_1}$ Let $V_C^{(1)}(t) = A\, e^{-t/\tau} + B$

RC circuit: example







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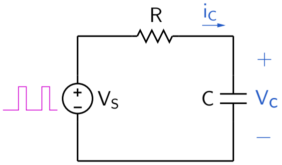

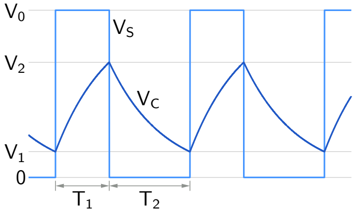

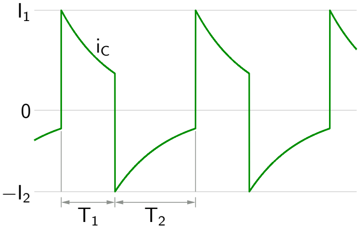

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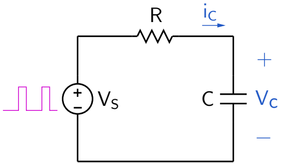
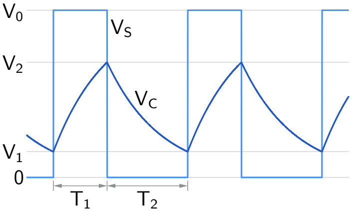
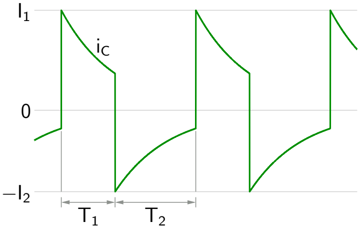
RC circuit: example







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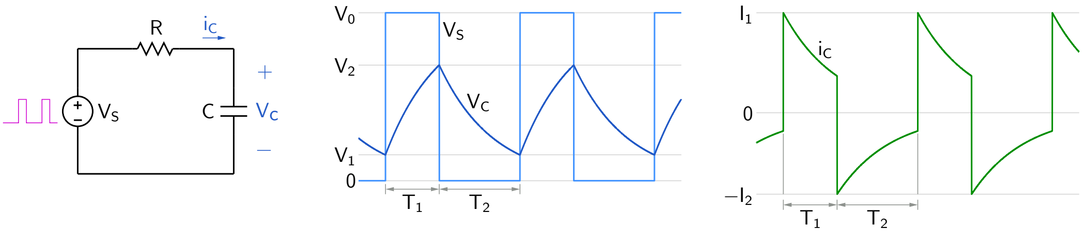

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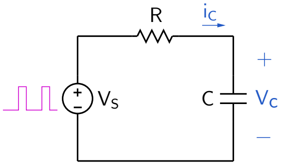

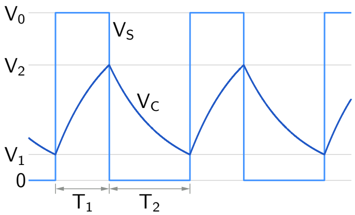

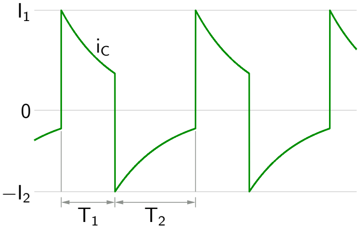

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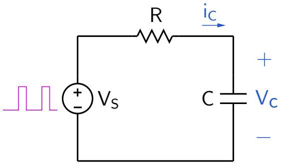
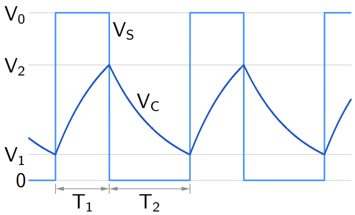
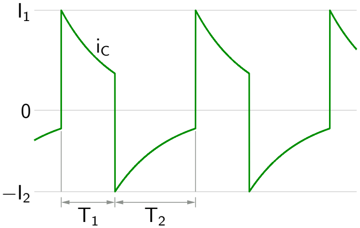
RC circuit: example







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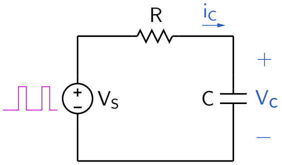
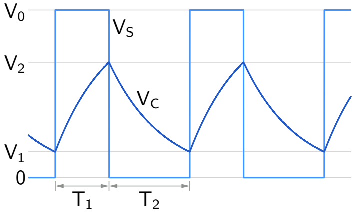
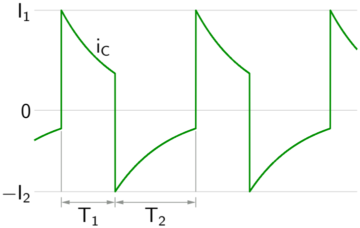
RC circuit: example







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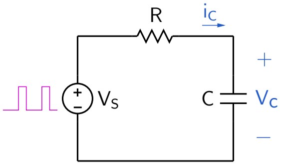
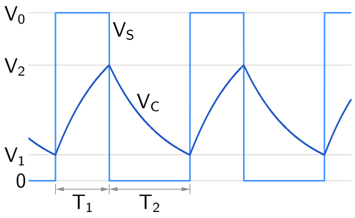
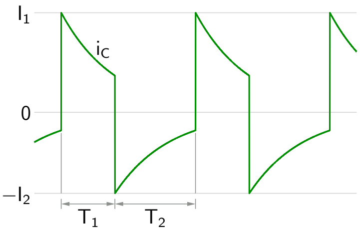
$$V_C^{(2)}(t) = V_2 e^{-(t-T_1)/\tau} \qquad (2)$$







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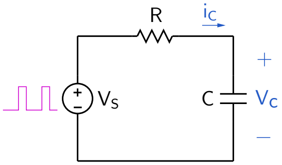

