2nd case: Limited Source Diffusion
In this case, fixed amount of impurities reside at the surface and then temperature -Time cycle starts.

From Diffusion Equation we have:

$$\frac{d^2 N(x,s)}{dx^2} - \frac{S N(x,s)}{D} = \frac{N(x,t \leq 0)}{D} = \frac{Q}{sD}$$

Let us have a Sheet charge with dose $Q$ (no/area) as fixed quality and is like a $\delta$ function at $x=0$.
For this case, initial condition is
\[ N(x, t \leq 0^-) = Q \cdot \delta \]

We have solution of diffusion equation as for earlier constant source case as
\[ N(x, s) = B(s) \exp \left\{ -\left(\frac{s}{D}\right)^{1/2} x \right\} \]

\[ \times \left[ N(x, 0^+) = \frac{Q}{s} \right] \]

\[ \Rightarrow \quad N(x, s) = \frac{Q}{\sqrt{s \cdot D}} \exp \left\{ -\left(\frac{s}{D}\right)^{1/2} x \right\} \]

\[ N(x, t) = \frac{Q}{\sqrt{\pi \cdot D \cdot t}} \exp \left( -\frac{x^2}{4 \cdot D \cdot t} \right) \]

This is essentially a Gaussian profile.
Some interesting points to note:

1. Since Impurity Dose was provided only once, it remains Constant.

2. However, subsequent time cycles (at 'A Temperature'), spreads the Impurity Profile as

\[ \int_{0}^{\infty} N(x) \, dx = Q \]
Since \( N(x,t) = \frac{Q}{\sqrt{\pi DT}} \exp \left[ -\frac{x^2}{4DT} \right] \)

we conclude that \( \sqrt{DE} \) Function decides \( N(x,t) \).

Next we look erf\( c \) profile again.

Evaluation of Dose of Impurities in Constant Source Diffusion Case

In this case \( N(x,t) = No \ \text{erfc} \left( \frac{x}{2\sqrt{DT}} \right) \)

We define \( y = \frac{x}{2\sqrt{DT}} \) : \( \frac{dy}{dx} = \frac{1}{2\sqrt{DT}} \)

From Fick's First Law

\( j(x,t) = -D \frac{\partial N}{\partial x} \)
At the surface \((x = 0)\) flux density is 

\[
j(0, t) = -D \frac{\partial N}{\partial x} \bigg|_{x=0}
\]

From erfc profile we get

\[
\frac{dN}{dx} \bigg|_{x=0} = N_0 \frac{d}{dx} \left[ \text{erfc} \left( \frac{x}{2\sqrt{D}t} \right) \right]
\]

\[
= N_0 \cdot \frac{d}{dy} \left[ \text{erfc} \left( y \right) \right] \cdot \frac{1}{2\sqrt{D}t}
\]

\[
= \frac{N_0}{2\sqrt{D}t} \frac{d}{dy} \left( 1 - \text{erf} \left( y \right) \right)
\]

\[
= -\frac{N_0}{2\sqrt{D}t} \frac{d}{dy} \left[ \text{erf} \left( y \right) \right]
\]

\[
= -\frac{N_0}{2\sqrt{D}t} \cdot 2 \cdot \exp \left( -y^2 \right)
\]
\[ j(0,t) = \frac{NoD}{\sqrt{\piDt}} \exp(-y^2) \bigg|_{y=0} \]

\[ = + No \sqrt{\frac{D}{\pi t}} \]

Now, dose \( Q = \int_{0}^{t_1} j(0,t) \, dt \)

where \( t_1 \) is the time for diffusion at temperature \( T_1 \). At \( T_1 \), \( D = D(T_1) = D_1 \)

\[ Q_{T_1, t_1} = No \sqrt{\frac{D_1}{\pi}} \left[ \int_{0}^{t_1} e^{-\frac{y^2}{2}} \, dy \right] \]

\[ = 2 No \sqrt{\frac{D_1 t_1}{\pi}} \]
Junction Formation:

Starting substrate has base concentration as $N_B$. It is one type of semiconductor ($n$ or $p$).

At Junction $N(x,t) - N_B = 0$
Let us take an example of diffusion of n-type of impurities in p-type substrate with concentration Nb. As n-type impurities enter p-water, electron concentration increases near surface and reduces as profile shows.

