Dynamic Programming

Other Topics

Objectives

- To explain the difference between discrete and continuous dynamic programming
- To discuss about multiple state variables
- To discuss the curse of dimensionality in dynamic programming

Discrete versus Continuous Dynamic Programming

- Discrete dynamic programming problems: Number of stages is finite
- When the number of stages tends to infinity then it is called a continuous dynamic programming problem
- Also called as infinite-stage problem
- Continuous dynamic problems are used to solve continuous decision problem
- The classical method of solving continuous decision problems is by the calculus of variations

Discrete versus Continuous Dynamic Programming …contd.

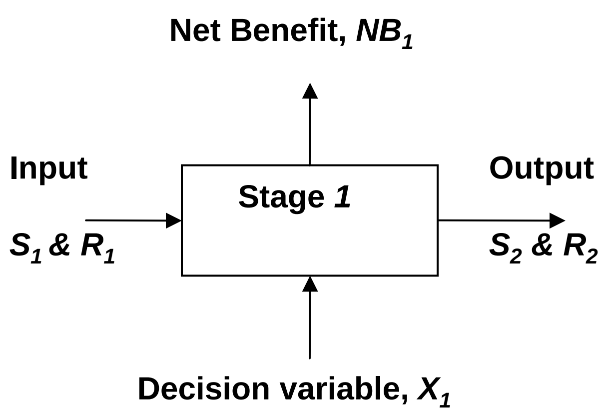
- However, the analytical solutions, using calculus of variations is applicable only for very simple problems
- The infinite-stage dynamic programming approach provides a very efficient numerical approximation procedure for solving continuous decision problems
- For discrete dynamic programming model, the objective function value is the sum of individual stage outputs
- For a continuous dynamic programming model, summation is replaced by integrals of the returns from individual stages
- Such models are useful when infinite number of decisions have to be made in finite time interval

Multiple State Problems

- Problems in which there are more than one state variable
- For example, consider a water allocation problem to *n* irrigated crops
- Let S_i be the units of water available to the remaining *n-i* crops
- Considering only the allocation of water, the problem can be solved as a single state problem, with S_i as the state variable
- Now, assume that *L* units of land are available for all these *n* crops
- Allocation of land also to be done to each crop after considering the units of water required for each unit of irrigated land containing each crop

Multiple State Problems …contd.

- Let R_i be the amount of land available for *n-i* crops
- An additional state variable R_i is included while sub-optimizing different stages
- Thus, in this problem two allocations need to be made: water and land.
- A single stage problem consisting of two state variables, S_1 & R_1 is shown below



Multiple State Problems …contd.

- In general, for a multistage decision problem of T stages, containing two state variables S_t and R_t, the objective function can be written as

$$f = \sum_{t=1}^{T} NB_t = \sum_{t=1}^{T} h\left(X_t, S_t, R_t\right)$$

where the transformation equations are given as

- $S_{t+1} = g(X_t, S_t)$ for $t = 1,2,\ldots, T$ &
- $R_{t+1} = g'(X_t, R_t)$ for $t = 1,2,\ldots, T$

Curse of Dimensionality

- Limitation of dynamic programming: Dimensionality restriction
- The number of calculations needed increases rapidly as the number of variables and stages increase
- Increases the computational effort
- Increase in the number of stage variables causes an increase in the number of combinations of discrete states to be examined at each stage
- For a problem consisting of 100 state variables and each variable having 100 discrete values, the sub-optimization table will contain 100^{100} entries

Curse of Dimensionality …contd.

- The computation of one table may take 100^{96} seconds (about 100^{92} years) even on a high speed computer
- Like this 100 tables have to be prepared for which computation is almost impossible
- This phenomenon as termed by Bellman, is known as *"curse of dimensionality" or "Problem of dimensionality"* of multiple state variable dynamic programming problems

Thank You