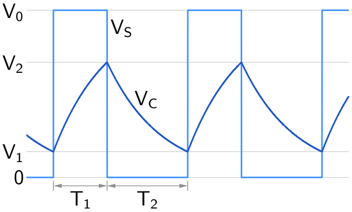

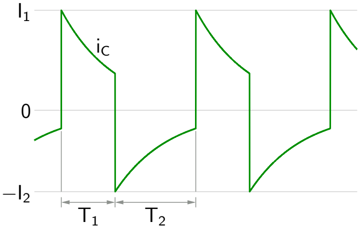

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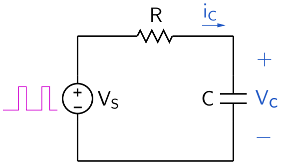
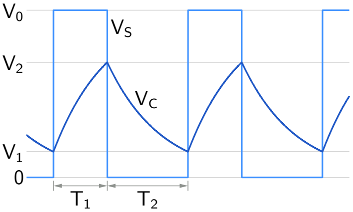
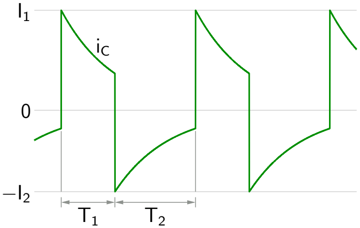
RC circuit: example







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$$V_2 = -(V_0 - V_1)e^{-T_1/\tau} + V_0 \qquad (3)$$

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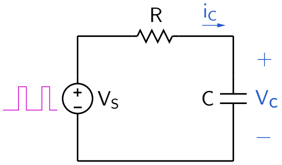

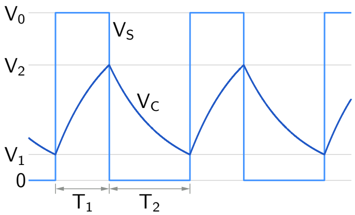

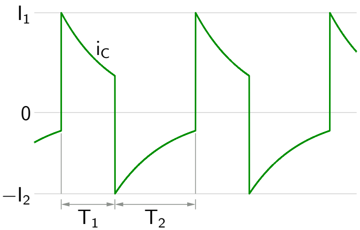

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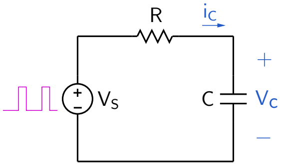
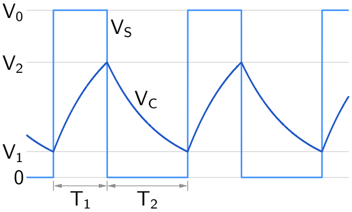
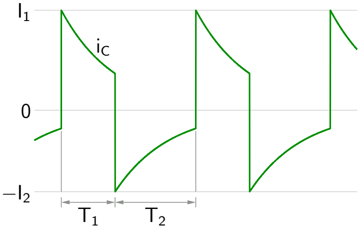
Rewrite with $a \equiv e^{-T_1/\tau}$, $b \equiv e^{-T_2/\tau}$.

RC circuit: example







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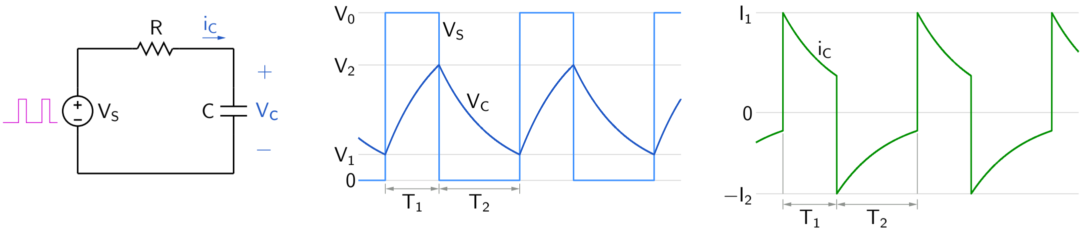

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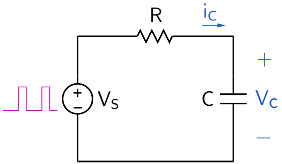
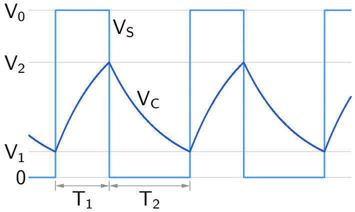
$$V_1 = V_2 e^{-(T_1+T_2-T_1)/\tau} = V_2 e^{-T_2/\tau} \qquad (4)$$

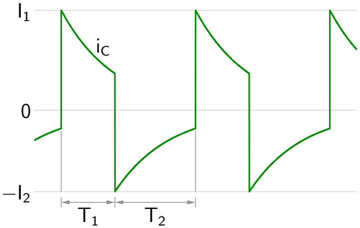
Rewrite with $a \equiv e^{-T_1/\tau}$, $b \equiv e^{-T_2/\tau}$.

$$V_2 = -(V_0 - V_1)a + V_0 \qquad (5)$$

$$V_1 = b\, V_2 \qquad (6)$$







$\boxed{0 < t < T_1}$ Let $V_C^{(1)}(t) = A\,e^{-t/\tau} + B$

$V_C^{(1)}(0) = V_1,\ V_C^{(1)}(\infty) = V_0$

$\rightarrow\ B = V_0,\ A = V_1 - V_0.$

$$V_C^{(1)}(t) = -(V_0 - V_1)e^{-t/\tau} + V_0 \qquad (1)$$

$\boxed{T_1 < t < T_2}$ Let $V_C^{(2)}(t) = A'\,e^{-t/\tau} + B'$

$V_C^{(2)}(T_1) = V_2,\ V_C^{(2)}(\infty) = 0$

$\rightarrow\ B' = 0,\ A' = V_2\,e^{T_1/\tau}.$

$$V_C^{(2)}(t) = V_2 e^{-(t-T_1)/\tau} \qquad (2)$$

Now use the conditions:

$V_C^{(1)}(T_1) = V_2,\ V_C^{(2)}(T_1 + T_2) = V_1.$

$$V_2 = -(V_0 - V_1)e^{-T_1/\tau} + V_0 \qquad (3)$$

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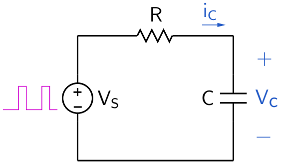
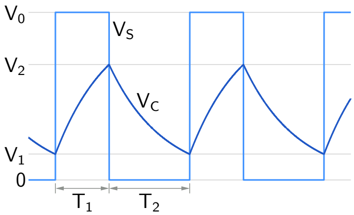
$$V_2 = -(V_0 - V_1)a + V_0 \qquad (5)$$

