



# Advanced Topics in Optimization

## Multilevel Optimization

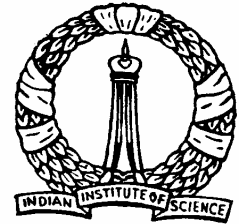
# Objectives

- To discuss about Multilevel Optimization
- To describe a decomposition method for nonlinear optimization problems, known as model-coordination method



# Introduction

- In practical situations, an optimization problem involves a large number of variables and constraints
- In multilevel optimization, such large sized problems are decomposed into smaller independent problems
- The overall optimum solution is obtained by solving each sub-problem independently



# Model Coordination Method

- Consider an minimization optimization problem  $F(x)$  consisting of  $n$  variables,  $x_1, x_2, \dots, x_n$

- subjected to constraints

$$g_j(x_1, x_2, \dots, x_n) \leq 0, \quad j = 1, 2, \dots, m$$

$$lx_i \leq x_i \leq ux_i \quad i = 1, 2, \dots, n$$

- where  $lx_i$  and  $ux_i$  represents the lower and upper bound of the decision variable  $x_i$
- Decision variable vector:  $X = \{x_1, x_2, \dots, x_n\}$



## ***Model Coordination Method ...contd.***

- For applying model coordination method, the vector  $X$  should be divided into two sub-vectors,  $Y$  and  $Z$
- $Y$  contains the coordination variables between the subsystems i.e., variables that are common to the sub-problems
- $Z$  vector contains the free or confined variables of sub-problems
- If the problem is partitioned into ' $P$ ' sub-problems, then vector  $Z$  can also be partitioned into ' $P$ ' variable sets, each set corresponding to each sub-problem

$$Z = \left\{ \begin{array}{l} Z_1 \\ Z_2 \\ \vdots \\ M \\ \vdots \\ Z_P \end{array} \right\}$$

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Optimization Methods: M8L3



## ***Model Coordination Method ...contd.***

- Thus the objective function  $F(x)$  can be partitioned into 'P' parts

$$F(x) = \sum_{k=1}^P f_k(Y, Z_k)$$

where  $f_k(Y, Z_k)$  denotes the objective function of the  $k^{\text{th}}$  sub-problem

- The coordination variable  $Y$  will appear in all sub-objective functions and  $Z_k$  will appear only in  $k^{\text{th}}$  sub-objective function



## ***Model Coordination Method ...contd.***

- Similarly the constraints are also decomposed as

$$g_k(Y, Z_k) \leq 0 \quad \text{for } k = 1, 2, \dots, P$$

- The lower and upper bound constraints are

$$lY \leq Y \leq uY$$

$$lZ_k \leq Z_k \leq uZ_k \quad \text{for } k = 1, 2, \dots, P$$

- The problem is decomposed and solved using a two level approach



## Model Coordination Method: Procedure

- First level:
- Fix the value of the coordination variables,  $Y$  at some value, say  $Y_{opt}$
- Solve each independent sub-problem and find the value of  $Z_k$

$$\text{Min } f_k(Y, Z_k)$$

subject to

$$g_k(Y, Z_k) \leq 0$$

$$lY \leq Y \leq uY$$

$$lZ_k \leq Z_k \leq uZ_k$$

for  $k = 1, 2, \dots, P$





## ***Model Coordination Method: Procedure ...contd.***

- Let the values of  $Z_k$  obtained by solving this problem be  $Z_{kopt}$
- Second level:
- Now consider the problem after substituting the  $Z_{kopt}$  values

$$\text{Min } f(Y) = \sum_{k=1}^P f_k(Y, Z_{kopt})$$

- subject to

$$lY \leq Y \leq uY$$



## ***Model Coordination Method: Procedure ...contd.***

- Solve this problem to find a new  $Y_{opt}$
- Again solve the first level problems
- This process is repeated until convergence is achieved



Thank You