Prof. Kulkarni
Lec-6 - 28/10/09

Lecture 6

Analysis of Linear Time Invariant ~~Sys~~ Dynamical Systems - An example.

RECAP

$$\dot{x} = A x$$

$$x(t) = \sum_{i=1}^{n} p_i \, e^{\lambda_i t} \, q_i^T x(0)$$

$\lambda_i \rightarrow$ eigenvalues $A_{n \times n}$.

$$\det(\lambda_i I - A) = 0$$

$x = Py$

$P^{-1} A P = \Lambda$ ← diagonal

↑

right eigenvectors

n distinct eigenvalues

A is diagonalizable

$q_i^T \longrightarrow$ rows of P^{-1}

$$\underbrace{q_i^T}_{\text{left eigenvectors}} A = \lambda_i q_i^T$$

A is not diagonalizable

A non-distinct e.v.

A <u>MAY NOT</u> be diagonalizable

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$\det(\lambda I - A) = 0$

$1, 1$ $\quad p_1$ & p_2

$P =$

$p_1 = \alpha p_2$ $\quad Ap_1 = p_1 \quad Ap_2 = p_2$.

$\lambda, \lambda, \lambda_3, \lambda_4$ $\qquad A$

$$P^{-1}AP = \begin{bmatrix} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \lambda_4 \end{bmatrix} = J$$

A diagonalizable

$$x(t) = P e^{\Lambda t} P^{-1} x(0).$$

$$e^{\Lambda t} = \begin{bmatrix} e^{\lambda_1 t} & & 0 \\ & e^{\lambda_2 t} & \\ 0 & & \ddots \\ & & & e^{\lambda_n t} \end{bmatrix}$$

$$x(t) = P\, e^{Jt}\, P^{-1}$$

$$e^{Jt} = \begin{bmatrix} e^{\lambda t} & te^{\lambda t} & 0 & 0 \\ 0 & e^{\lambda t} & 0 & 0 \\ 0 & 0 & e^{\lambda_3 t} & 0 \\ 0 & 0 & 0 & e^{\lambda_4 t} \end{bmatrix}$$

K. Ogata

State Space Analysis of Control System (1967)

Prentice Hall.

$$\dot{x} = Ax + bu$$

$$y = cx + du$$

u & y are scalar

SISO

$$\rightarrow x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)} b\, u(\tau)\, dt$$

$$\rightarrow y(t) = c\,x(t) + d\,u(t)$$

$$e^{At} = P e^{\Lambda t} P^{-1} \leftarrow \text{expand}$$

$$= P e^{Jt} P^{-1}$$

$$y(t) = C \sum_{i=1}^{n} p_i e^{\lambda_i t} q_i^T x(0)$$

$$+ \cancel{\text{other terms}} \rightarrow u.$$

$$\underline{C p_i = 0} \qquad e^{\lambda_i t} \rightarrow \text{not visible in } y(t)$$

$$x(t) = \sum_{i=1}^{n} p_i e^{\lambda_i t} q_i^T x(0) + \int_0^t \sum_{i=1}^{n} p_i e^{\lambda_i (t-\tau)} \times q_i^T b \, u(\tau)\, d\tau$$

$$= \sum_{i=1}^{n} p_i e^{\lambda_i t} x(0) + \sum_{i=1}^{n} p_i e^{\lambda_i t} \left[\int_0^t e^{-\lambda_i \tau} q_i^T b \, u(\tau)\, d\tau \right]$$

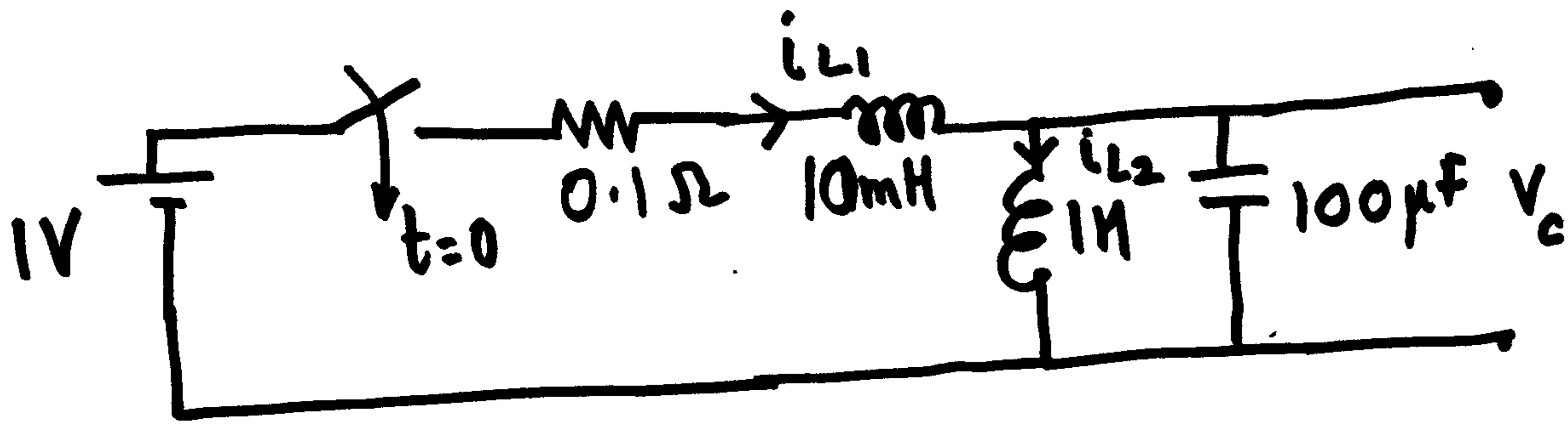
$q_i^T b = 0$ $\quad e^{\lambda_i t} \rightarrow$ only on i/c.