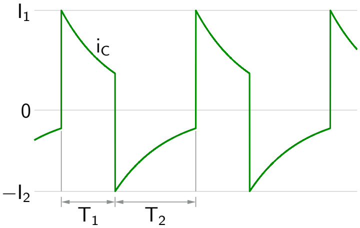
$$V_1 = b\,V_2 \qquad (6)$$

Solve to get

$$\boxed{V_1 = b\,V_0\,\frac{1-a}{1-ab},\ V_2 = V_0\,\frac{1-a}{1-ab}}$$

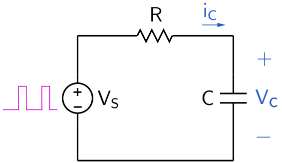


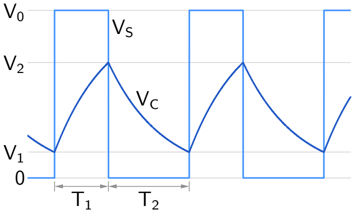


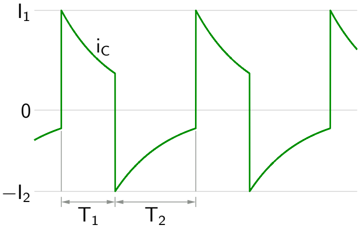


$$V_1 = b\,V_0\,\frac{1-a}{1-ab},\quad V_2 = V_0\,\frac{1-a}{1-ab},\quad \text{with}\quad a = e^{-T_1/\tau},\quad b = e^{-T_2/\tau}.$$

RC circuit: example







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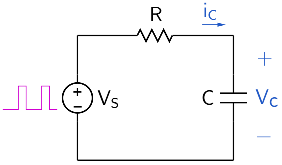
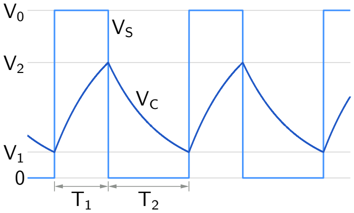
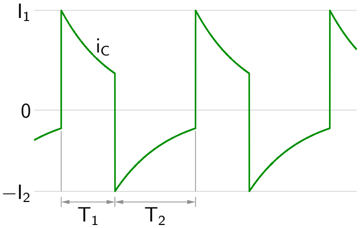
RC circuit: example







$$V_1 = b\, V_0 \frac{1-a}{1-ab}, \quad V_2 = V_0 \frac{1-a}{1-ab}, \quad \text{with} \quad a = e^{-T_1/\tau}, \quad b = e^{-T_2/\tau}.$$

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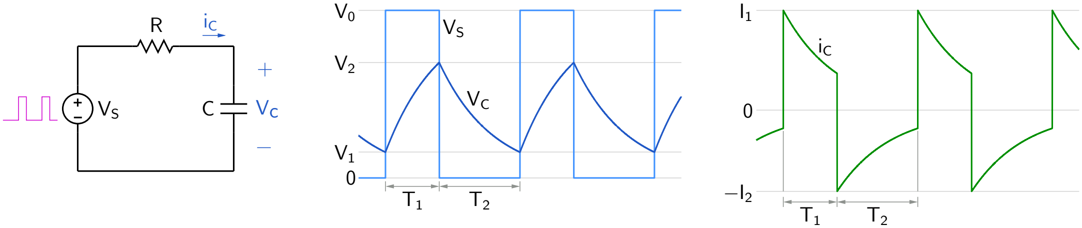
Current calculation:

RC circuit: example



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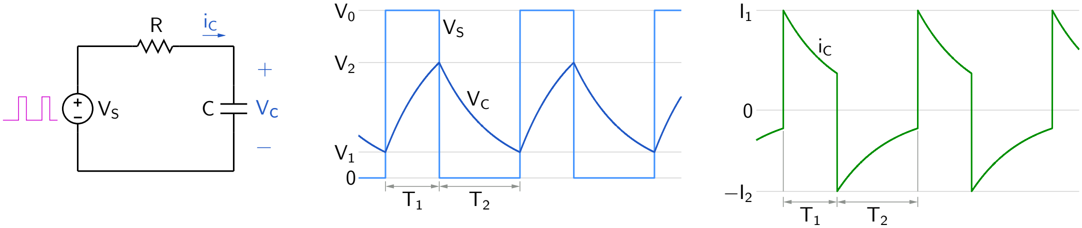
Current calculation:

Method 1:

$$i_C(t) = C\,\frac{dV_C}{dt}\ \text{(home work)}$$

RC circuit: example



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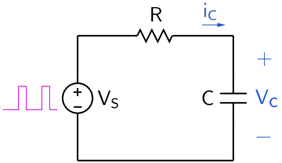
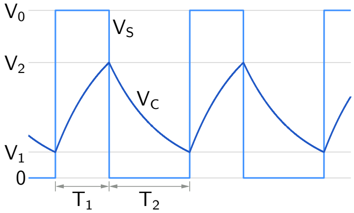
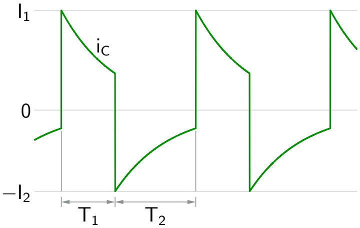
Method 2:

Start from scratch!

