Assignment 7

1) If $L$ is a positive definite, self-adjoint operator and $L\Phi = g$ has a solution $\Phi_0$, then the function $I$ is

$$ I = \langle L\Phi, \Phi \rangle + 2\langle \Phi, g \rangle $$

or

$$ I = \langle L\Phi, \Phi \rangle - 2\langle \Phi, g \rangle $$

or

$$ I = \langle L\Phi, \Phi \rangle + 2\langle \Phi, g \rangle + \Phi_0 $$

or

$$ I = \langle L\Phi, \Phi \rangle - 2\langle \Phi, g \rangle + \Phi_0 $$

Accepted Answers:

$I = \langle L\Phi, \Phi \rangle - 2\langle \Phi, g \rangle$

2) The function that minimizes the functional $I(\phi)$, where

$$ I(\Phi) = \frac{1}{2} \int \left[ (\nabla \Phi)^2 + k^2 \Phi^2 + 2g\Phi \right] dS $$

is a solution to

- Poisson equation
- Laplace equation
- Inhomogeneous Helmholtz equation
- Advection equation

Accepted Answers:

Inhomogeneous Helmholtz equation

3) Consider the equation

$$ \frac{\partial^2 \Phi}{\partial x^2} + x \frac{\partial \Phi}{\partial x} + \Phi = 2x $$

subject to $\Phi(0) = 1$, $\Phi(1) = 0$. Using Galerkin's method with first, second and third approximation, which of the following statement(s) is/are true for basis function $u_n$?

1. $u_1 = x(1-x)$ for $N = 1$
2. $u_2 = x(1+x)$ and $u_2 = x^2(1+x)$ for $N = 2$
3. $u_3 = x(1-x)$, $u_2 = x^2(1-x)$, $u_3 = x^3 - x^2$ for $N = 3$
4) Consider the following equation which relates potential within element \( V_e \) and potential at nodes in elements \( V_{ei} \).

\[
V_e = \sum_{i=1}^{n} \alpha_i(x, y) V_{ei}
\]

Which of the following statement(s) is/are true for the element shape function \( \alpha_i(x, y) \)?

1. \( \alpha_i = 1 \) when \( i=j; 0 \) otherwise.
2. \( \sum_{i=1}^{n} \alpha_i(x, y) = 1 \)
3. \( \sum_{i=1}^{n} \alpha_i(x, y) = 0 \)

**Accepted Answers:**

1 and 3 are correct, but not 2.

5) In finite element method, the material matrix tells about

1. Number of global nodes.
2. Number of local nodes.
3. Number of elements.

**Accepted Answers:**

All of them are correct.

6) In finite element method, which of the following statement(s) is/are true regarding element coefficient matrix and global coefficient matrix?

1. Element coefficient matrix which will give coupling between nodes within an element.

**Accepted Answers:**

All of them are correct.
2. Global coefficient matrix which contains transfer values of local nodes into respective values global nodes.
3. Element coefficient matrix and global coefficient matrix are equal for a single triangular element.

- Only 1 is correct, but not others.
- Only 2 is correct, but not others.
- Only 3 is correct, but not others.
- 1 and 2 are correct, but not 3.
- 1 and 3 are correct, but not 2.
- 2 and 3 are correct, but not 1.
- All of them are correct.
- None of them are correct.

**Accepted Answers:**
All of them are correct.

7) Consider two functions given by, \( u(x) = 1 - x \) and \( v(x) = 2x \). In the interval \((0, 1)\), their inner product is

- 0.125
- 0.215
- 0.333
- 0.500
- -0.125
- -0.215

**Accepted Answers:**
0.333

8) Consider the wave equation
\[ \nabla^2 \Phi + k^2 \Phi = g \]
Which of the following statement(s) is/are true?

1. \( k = 0 \Rightarrow g \); Laplace equation.
2. \( k = 0 \); Poisson equation.
3. \( k \) is an unknown, \( g = 0 \); homogeneous, scalar Helmholtz equation.

- Only 1 is correct, but not others.
- Only 2 is correct, but not others.
- Only 3 is correct, but not others.
- 1 and 2 are correct, but not 3.
- 1 and 3 are correct, but not 2.
- 2 and 3 are correct, but not 1.
- All of them are correct.
- None of them are correct.

**Accepted Answers:**
All of them are correct.

9) **Common data for questions 9 and 10**

The 2D co-ordinates of the quadrilateral element ABCD are given below.
Node A: (2, 2), Node B: (10, 3), Node C: (7, 9) and Node D: (3, 11)
What is the closest approximation to the area of the quadrilateral element ABCD?

- 26.5
- 45.5
- 44.5
- 50.5
- 65.5
- 100

**Accepted Answers:**
- 44.5

10. What is the closest approximation to the length of the diagonal AC?

- 6.7
- 6.8
- 7.6
- 8.6
- 10.8

**Accepted Answers:**
- 8.6