Module
4
Analysis of Statically Indeterminate Structures by the Direct Stiffness Method

Lesson 26

The Direct Stiffness Method: Temperature Changes and Fabrication Errors in Truss Analysis

Instructional Objectives

After reading this chapter the student will be able to
1. Compute stresses developed in the truss members due to temperature changes.
2. Compute stresses developed in truss members due to fabrication members.
3. Compute reactions in plane truss due to temperature changes and fabrication errors.

26.1 Introduction

In the last four lessons, the direct stiffness method as applied to the truss analysis was discussed. Assembly of member stiffness matrices, imposition of boundary conditions, and the problem of inclined supports were discussed. Due to the change in temperature the truss members either expand or shrink. However, in the case of statically indeterminate trusses, the length of the members is prevented from either expansion or contraction. Thus, the stresses are developed in the members due to changes in temperature. Similarly the error in fabricating truss members also produces additional stresses in the trusses. Both these effects can be easily accounted for in the stiffness analysis.

26.2 Temperature Effects and Fabrication Errors

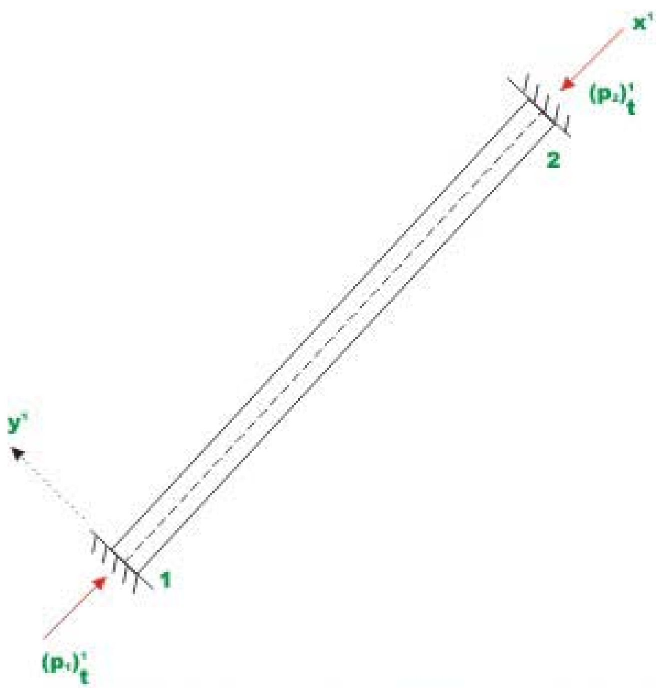

Fig.26.1 Truss member subjected to temperature loads

Consider truss member of length *L*, area of cross section *A* as shown in Fig.26.1.The change in length Δl is given by

$$\Delta l = \alpha L \Delta T \tag{26.1}$$

where α is the coefficient of thermal expansion of the material considered. If the member is not allowed to change its length (as in the case of statically indeterminate truss) the change in temperature will induce additional forces in the member. As the truss element is a one dimensional element in the local coordinate system, the thermal load can be easily calculated in global co-ordinate system by

$$(p_1')_t = AE\Delta L \tag{26.2a}$$

$$(p_2')_t = -AE\Delta L \tag{26.2b}$$

or

$$\{(p')_t\} = AE\Delta L \begin{Bmatrix} +1 \\ -1 \end{Bmatrix} \tag{26.3}$$

The equation (26.3) can also be used to calculate forces developed in the truss member in the local coordinate system due to fabrication error. ΔL will be considered positive if the member is too long. The forces in the local coordinate system can be transformed to global coordinate system by using the equation,

$$\begin{Bmatrix} (p_1)_t \\ (p_2)_t \\ (p_3)_t \\ (p_4)_t \end{Bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & \cos\theta \\ 0 & \sin\theta \end{bmatrix} \begin{Bmatrix} (p_1')_t \\ (p_2')_t \end{Bmatrix} \tag{26.4a}$$

where $(p_1)_t, (p_2)_t$ and $(p_3)_t, (p_4)_t$ are the forces in the global coordinate system at nodes 1 and 2 of the truss member respectively Using equation (26.3), the equation (26.4a) may be written as,

$$\begin{Bmatrix} (p_1)_t \\ (p_2)_t \\ (p_3)_t \\ (p_4)_t \end{Bmatrix} = AE\Delta L \begin{Bmatrix} \cos\theta \\ \sin\theta \\ -\cos\theta \\ -\sin\theta \end{Bmatrix} \tag{26.4b}$$