RC circuit: example







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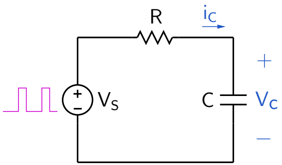
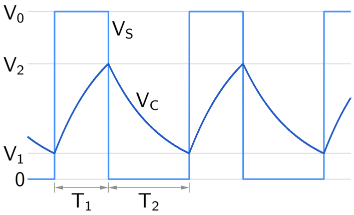
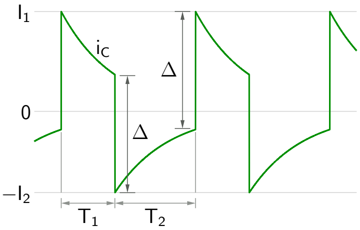
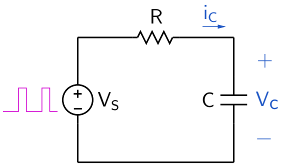
RC circuit: example

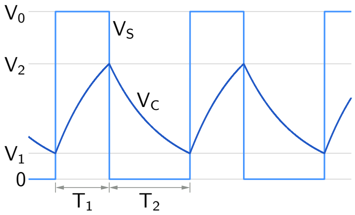


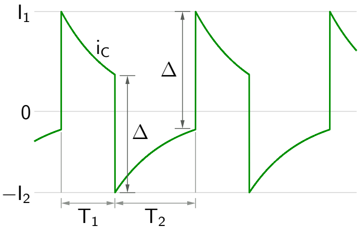




RC circuit: example







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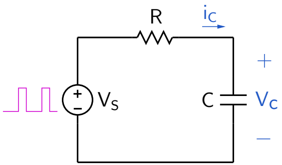

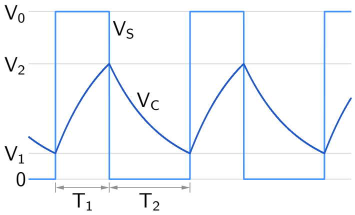

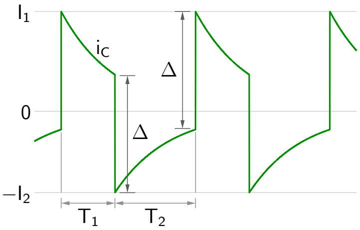

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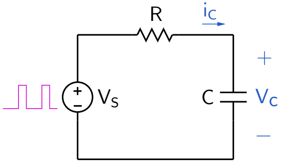
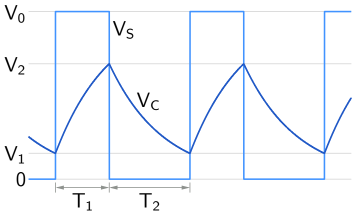
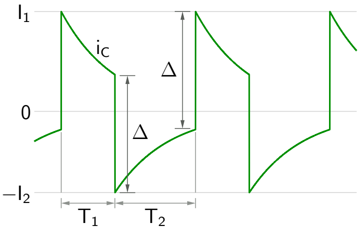
RC circuit: example







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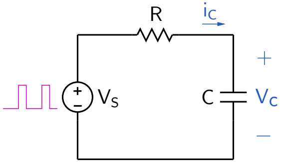
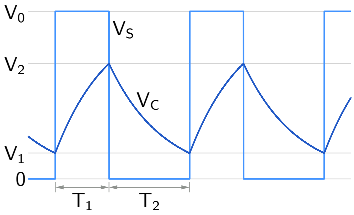
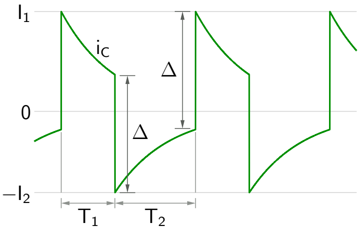
RC circuit: example







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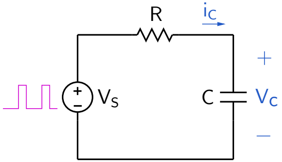
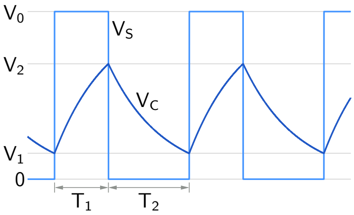
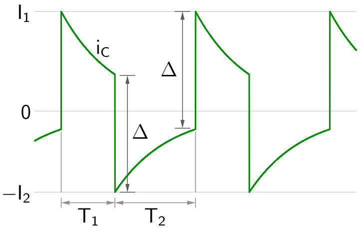
RC circuit: example







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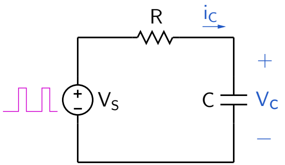

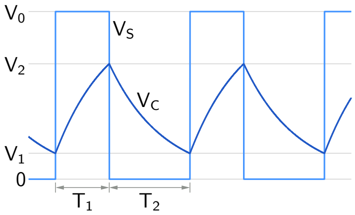

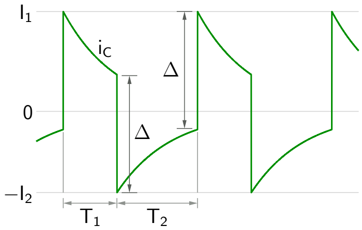

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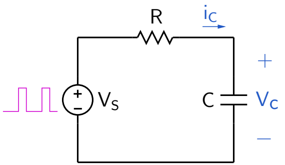

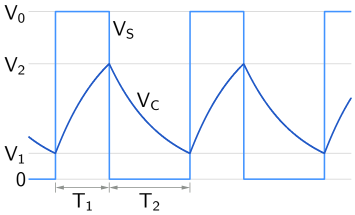

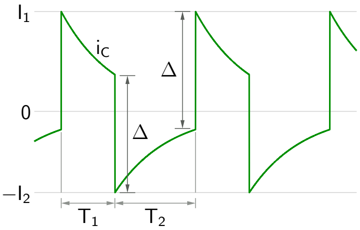

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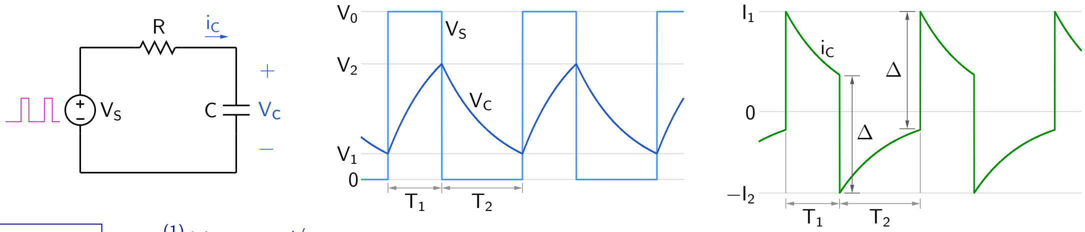
RC circuit: example