At $x = x_j$, n-type dopants = p-type dopants (compensation)

For constant source diffusion:

$$N(x, t) = N_0 \text{erfc} \left( \frac{x}{2 \sqrt{Dt}} \right)$$

If diffusion is performed at temperature $T_i$, then,

$$D(T_i) = D_1$$

and $t = t_i$, time of diffusion.

$$Q(T_i, t_i) = 2N_0 \sqrt{\frac{D_i t_i}{\pi}}$$
At Junction, net conc. is zero. Hence we say incoming impurity conc is exactly same as \( N_B \)

\[ N (x_j, t_i) = N_B \]

\[ \alpha = N_0 \text{erfc} \left( \frac{x_j}{2\sqrt{D_i t_i}} \right) = N_B \]

\[ x_j = 2\sqrt{D_i t_i} \text{erfc}^{-1} \left( \frac{N_B}{N_0} \right) \]

In specific case, \( T_i = 1000^\circ \text{C} \) and diffusion of phosphorus takes place in p-doped wafer with conc \( N_B = 5 \times 10^{15} / \text{cc} \) for total time of 60 minutes (3600 seconds).

At \( T = T_i, N_0 = 3 \times 10^{20} / \text{cc} \) for phosphorus

\[ \alpha = 8 \times 10^{14} \text{cm}^2/\text{sec} \]
\[
\sqrt{D_{D1}} = \sqrt{3 \times 10^{14} \times 3600} = 1.04 \times 10^5 \text{ cm}
\]

Further \( \text{erfc}^{-1} \left( \frac{N_b}{N_0} \right) = \text{erfc}^{-1} \left( \frac{5 \times 10^{15}}{3 \times 10^{20}} \right) = \text{erfc}^{-1} \left( 1.66 \times 10^{-5} \right) = x \)

As \( \text{erfc}(x) = 1 - \text{erf}(x) \).

\[
\therefore x_j = 2\sqrt{D_{D1}} \times 3.06
\]

\[
x_j = 2 \times 1.04 \times 10^5 \times 3.06
\]

\[x_j = 0.636 \ \mu\text{m}\]

Let us say \( x = 1.66 \times 10^{-5} \).

\[
\text{Let us define, } \text{erfc} x = 1.66 \times 10^{-5} \text{ or } \text{erf} x = 1 - 1.66 \times 10^{-5} = 0.999984
\]

\[
\therefore x = \text{erf}^{-1} (0.999984) = 3.06 \quad x = \text{erf}^{-1} x
\]
In practice, the diffusion of impurities is a two-step process:

(i) Predereposition

(ii) Drive-in

1. Predereposition:

Here we perform constant source diffusion at temperature $T_1$ for time $t_1$:

$$D_i = D_i(T_1), \quad N_{01} = N_0(T_1)$$

$$N(x, t_1) = N_{01} \text{erfc} \left( \frac{x}{2\sqrt{D_i t_1}} \right)$$

And after time $t_1$, the dose of impurities $Q(t_1)$ at surface:

$$= 2 \cdot N_{01} \sqrt{D_i t_1}$$
(ii) After Predeposition step, source of impurities are stopped. Then the wafers are reintroduced in the furnace at Temperature $T_2$ for time $t_2$ this process is continued. This process is called Drive-In. 

At $T_2$, $D_2(T_2) = D_2$, \therefore Dt = D_2 t_2$

In Drive-In, no new impurities are introduced, but available ones from Predeposition are Re-distributed ($Q$ dose). This is the case of Limited Source Diffusion giving Gaussian Profile:

$$N(x, t_1, t_2) = \frac{Q}{\sqrt{\pi D_2 t_2}} \exp \left(-\frac{x^2}{4 D_2 t_2}\right)$$

$$\propto N(x, t_1, t_2) = \frac{2 N_0}{\pi} \sqrt{\frac{D_1 t_1}{D_2 t_2}} \exp \left(-\frac{x^2}{4 D_2 t_2}\right)$$
Fig. 4.7 Average diffusion coefficients for substitutional diffusers in silicon.

formed. This experimental technique essentially duplicates the fabrication process in monocrystalline wafers. Consequently, the results are reasonably accurate.
Fig. 6.12: Projected range of boron, phosphorus, arsenic, and antimony.
Adapted from Gibbons, Johnson, and Myrolle [9].
4.5.1.2 Diffusion from a Limited Source

Here a finite quantity of the diffusing matter is first placed on the wafer. It is assumed that the source is a thin film of the diffusing atoms, and it is assumed that all of the diffusing matter is contained within the source.
Fig. 1.8. Diffusivities of boron in silicon. From R. Colless, Microelectronics: Practice, 1970. Reprinted with permission of the publisher, John Wiley & Sons, Inc.
Fig. 4.11: Solubility limits for boron, arsenic, and phosphorus. From H. Golab, Microelectronics: Processing and Device Design [10], 1988. Reprinted with permission of the publisher, John Wiley & Sons.
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