The force displacement equation for the entire truss may be written as,

$$\{p\} = [k]\{u\} + \{(p)_t\} \qquad (26.5)$$

where , $\{p\}$ is the vector of external joint loads applied on the truss and $\{(p)_t\}$ is the vector of joint loads developed in the truss due to change in temperature/fabrication error of one or more members. As pointed out earlier. in the truss analysis, some joint displacements are known due to boundary conditions and some joint loads are known as they are applied externally.Thus,one could partition the above equation as,

$$\begin{Bmatrix} p_k \\ p_u \end{Bmatrix} = \begin{bmatrix} [k_{11}] & [k_{12}] \\ [k_{21}] & [k_{22}] \end{bmatrix} \begin{Bmatrix} \{u_u\} \\ \{u_k\} \end{Bmatrix} + \begin{Bmatrix} (p_k)_t \\ (p_u)_t \end{Bmatrix} \qquad (26.6)$$

where subscript u is used to denote unknown quantities and subscript k is used to denote known quantities of forces and displacements. Expanding equation (26.6),

$$\{p_k\} = [k_{11}]\{u_u\} + [k_{12}]\{u_k\} + \{(p_k)_t\} \qquad (26.7a)$$

$$\{p_u\} = [k_{21}]\{u_u\} + [k_{22}]\{u_k\} + \{(p_u)_t\} \qquad (26.7b)$$

If the known displacement vector $\{u_k\} = \{0\}$ then using equation (26.2a) the unknown displacements can be calculated as

$$\{u_u\} = [k_{11}]^{-1}(\{p_k\} - \{(p_k)_t\}) \qquad (26.8a)$$

If $\{u_k\} \neq 0$ then

$$\{u_u\} = [k_u]^{-1}(\{p_k\} - [k_{12}]\{u_k\} - \{(p_k)_t\}) \qquad (26.8b)$$

After evaluating unknown displacements, the unknown force vectors are calculated using equation (26.7b).After evaluating displacements, the member forces in the local coordinate system for each member are evaluated by,

$$\{p'\} = [k'][T]\{u\} + \{p'\}_t \qquad (26.9a)$$

or

$$\begin{Bmatrix} p_1^{'} \\ p_2^{'} \end{Bmatrix} = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix} + \begin{Bmatrix} (p_1^{'})_t \\ (p_2^{'})_t \end{Bmatrix}$$

Expanding the above equation, yields

$$\{p_1'\} = \frac{AE}{L}\{\cos\theta \quad \sin\theta \quad -\cos\theta \quad -\sin\theta\}\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix} + AE\Delta L \qquad (26.10a)$$

And,

$$\{p_2'\} = \frac{AE}{L}\{-\cos\theta \quad -\sin\theta \quad \cos\theta \quad \sin\theta\}\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix} - AE\Delta L \qquad (26.10b)$$

Few problems are solved to illustrate the application of the above procedure to calculate thermal effects /fabrication errors in the truss analysis:-

Example 26.1

Analyze the truss shown in Fig.26.2a, if the temperature of the member (2) is raised by $40^\circ C$.The sectional areas of members in square centimeters are shown in the figure. Assume $E = 2\times10^5\, N/mm^2$ and $\alpha = 1/75{,}000$ per $^\circ C$.