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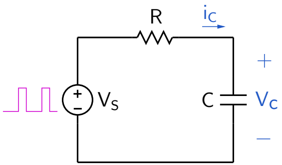
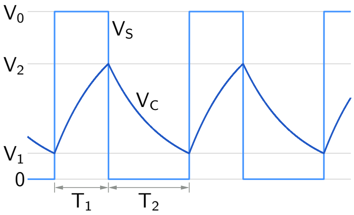
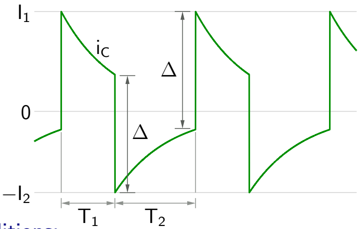
RC circuit: example







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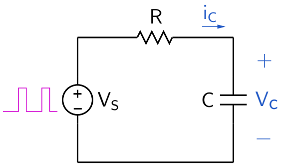
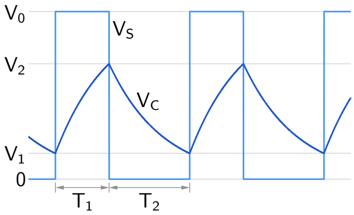
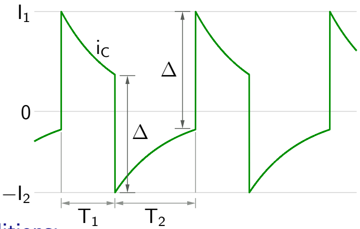
Now use the conditions:

$i_C^{(1)}(T_1) - i_C^{(2)}(T_1) = \Delta = V_0/R,$

RC circuit: example







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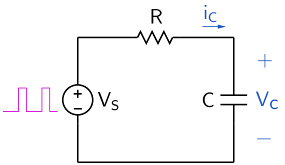
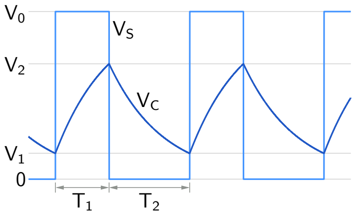
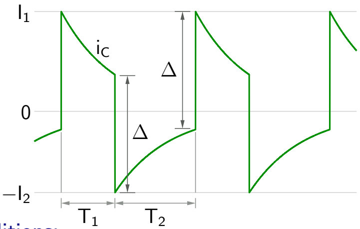
$i_C^{(1)}(0) - i_C^{(2)}(T_1 + T_2) = \Delta = V_0/R.$

RC circuit: example







$\boxed{0 < t < T_1}$ Let $i_C^{(1)}(t) = A\,e^{-t/\tau} + B$

$i_C^{(1)}(0) = I_1,\ i_C^{(1)}(\infty) = 0$

$\rightarrow\ B = 0,\ A = I_1.$

$$i_C^{(1)}(t) = I_1 e^{-t/\tau} \qquad (1)$$

$\boxed{T_1 < t < T_2}$ Let $i_C^{(2)}(t) = A'\,e^{-t/\tau} + B'$

$i_C^{(2)}(T_1) = -I_2,\ i_C^{(2)}(\infty) = 0$

$\rightarrow\ B' = 0,\ A' = -I_2\,e^{T_1/\tau}.$

$$i_C^{(2)}(t) = -I_2 e^{-(t-T_1)/\tau} \qquad (2)$$

Now use the conditions:

$i_C^{(1)}(T_1) - i_C^{(2)}(T_1) = \Delta = V_0/R,$

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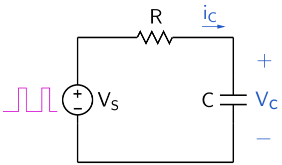
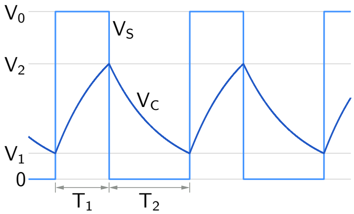
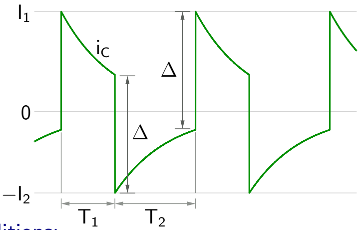
$$I_1 e^{-T_1/\tau} - (-I_2) = \Delta \qquad (3)$$







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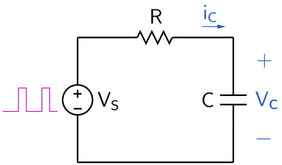
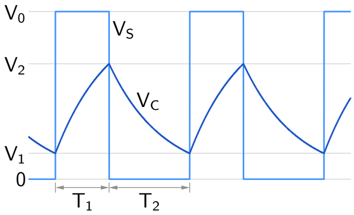
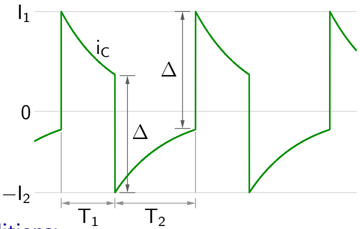
$$I_1 e^{-T_1/\tau} - (-I_2) = \Delta \qquad (3)$$

$$I_1 - (-I_2 e^{-(T_1+T_2-T_1)/\tau}) = \Delta \qquad (4)$$

RC circuit: example







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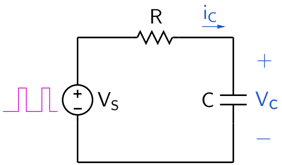
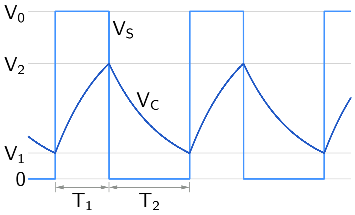
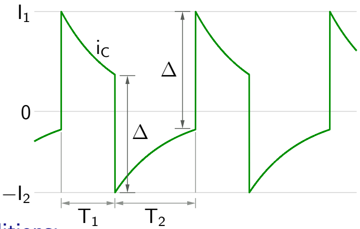
$$a\,I_1 + I_2 = \Delta \qquad (5)$$







