Unit 7 - Week 5:
Variational Methods

Assignment 5

1) Rayleigh-Ritz method is applicable
   1. When a suitable functional exists.
   2. When a nonlinear functional exists.
   3. For time-dependent problems.
   4. For time-independent problems.
   - Only 1 is correct, but not others.
   - Only 2 is correct, but not others.
   - Only 3 is correct, but not others.
   - Only 4 is correct, but not others.
   - 1 and 2 are correct, but not others.
   - 1 and 3 are correct, but not others.
   - 1 and 4 are correct, but not others.
   - 2 and 3 are correct, but not others.
   - 2 and 4 are correct, but not others.
   - 1, 2, and 3 are correct, but not 4.
   - 1, 2, and 4 are correct, but not 3.
   - 1, 3, and 4 are correct, but not 2.
   - None of them are correct.

Accepted Answers:
1, 3, and 4 are correct, but not 2.

2) For any linear operator and linear functions $u$, $v$, and $g$ which of the following is/are correct?
   - 1. is self-adjoint when $\langle Lu, g \rangle = \langle u, Lg \rangle$
   - 2. $L$ is positive definite when $\langle Lu, u \rangle > 0$
   - 3. If $\langle u, v \rangle > 0$ then $u$ and $v$ are orthogonal.
   - 4. if $\langle u, v \rangle$ is a large number then $u$ and $v$ are not related.
   - Only 1 is correct, but not others.
   - Only 2 is correct, but not others.
   - Only 3 is correct, but not others.
1) Only 4 is correct, but not others.
2) 1 and 2 are correct, but not others.
3) 1 and 3 are correct, but not others.
4) 1 and 4 are correct, but not others.
5) 2 and 3 are correct, but not others.
6) 2 and 4 are correct, but not others.
7) 3 and 4 are correct, but not others.
8) 1, 2, and 3 are correct, but not 4.
9) 1, 3, and 4 are correct, but not 2.
10) 2, 3, and 4 are correct, but not 1.
11) All of them are correct.
12) None of them are correct.

**Accepted Answers:**

- Only 2 are correct, but not others.

2) For a 1D problem, a function \( \Phi(x) \) is defined at \( N \) different nodes as below.

\[
\Phi(x) = \sum_{n=1}^{N} A_n \Phi_n(x)
\]

The accuracy and computational time of Rayleigh-Ritz method crucially depends on

1. Function \( \Phi_n(x) \)
2. \( A_n \)
3. \( N \)

- Only 1 is correct, but not others.
- Only 2 is correct, but not others.
- Only 3 is correct, but not others.
- 1 and 2 are correct, but not 3.
- 1 and 3 are correct, but not 2.
- 2 and 3 are correct, but not 1.
- All of them are correct.
- None of them are correct.

**Accepted Answers:**

- 1 and 3 are correct, but not 2.

4) Which of the following statement(s) is/are true for the method of weighted residuals?

1. It solves the partial differential equations analytically.
2. It does not require a suitable functional to exist.
3. It needs matrix inversion.
4. It needs no matrix inversion.

- Only 1 is correct, but not others.
- Only 2 is correct, but not others.
- Only 3 is correct, but not others.
- Only 4 is correct, but not others.
- 1 and 2 are correct, but not others.
- Only 4 is correct, but not others.
- 1 and 3 are correct, but not others.
- 1 and 4 are correct, but not others.
- 2 and 3 are correct, but not others.
- 2 and 4 are correct, but not others.
- None of them are correct.
Accepted Answers:
2 and 3 are correct, but not others.

5) Common data for questions 5 and 6

Consider the equation \(-\frac{d^2f}{dx^2} = 1 + x^2\) for \(0 \leq x \leq 1\)
subject to boundary conditions, \(f(0) = f(1) = 0\).

Using Galerkin’s method with first and second approximation, the solution is
(Hint: Use basis function as \(x - x^n\))

1. \(f^{(1)}(x) = \frac{13}{30} (x - x^2)\) for \(N=1\).
2. \(f^{(1)}(x) = \frac{20}{13} (x - x^2)\) for \(N=1\).
3. \(f^{(2)}(x) = \frac{1}{3} (x - x^2) + \frac{1}{6} (x - x^3)\) for \(N=2\).
4. \(f^{(2)}(x) = \frac{1}{5} (x - x^2) + \frac{1}{2} (x - x^3)\) for \(N=2\).

- Only 1 is correct, but not others.
- Only 2 is correct, but not others.
- Only 3 is correct, but not others.
- Only 4 is correct, but not others.
- 1 and 3 are correct, but not others.
- 1 and 4 are correct, but not others.
- 2 and 3 are correct, but not others.
- 2 and 4 are correct, but not others.
- None of them are correct.

Accepted Answers:
1 and 3 are correct, but not others.

6) For the data given above, what are the residuals of \(f(x)\) for \(N = 2\) at \(x = 0, 0.5,\) and 1? 0.31 points

- -0.05, 0.2, and 0.04 respectively.
- 0.1, -0.05, and 0.2 respectively.
- 0.04, 0.2, and 0.05 respectively.
- 0.2, -0.05, and 0.2 respectively.
- None of the above.

Accepted Answers:
0.2, -0.05, and 0.2 respectively.

7) Consider two functions given by, \(u(x) = 1 - x^2\) and \(v(x) = x\). In the interval (0, 1), their 0.31 points
inner product is

- 0.33
- 0.25
- 0
- -0.25
- -0.33

Accepted Answers:
0.25

8)
If \( u \) and \( v \) are two real functions, \( \alpha \) and \( \beta \) are two scalars, and \( \langle u, v \rangle \) denotes inner product of \( u \) and \( v \), then which of the following correctly represents the properties of inner product?

\[
\langle u + \beta v, w \rangle = \alpha \langle u, w \rangle + \beta \langle v, w \rangle
\]

\[
\langle u, v \rangle^* = \langle v, u \rangle
\]

\[
\langle u, u \rangle > 0 \text{ if } u \neq 0
\]

\[
\langle u, u \rangle = 0 \text{ if } u = 0
\]

None of the above.

**Accepted Answers:**

\[
\langle u, v \rangle = \langle v, u \rangle^*
\]

\[
\langle u + \beta v, w \rangle = \alpha \langle u, w \rangle + \beta \langle v, w \rangle
\]

\[
\langle u, u \rangle > 0 \text{ if } u \neq 0
\]

\[
\langle u, u \rangle = 0 \text{ if } u = 0
\]

9) Consider a functional defined as \( I(y) = \int_0^1 F(x, y, y') dx \).

For \( F(x, y, y') = \frac{1}{2} \left( \frac{dy}{dx} \right)^2 - 1 \), the corresponding Euler equation is

\[
\frac{d^2 y}{dx^2} = 2
\]

\[
\frac{d^2 y}{dx^2} = 0
\]

\[
\frac{d^2 y}{dx^2} = -1
\]

\[
\frac{d^2 y}{dx^2} = 1
\]

**Accepted Answers:**

\[
\frac{d^2 y}{dx^2} = -1
\]

10) Minimizing the energy functional \( I = \frac{1}{2} \int (\nabla V)^2 dv \) is equivalent to solving

Laplace equation

Helmholtz equation

Poisson equation

None of the above.

**Accepted Answers:**
Accepted Answers:
Laplace equation

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