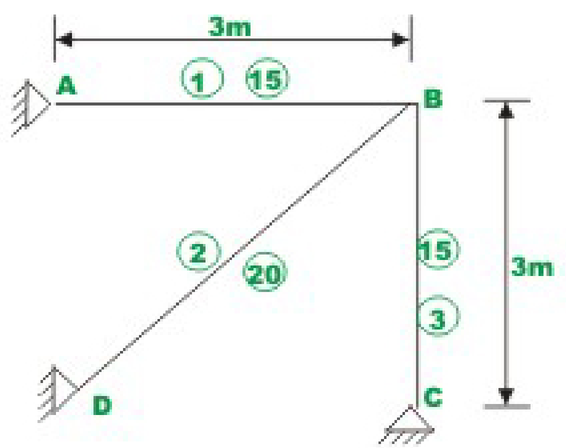

Fig 26.2a Example 26.2

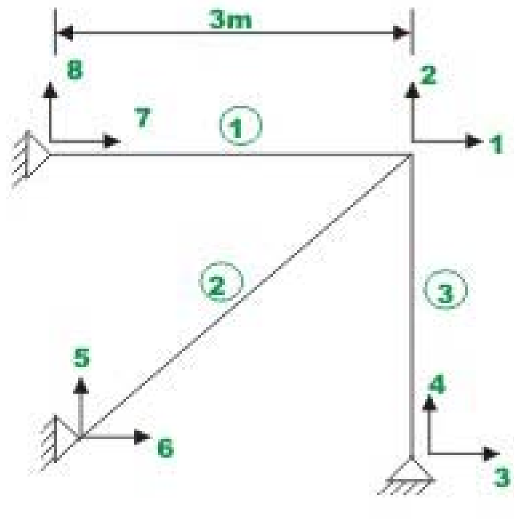

Fig 26.2b Node and Members numbering

The numbering of joints and members are shown in Fig.26.2b. The possible global displacement degrees of freedom are also shown in the figure. Note that lower numbers are used to indicate unconstrained degrees of freedom. From the figure it is obvious that the displacements $u_3 = u_4 = u_5 = u_6 = u_7 = u_8 = 0$ due to boundary conditions.

The temperature of the member (2) has been raised by $40^\circ C$. Thus,

$$\Delta L = \alpha L \Delta T$$

$$\Delta L = \frac{1}{75000}\left(3\sqrt{2}\right)\left(40\right) = 2.2627 \times 10^{-3} \text{ m} \qquad (1)$$

The forces in member (2) due to rise in temperature in global coordinate system can be calculated using equation (26.4b).Thus,

$$\begin{Bmatrix} (p_5)_t \\ (p_6)_t \\ (p_1)_t \\ (p_2)_t \end{Bmatrix} = AE\Delta L \begin{Bmatrix} \cos\theta \\ \sin\theta \\ -\cos\theta \\ -\sin\theta \end{Bmatrix} \qquad (2)$$

For member (2),

$$A = 20cm^2 = 20 \times 10^{-4} m^2 \text{ and } \theta = 45^\circ$$

$$\begin{Bmatrix} (p_5)_t \\ (p_6)_t \\ (p_1)_t \\ (p_2)_t \end{Bmatrix} = 20\times10^{-4}\times2\times10^{11}\times2.2627\times10^{-3}/10^3 \begin{Bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{Bmatrix} \qquad (3)$$

$$\begin{Bmatrix} (p_5)_t \\ (p_6)_t \\ (p_1)_t \\ (p_2)_t \end{Bmatrix} = 150.82 \begin{Bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{Bmatrix} kN \qquad (4)$$

In the next step, write stiffness matrix of each member in global coordinate system and assemble them to obtain global stiffness matrix

Element (1): $\theta = 0^0, L = 3m, A = 15\times10^{-4}m^2$,nodal points 4-1

$$\left[k^{'}\right] = \frac{15\times10^{-4}\times2\times10^{11}}{3\times10^3}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (5)$$

Member (2): $\theta = 45^\circ, L = 3\sqrt{2}m, A = 20\times10^{-4}m^2$, nodal points 3-1

$$\left[k^2\right] = \frac{20\times10^{-4}\times2\times10^{11}}{3\sqrt{2}}\begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix} \qquad (6)$$

Member (3): $\theta = 90^\circ, A = 15\times10^{-4}m^2, L = 30m,$ nodal points 2-1

$$\left[k^3\right]=\frac{15\times10^{-4}\times2\times10^{11}}{3\text{ x }10^3\times10^3}\begin{bmatrix}0&0&0&0\\0&1&0&-1\\0&0&0&0\\0&-1&0&1\end{bmatrix} \quad (7)$$

The global stiffness matrix is of the order 8×8 ,assembling the three member stiffness matrices, one gets

$$[k]=10^3\begin{bmatrix}147.14&47.14&0&0&-47.14&-47.14&-100&0\\47.14&147.14&0&-100&-47.14&-47.14&0&0\\0&0&0&0&0&0&0&0\\0&-100&0&100&0&0&0&0\\-47.14&-47.14&0&0&47.14&47.14&0&0\\-47.14&-47.14&0&0&47.14&47.14&0&0\\-100&0&0&0&0&0&100&0\\0&0&0&0&0&0&0&0\end{bmatrix} \quad (8)$$