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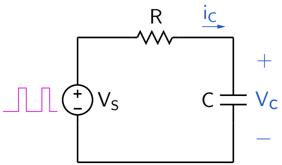
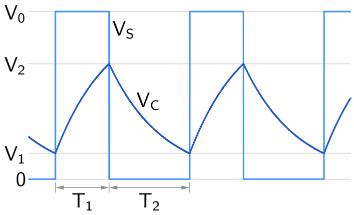
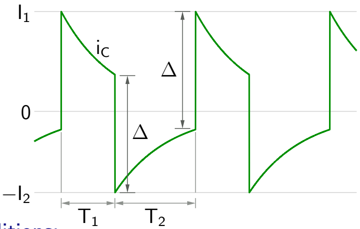
$$I_1 + b\,I_2 = \Delta \qquad (6)$$

RC circuit: example







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$$a\,I_1 + I_2 = \Delta \qquad (5)$$

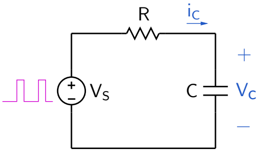
$$I_1 + b\,I_2 = \Delta \qquad (6)$$

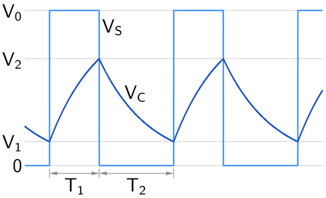
Solve to get

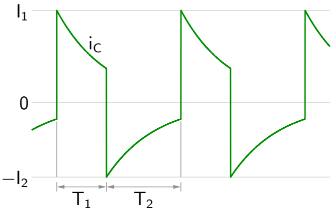
$$\boxed{I_1 = \Delta\,\frac{1-b}{1-ab},\; I_2 = \Delta\,\frac{1-a}{1-ab}}$$

$(a = e^{-T_1/\tau},\;\; b = e^{-T_2/\tau})$

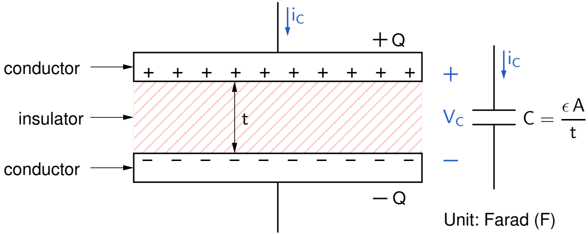
RC circuit: example





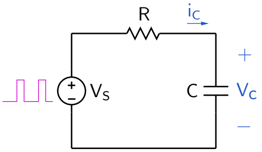


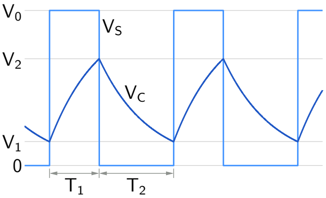
Charge conservation:

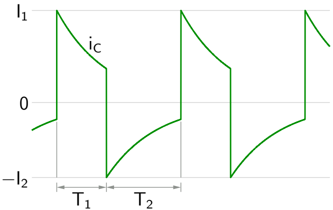


Unit: Farad (F)

$$i_C = \frac{dQ}{dt} = C\,\frac{dV_C}{dt}$$

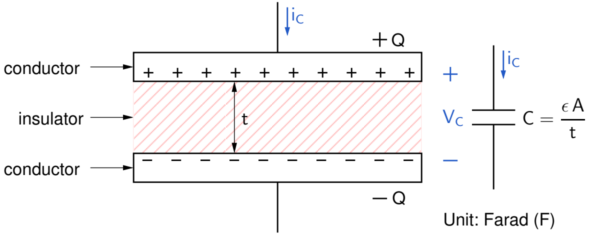






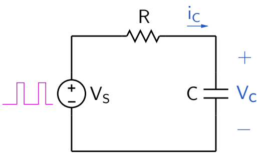
Charge conservation:

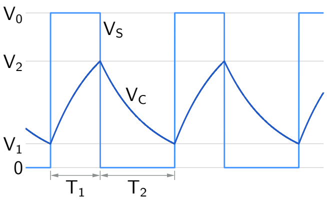
Periodic steady state: All quantities are periodic, i.e., $x(t_0 + T) = x(t_0)$

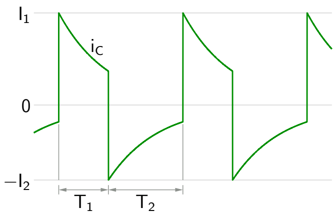


$$i_C = \frac{dQ}{dt} = C\,\frac{dV_C}{dt}$$

RC circuit: example



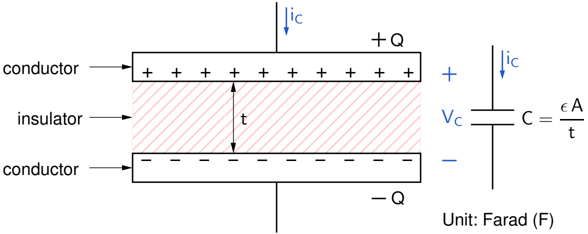




Charge conservation:

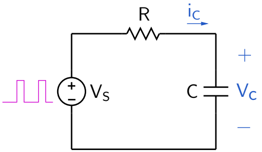
Periodic steady state: All quantities are periodic, i.e.,
$x(t_0 + T) = x(t_0)$

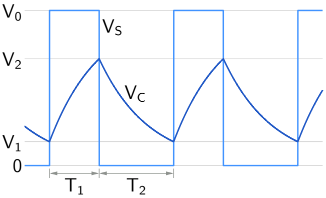
Capacitor charge: $Q(t_0 + T) = Q(t_0)$

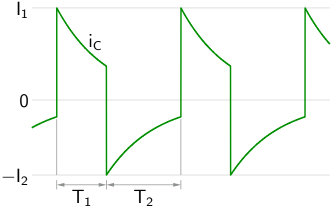


$$i_C = \frac{dQ}{dt} = C\,\frac{dV_C}{dt}$$

RC circuit: example





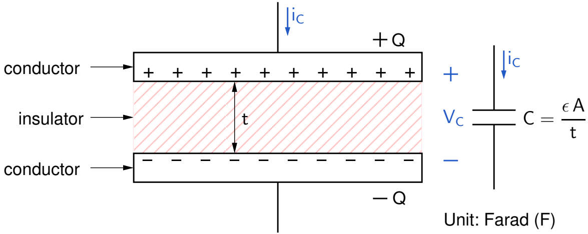


Charge conservation:

