Advanced Topics in Optimization

Multilevel Optimization

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Objectives

- To discuss about Multilevel Optimization
- To describe a decomposition method for nonlinear optimization problems, known as model-coordination method

Introduction

- In practical situations, an optimization problem involves a large number of variables and constraints
- In multilevel optimization, such large sized problems are decomposed into smaller independent problems
- The overall optimum solution is obtained by solving each sub-problem independently

Model Coordination Method

- Consider an minimization optimization problem $F(x)$ consisting of *n* variables, $x_1, x_2, \ldots, x_n$
- subjected to constraints

$$g_j(x_1, x_2, \ldots, x_n) \le 0, \qquad j = 1,2,\ldots,m$$
$$lx_i \le x_i \le ux_i \qquad i = 1,2,\ldots,n$$

- where lx_i and ux_i represents the lower and upper bound of the decision variable x_i
- Decision variable vector: $X = \{x_1, x_2, \ldots, x_n\}$

Model Coordination Method …contd.

- For applying model coordination method, the vector *X* should be divided into two sub-vectors, *Y* and *Z*
- *Y* contains the coordination variables between the subsystems i.e., variables that are common to the sub-problems
- *Z* vector contains the free or confined variables of sub-problems
- If the problem is partitioned into *'P'* sub-problems, then vector *Z* can also be partitioned into *'P'* variable sets, each set corresponding to each sub-problem

$$Z = \begin{Bmatrix} Z_1 \\ Z_2 \\ \vdots \\ Z_P \end{Bmatrix}$$

Model Coordination Method …contd.

- Thus the objective function $F(x)$ can be partitioned into 'P' parts

$$F(x)=\sum_{k=1}^{P} f_k\left(Y, Z_k\right)$$

where $f_k\left(Y, Z_k\right)$ denotes the objective function of the k^{th} sub-problem

- The coordination variable Y will appear in all sub-objective functions and Z_k will appear only in k^{th} sub-objective function

Model Coordination Method …contd.

- Similarly the constraints are also decomposed as

$$g_k(Y, Z_k) \leq 0 \qquad\qquad for \;\; k = 1,2,\ldots,P$$

- The lower and upper bound constraints are

$$lY \leq Y \leq uY$$

$$lZ_k \leq Z_k \leq uZ_k \qquad\qquad for\; k = 1,2,\ldots,P$$

- The problem is decomposed and solved using a two level approach

Model Coordination Method: Procedure

- First level:
- Fix the value of the coordination variables, Y at some value, say Y_{opt}
- Solve each independent sub-problem and find the value of Z_k

$$Min\ f_k(Y, Z_k)$$

subject to

$$g_k(Y, Z_k) \leq 0$$
$$lY \leq Y \leq uY$$
$$lZ_k \leq Z_k \leq uZ_k \qquad\qquad for\ k = 1,2,\ldots,P$$

Model Coordination Method: Procedure …contd.

- Let the values of Z_k obtained by solving this problem be Z_{kopt}
- Second level:
- Now consider the problem after substituting the Z_{kopt} values

$$Min\ f(Y) = \sum_{k=1}^{P} f_k\left(Y, Z_{k\ opt}\right)$$

- subject to

$$lY \le Y \le uY$$

Model Coordination Method: Procedure …contd.

- Solve this problem to find a new Y_{opt}
- Again solve the first level problems
- This process is repeated until convergence is achieved

Thank You