$$\det(\lambda I - A) = 0$$

<u>Iterative method</u>

'Power' method 'Q-R' method.

<u>Software</u> : SCILAB , MATLAB



i_{L1} , i_{L2} , V_C.

when is $\frac{di_{L1}}{dt} = 0$ $\frac{di_{L2}}{dt} = 0$

Equilibrium
if i/p $V_i = 0$

$\frac{dV_C}{dt} = 0$?

$$v_C = 0 \quad , \quad i_{L_1} = 0 \quad , \quad i_{L_2} = 0$$

for $u = V_i = 0$.

$$10^{-2} \frac{di_{L1}}{dt} = -0.1\, i_{L_1} - V_C + V_i$$

$$1 \frac{di_{L_2}}{dt} = V_C$$

$$100 \times 10^{-6} \frac{dV_c}{dt} = i_{L_1} - i_{L_2}.$$

$$\begin{bmatrix} \frac{di_{L_1}}{dt} \\ \frac{di_{L_2}}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} -10 & 0 & -100 \\ 0 & 0 & 1 \\ 10000 & -10000 & 0 \end{bmatrix}}_{A} \begin{bmatrix} i_{L_1} \\ i_{L_2} \\ V_c \end{bmatrix} + \underbrace{\begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}}_{bu.}$$

$t > 0$ $\quad \left\{ t=0 \quad i/c \quad i_{L_1} = i_{L_2} = V_c = 0 \right\}$

$$\begin{bmatrix} \dot{i}_{L_1} \\ \dot{i}_{L_2} \\ v_C \end{bmatrix} = \underset{\substack{\downarrow \\ P^{-1} e^{\Lambda t} P}}{e^{At}} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \int_0^t e^{A(t-\tau)} \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} d\tau .$$

$$\Lambda = \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{bmatrix} = \left[\underline{I}_3 - e^{+At} \right] \begin{bmatrix} 10 \\ 10 \\ 0 \end{bmatrix}$$

$$e^{\Lambda t} = \begin{bmatrix} e^{\lambda_1 t} & & \\ & e^{\lambda_2 t} & \\ & & e^{\lambda_3 t} \end{bmatrix}$$

$$\left[I_3 - P \begin{bmatrix} e^{\lambda_1 t} & 0 & 0 \\ 0 & e^{\lambda_2 t} & 0 \\ 0 & 0 & e^{\lambda_3 t} \end{bmatrix} P^{-1} \right] \begin{bmatrix} 10 \\ 10 \\ 0 \end{bmatrix}$$

$$\left.\begin{matrix} \lambda_1, \lambda_2, \lambda_3 \\ P \end{matrix}\right\} A.$$

www.scilab.org.

$\lambda_1 \approx -5 + j1005$
$\lambda_2 \approx -5 - j1005$
$\lambda_3 \approx -0.1$.

$\underline{Re(\lambda) < 0}$.

'A' real $\rightarrow$ real and/or complex conj pairs.

$$P = \left[\, p_1 \;\middle|\; p_2 \;\middle|\; p_3 \,\right]$$

$$\lambda_1 = -5 + j1005$$

$$\lambda_2 = -5 - j1005$$

$$\lambda_3 = -0.1$$

"slow"

$$\begin{array}{cc} & \\ 2^{nd} \rightarrow & \\ & \\ \end{array} \approx \begin{bmatrix} j0.1 & -j0.1 & -0.7 \\ \underline{0} & \underline{0} & -0.7 \\ 1 & 1 & 0.06 \end{bmatrix}$$

$$\begin{array}{ccc} \uparrow & \uparrow & \\ \xi_1 & \xi_2 & \end{array}$$

$$\underline{\underline{P}}$$

$$e^{\lambda_1 t}, \; e^{\lambda_2 t}$$

$$\begin{bmatrix} i_{L_1}(t) \\ i_{L_2}(t) \\ v_c(t) \end{bmatrix} = \begin{bmatrix} 10 - 10\,e^{-0.1t} + 0.1\,e^{-5t}\sin(1005t) \\ 10 - 10\,e^{-0.1t} \\ e^{-0.1t} - e^{-5t}\cos(1005t) \end{bmatrix}$$

$$\frac{e^{j\omega t} + e^{-j\omega t}}{2} = \cos\omega t$$

$$\frac{e^{+j\omega t} - e^{-j\omega t}}{2j} = \sin\omega t$$