Periodic steady state: All quantities are periodic, i.e., $x(t_0 + T) = x(t_0)$

Capacitor charge: $Q(t_0 + T) = Q(t_0)$

$$i_C = \frac{dQ}{dt} \rightarrow Q = \int i_C \, dt.$$



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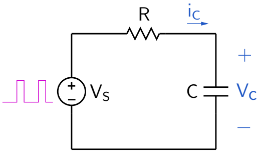
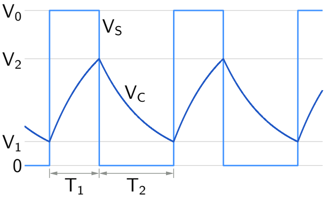
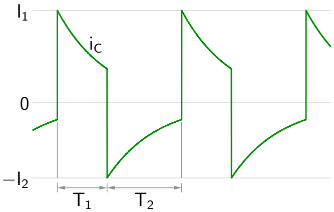
RC circuit: example







Charge conservation:

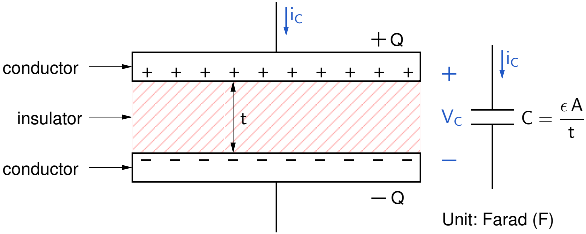
Periodic steady state: All quantities are periodic, i.e.,
$x(t_0 + T) = x(t_0)$

Capacitor charge: $Q(t_0 + T) = Q(t_0)$

$$i_C = \frac{dQ}{dt} \rightarrow Q = \int i_C \, dt.$$

$$Q(t_0 + T) = Q(t_0) \rightarrow Q(t_0 + T) - Q(t_0) = 0$$

$$\rightarrow \int_{t_0}^{t_0+T} i_C \, dt = 0.$$



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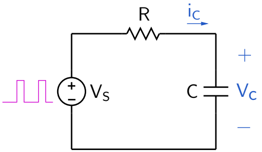
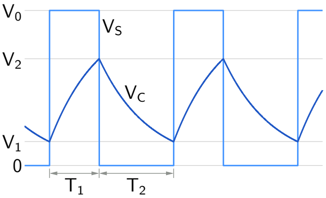
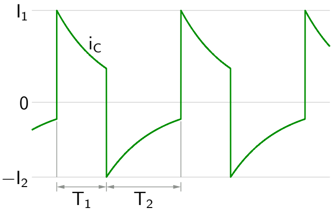
RC circuit: example







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Capacitor charge: $Q(t_0+T)=Q(t_0)$

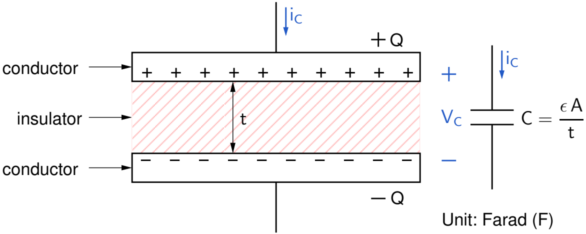
$$i_C=\frac{dQ}{dt}\rightarrow Q=\int i_C\,dt.$$

$$Q(t_0+T)=Q(t_0)\rightarrow Q(t_0+T)-Q(t_0)=0$$

$$\rightarrow \int_{t_0}^{t_0+T} i_C\,dt=0.$$

$$\int_0^T i_C\,dt=0\rightarrow \int_0^{T_1} i_C\,dt+\int_{T_1}^{T_1+T_2} i_C\,dt=0$$

$$\rightarrow \boxed{\int_{T_1}^{T_1+T_2} i_C\,dt=-\int_0^{T_1} i_C\,dt.}$$



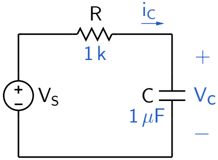
Unit: Farad (F)

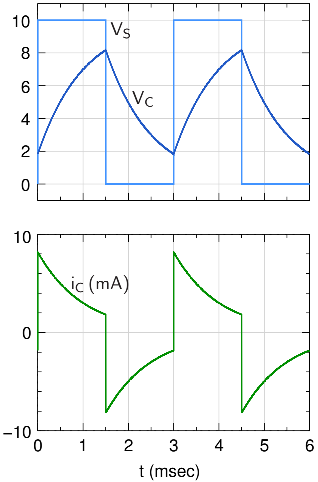
$$i_C=\frac{dQ}{dt}=C\,\frac{dV_C}{dt}$$

$T_1 = 1.5$ msec, $T_2 = 1.5$ msec.

$V_1 = 1.8$ V, $V_2 = 8.2$ V.

$I_1 = 8.2$ mA, $I_2 = 8.2$ mA.

