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Lec 8
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$\dot{x} = Ax \longrightarrow$

$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 \cdots$

$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 \cdots$

$\dot{x} = g(x)$

$\dot{x}_1 = g_1(x_1, x_2 \ldots x_n)$

$\dot{x}_2 = g_2(x_1, x_2 \ldots x_n)$

$\vdots$

$\dot{x}_n = g_n(x_1, x_2 \ldots x_n)$

$\left.\begin{array}{l} x_3 \\ x_4 \\ \vdots \\ x_n \end{array}\right\}$ fast transients

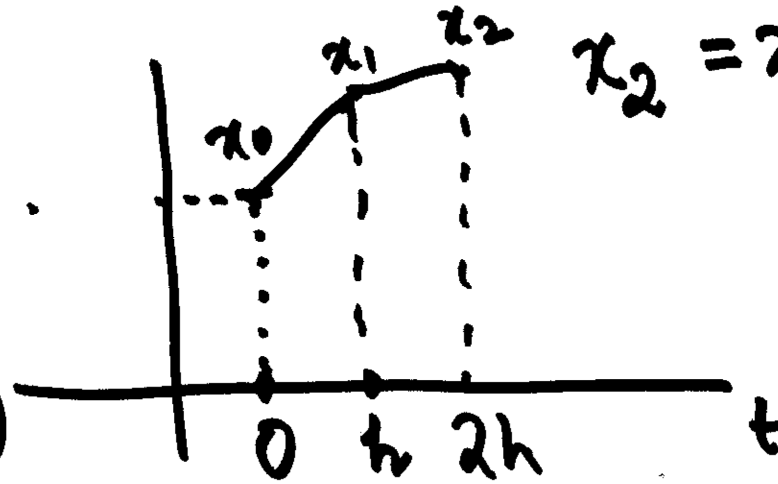
$$
\left.
\begin{aligned}
\dot{x}_1 &= g_1(x_1, x_2 \cdots x_n) \\
\dot{x}_2 &= g_2(x_1, x_2, \cdots x_n) \\
0 &= g_3(x_1, x_2 \cdots x_n) \\
0 &= g_4(x_1 \cdots - - x_n) \\
&\;\vdots \\
0 &= g_n(x_1, x_2 - - - x_n)
\end{aligned}
\right]
$$

Liun $\begin{matrix} \dot{x}_1 \\ \dot{x}_2 \end{matrix}$

$$\dot{x} = f(x, t).$$

$$x_{k+1} = x_k + h f(x_k, t_k)$$

$$\frac{x_{k+1} - x_k}{h} = f(x_k, t_k)$$



$x_2 = x(2h)$

$k = 0$

$x_1 \Rightarrow x(h)$

$$\dot{x} = a\bar{x}$$

$$x(t) = e^{at}x(0)$$

CORRECT

$$x_k = e^{akh}x(0)$$

$$\frac{x_{k+1} - x_k}{h} = a\,x_k$$

$$x_{k+1} = (1+ah)\, x_k .$$

$$x_k = (1+ah)^k\, x(0)$$

$$x_k = e^{akh}\, x(0)$$

$h > 2/a$

$\cancel{a}$ $a < 0$

$|1+ah| < 1$ Euler method

$$|1 + ah| < 1 \implies \text{System is stable}$$

$$|1 - 5h| < 1 \qquad h > \left|\frac{2}{a}\right|$$

$$\dot{x} = -5x \quad \text{IS STABLE}$$

$$|1 - 5h| > 1 \qquad h > \left|\frac{2}{-5}\right| \leftarrow 0.4$$

✓ 1. Order

✓ 2. Explict / Implicit Method ←

✓ 3. Single-step or Multistep.

1] Truncation

2] Round-off

$$\dot{x} = a x$$

$$\frac{x_{k+1} - x_k}{h} = g(x_k)$$

EULER

$$\frac{x_{k+1} - x_k}{h} = a x_k$$

BACKWARD EULER

$$\frac{x_{k+1} - x_k}{h} = a x_{k+1}$$

Nonlinear

$$\frac{x_{k+1} - x_k}{h} = g(x_{k+1})$$

$$\dot{x} = g(x)$$

$$\frac{x_{k+1} - x_k}{h} = \frac{g(x_k) + g(x_{k+1})}{2}.$$

$$\left.\frac{\text{"Trapezoidal Rule"}}{\text{Backward Euler}}\right\}$$

$$x(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2$$

Trapezoidal

$$\dot{x} = f(x, t)$$

Runge Kutta 4th order

$$K_1 = f(x_k, t_k)$$

$$K_2 = f\left(x_k + \frac{h}{2} K_1, t_k + h/2\right)$$

$$K_3 = f\left(x_k + \frac{h}{2} K_2, t_k + h/2\right)$$

$$K_4 = f(x_k + h K_3, t_k + h)$$

$$x_{k+1} = x_k + \frac{h}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

4th order

SINGLE

~~IMPLICIT~~

EXPLICIT

$$x(t) = \alpha_0 + \alpha_1 t \cdots$$

$$\dot{x} = \alpha_1$$

CORRECT SOLN.

FIRST ORDER

$$\dot{x} = ax \qquad x(t) = e^{at} x(0)$$

→ C. W. Gear, Numerical Initial Value problems in Ordinary Differential Equations, Printice Hall, Englewood Cliffs, 1971

→ L. Lapidus & J. H. Seinfeld, Numerical Solution of ODE, Academic Press, New York 1971