Graph Theory: Lecture No. 19

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The least integer $k$ such that $G$ has an edge coloring from any family of lists of size $k$ is the list chromatic index $\text{ch}'(G)$ of $G$. That is $\text{ch}'(G) = \text{ch}(L(G))$ where $L(G)$ is the line graph of $G$. Clearly $\text{ch}'(G) \geq \chi'(G)$
The List Coloring Conjecture: Every graph $G$ satisfies $ch'(G) = \chi'(G)$. 
Let $D$ be a directed graph. An independent set $U \subseteq V(D)$, such that for every vertex $v \in D - U$, there is an edge in $D$ directed from $v$ to a vertex in $U$, is called a kernel of $D$. 
Let \( H \) be a graph and \((S_v)_{v \in V(H)}\) be a family of lists. If \( H \) has an orientation \( D \) with \( d^+(v) < |S_v| \) for every vertex \( v \) and such that every induced subgraph of \( D \) has a kernel, then \( H \) can be colored from the list \( S_v \).
Let a family \((\leq_v)_{v \in V}\) of linear orderings \(\leq_v\) on \(E(v)\) a set of preferences for \(G\). Then call a matching \(M\) in \(G\) stable if for every edge \(e \in E - M\), there exists an edge \(f \in M\) such that \(e\) and \(f\) have a common vertex \(v\) with \(e \leq_v f\).
For every set of preferences, $G$ has a stable matching.
Every bipartite graph $G$ satisfies,
$ch'(G) = \chi'(G)$. 
A matching $M$ in $G$ is better than a matching $M' \neq M$ if $M$ makes the vertices in $B$ happier than $M'$ does, i.e. if every vertex $b$ in an edge $f' \in M'$ is incident also with some $f \in M$ such that $f' \leq_b f$. 
Given a matching $M$, call a vertex $a \in A$ acceptable to $b \in B$ if $e = ab \in E - M$ and any edge $f \in M$ at $b$ satisfies $f \leq_b e$. 
$a \in A$ is happy with $M$ if $a$ is unmatched or its matching edge $f \in M$ satisfies $f >_a e$ for all edges $e = ab$ such that $a$ is acceptable to $b$. 