Writing the load displacement equation for the truss

$$\begin{Bmatrix}p_1\\p_2\\p_3\\p_4\\p_5\\p_6\\p_7\\p_8\end{Bmatrix}=10^3\begin{bmatrix}147.14&47.14&0&0&-47.14&-47.14&-100&0\\47.14&147.14&0&-100&-47.14&-47.14&0&0\\0&0&0&0&0&0&0&0\\0&-100&0&100&0&0&0&0\\-47.14&-47.14&0&0&47.14&47.14&0&0\\-47.14&-47.14&0&0&47.14&47.14&0&0\\-100&0&0&0&0&0&100&0\\0&0&0&0&0&0&0&0\end{bmatrix}\begin{Bmatrix}u_1\\u_2\\u_3\\u_4\\u_5\\u_6\\u_7\\u_8\end{Bmatrix}+640\begin{Bmatrix}-1\\-1\\0\\0\\1\\1\\0\\0\end{Bmatrix} \quad (9)$$

In the present case, the displacements u_1 and u_2 are not known. All other displacements are zero. Also $p_1=p_2=0$ (as no joint loads are applied).Thus,

$$\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \end{Bmatrix} = = \begin{bmatrix} 147.14 & 47.14 & 0 & 0 & -47.14 & -47.14 & -100 & 0 \\ 47.14 & 147.14 & 0 & -100 & -47.14 & -47.14 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -100 & 0 & 100 & 0 & 0 & 0 & 0 \\ -47.14 & -47.14 & 0 & 0 & 47.14 & 47.14 & 0 & 0 \\ -47.14 & -47.14 & 0 & 0 & 47.14 & 47.14 & 0 & 0 \\ -100 & 0 & 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \end{Bmatrix} + 640 \begin{Bmatrix} -1 \\ -1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{Bmatrix} \tag{10}$$

Thus unknown displacements are

$$\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \frac{1}{10^3} \begin{bmatrix} 147.14 & 47.14 \\ 47.14 & 147.14 \end{bmatrix}^{-1} \left(\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - 150.82 \begin{Bmatrix} -1 \\ -1 \end{Bmatrix} \right) \tag{11}$$

$$u_1 = 7.763 \times 10^{-4} m$$

$$u_2 = 7.763 \times 10^{-4} m$$

Now reactions are calculated as

$$\begin{Bmatrix} p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \end{Bmatrix} = 10^3 \begin{bmatrix} 0 & 0 \\ 0 & -100 \\ -47.14 & -47.14 \\ -47.14 & -47.14 \\ -100 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 0 & 0 \\ 0 & 0 & 47.14 & 47.14 & 0 & 0 \\ 0 & 0 & 47.14 & 47.14 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + 640 \begin{Bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -77.63 \\ 77.63 \\ 77.63 \\ -77.63 \\ 0 \end{Bmatrix} \text{kN} \tag{12}$$

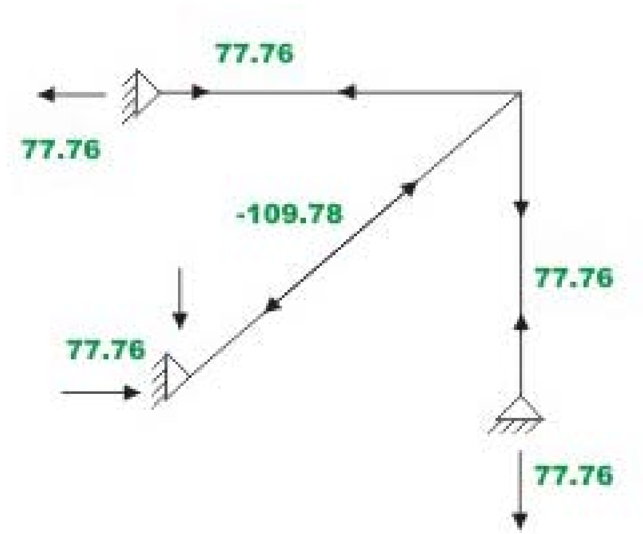

Fig 26.2c Force in members

The support reactions are shown in Fig.26.2c.The member forces can be easily calculated from reactions. The member end forces can also be calculated by using equation (26.10a) and (26.10b). For example, for member (1),

$\theta = 0^\circ$

$$p_2' = 10^3 \times 100 \begin{bmatrix} -1 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 7.763 \times 10^{-4} \\ 7.763 \times 10^{-4} \end{Bmatrix} \tag{13}$$

= 77.763 kN. Thus the member (1) is in tension.