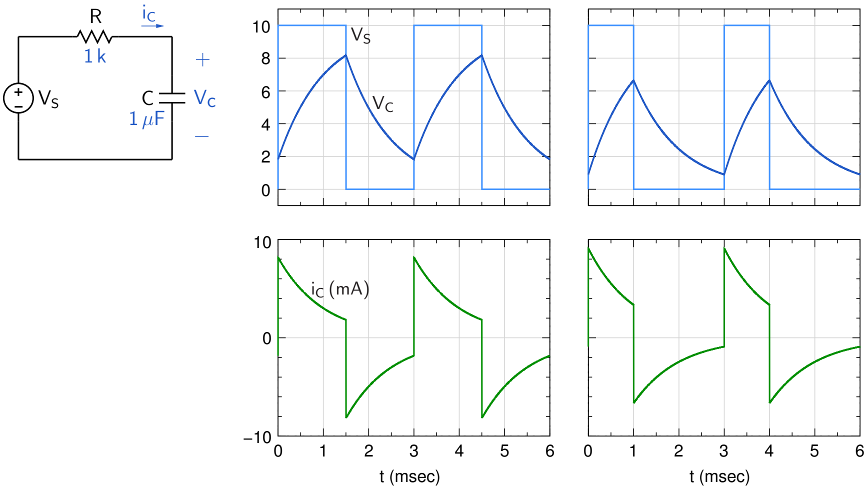


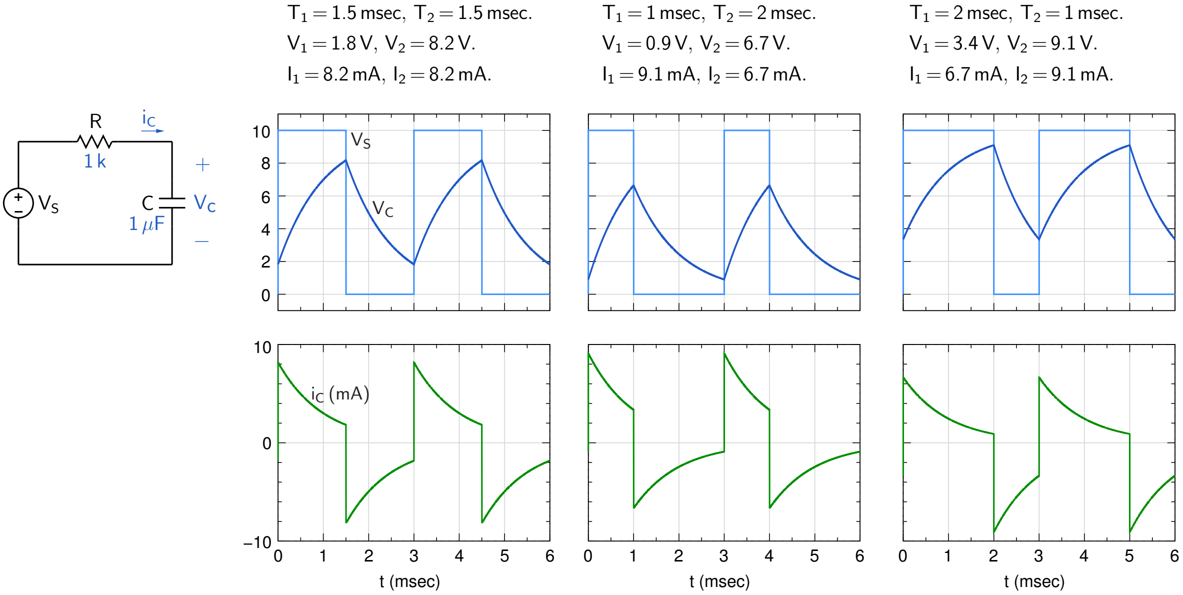


$T_1 = 1.5$ msec, $T_2 = 1.5$ msec. $T_1 = 1$ msec, $T_2 = 2$ msec.

$V_1 = 1.8$ V, $V_2 = 8.2$ V. $V_1 = 0.9$ V, $V_2 = 6.7$ V.

$I_1 = 8.2$ mA, $I_2 = 8.2$ mA. $I_1 = 9.1$ mA, $I_2 = 6.7$ mA.

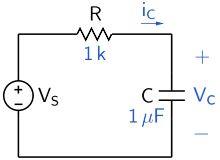


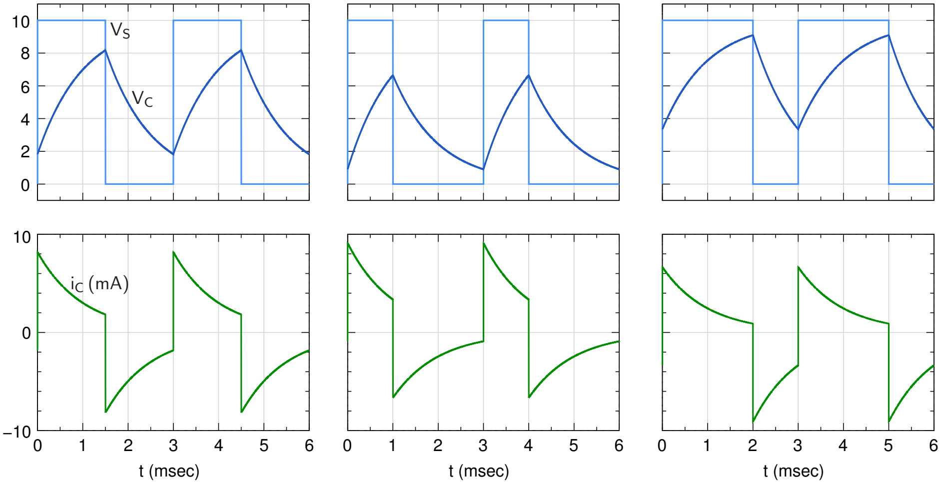

$T_1 = 1.5$ msec, $T_2 = 1.5$ msec.
$V_1 = 1.8$ V, $V_2 = 8.2$ V.
$I_1 = 8.2$ mA, $I_2 = 8.2$ mA.
$T_1 = 1$ msec, $T_2 = 2$ msec.
$V_1 = 0.9$ V, $V_2 = 6.7$ V.
$I_1 = 9.1$ mA, $I_2 = 6.7$ mA.
$T_1 = 2$ msec, $T_2 = 1$ msec.
$V_1 = 3.4$ V, $V_2 = 9.1$ V.
$I_1 = 6.7$ mA, $I_2 = 9.1$ mA.
R
i_C
1 k
V_S
C
1 μF
V_C
+
−
V_S
V_C
i_C (mA)
t (msec)
t (msec)
t (msec)

$T_1 = 1.5$ msec, $T_2 = 1.5$ msec.
$V_1 = 1.8$ V, $V_2 = 8.2$ V.
$I_1 = 8.2$ mA, $I_2 = 8.2$ mA.

$T_1 = 1$ msec, $T_2 = 2$ msec.
$V_1 = 0.9$ V, $V_2 = 6.7$ V.
$I_1 = 9.1$ mA, $I_2 = 6.7$ mA.

$T_1 = 2$ msec, $T_2 = 1$ msec.
$V_1 = 3.4$ V, $V_2 = 9.1$ V.
$I_1 = 6.7$ mA, $I_2 = 9.1$ mA.





SEQUEL file: `ee101_rc1b.sqproj`

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$T_1 = 1\,\text{msec},\ T_2 = 2\,\text{msec}.$

$V_1 \approx 0\,\text{V},\ V_2 = 10\,\text{V}.$

$I_1 = 100\,\text{mA},\ I_2 = 100\,\text{mA}.$