Member (2)

$\theta = 45^\circ$

$$p_2' = 10^3 \times 94.281 [-0.707\ -0.707\ 0.707\ 0.707] \begin{Bmatrix} 0 \\ 0 \\ 3.2942 \times 10^{-3} \\ 3.2942 \times 10^{-3} \end{Bmatrix}$$

$$p_2' = -109.78 \text{ kN.}$$

Thus member (2) is in compression

Example 26.2

Analyze the truss shown in Fig.26.3a, if the member BC is made 0.01m too short before placing it in the truss. Assume *AE=300* kN for all members.

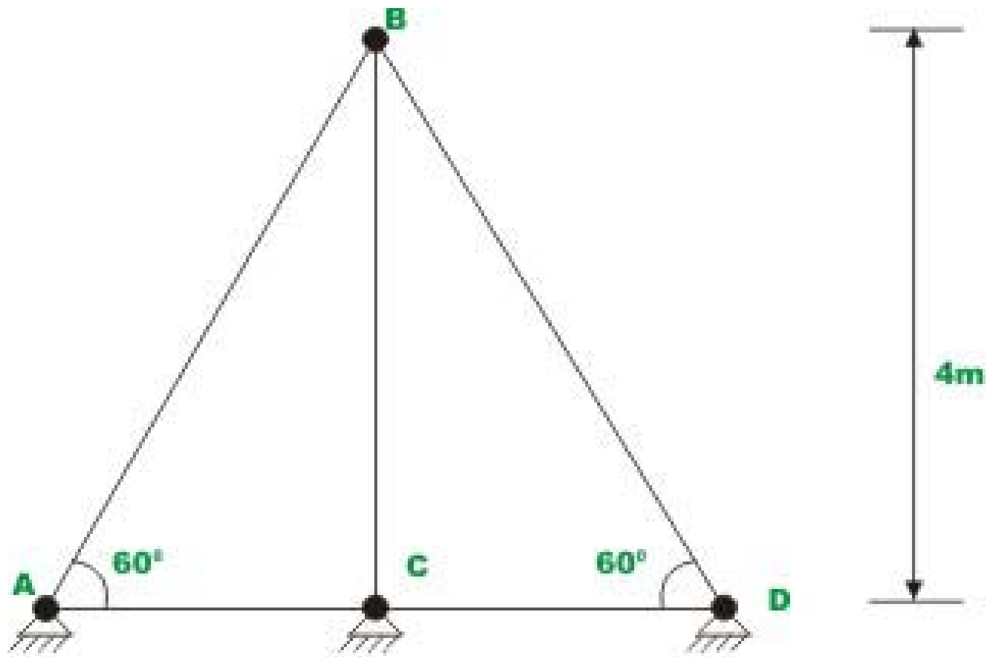

Fig 26.3a (Example 26.2)

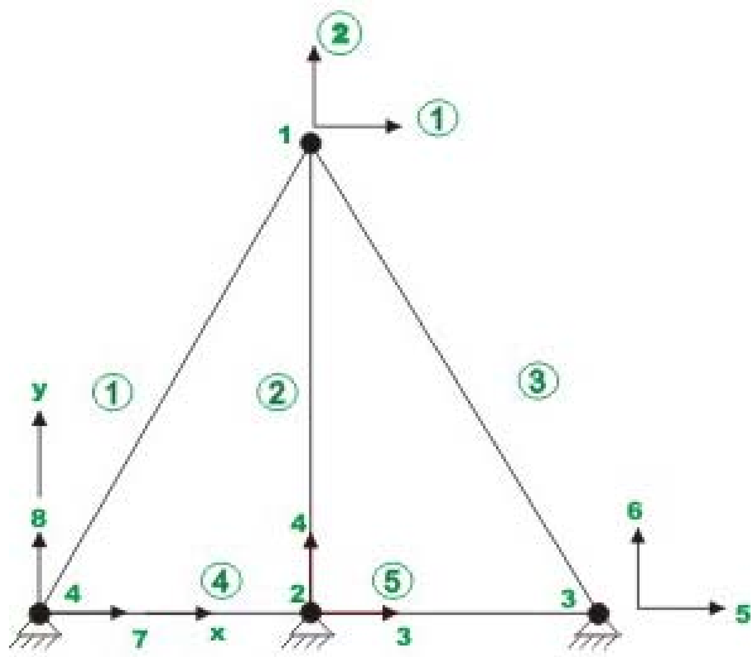

Fig. 26.3b Node and member numbering

Solution

A similar truss with different boundary conditions has already been solved in example 25.1. For the sake of completeness the member of nodes and members are shown in Fig.26.3b.The displacements u_3, u_4, u_5, u_6, u_7 and u_8 are zero due to boundary conditions. For the present problem the unconstrained degrees of freedom are u_1 and u_2.The assembled stiffness matrix is of the order 8×8 and is available in example 25.1.

In the given problem the member (2) is short by 0.01m.The forces developed in member (2) in the global coordinate system due to fabrication error is

$$\begin{Bmatrix} (p_3)_0 \\ (p_4)_0 \\ (p_1)_0 \\ (p_2)_0 \end{Bmatrix} = \frac{AE(-0.01)}{4} \begin{Bmatrix} \cos\theta \\ \sin\theta \\ -\cos\theta \\ -\sin\theta \end{Bmatrix}$$

$$= \begin{Bmatrix} 0 \\ -0.75 \\ 0 \\ 0.75 \end{Bmatrix} \text{kN} \qquad (1)$$

Now force-displacement relations for the truss are

$$\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \end{Bmatrix} = AE \begin{bmatrix} 0.108 & 0 & 0 & 0 & -0.054 & 0.094 & -0.054 & -0.094 \\ 0 & 0.575 & 0 & -0.25 & 0.094 & -0.162 & -0.094 & -0.162 \\ 0 & 0 & 0.866 & 0 & -0.433 & 0 & -0.433 & 0 \\ 0 & -0.25 & 0 & 0.25 & 0 & 0 & 0 & 0 \\ -0.054 & 0.094 & -0.433 & 0 & 0.487 & -0.094 & 0 & 0 \\ 0.094 & -0.162 & 0 & 0 & -0.094 & 0.162 & 0 & 0 \\ -0.054 & -0.094 & -0.433 & 0 & 0 & 0 & 0.487 & 0.0934 \\ -0.094 & -0.162 & 0 & 0 & 0 & 0 & 0.0934 & 0.162 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0.75 \\ 0 \\ -0.75 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{2}$$

Note that $u_3 = u_4 = u_5 = u_6 = u_7 = u_8 = 0$
Thus, solving

$$\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \frac{1}{AE} \begin{bmatrix} 0.108 & 0 \\ 0 & 0.575 \end{bmatrix}^{-1} \left(\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0.75 \end{Bmatrix} \right) \tag{3}$$

$u_1 = 0$

and, $u_2 = -4.3478 \times 10^{-3} \, m$ (4)

Reactions are calculated as,

$$\begin{Bmatrix} p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \end{Bmatrix} = AE \begin{bmatrix} 0 & 0 \\ 0 & -0.25 \\ -0.054 & 0.094 \\ 0.094 & -0.162 \\ -0.054 & -0.094 \\ -0.094 & -0.162 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -0.75 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{5}$$

$$\begin{Bmatrix} p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -0.424 \\ -0.123 \\ 0.211 \\ 0.123 \\ 0.211 \end{Bmatrix} \tag{6}$$

The reactions and member forces are shown in Fig.26.3c. The member forces can also be calculated by equation (26.10a) and (26.10b). For example, for member (2),

$\theta = 90^\circ$

$$p_2' = \frac{300}{4}[0\ -1\ 0\ 1]\begin{Bmatrix} u_3 \\ u_4 \\ u_1 \\ u_2 \end{Bmatrix} - \frac{AE\Delta L}{L}$$

$$= \frac{300}{4}\left(-4.3478\times10^{-3}\right) - \frac{300(-0.01)}{4}$$

$$= 0.4239 \cong 0.424 \text{ kN} \qquad (7)$$

Example 26.3

Evaluate the member forces of truss shown in Fig.26.4a.The temperature of the member BC is raised by $40^{\circ}C$ and member BD is raised by $50^{\circ}C$.Assume AE=300KN for all members and $\alpha = \frac{1}{75000}$ per $^{\circ}$C.

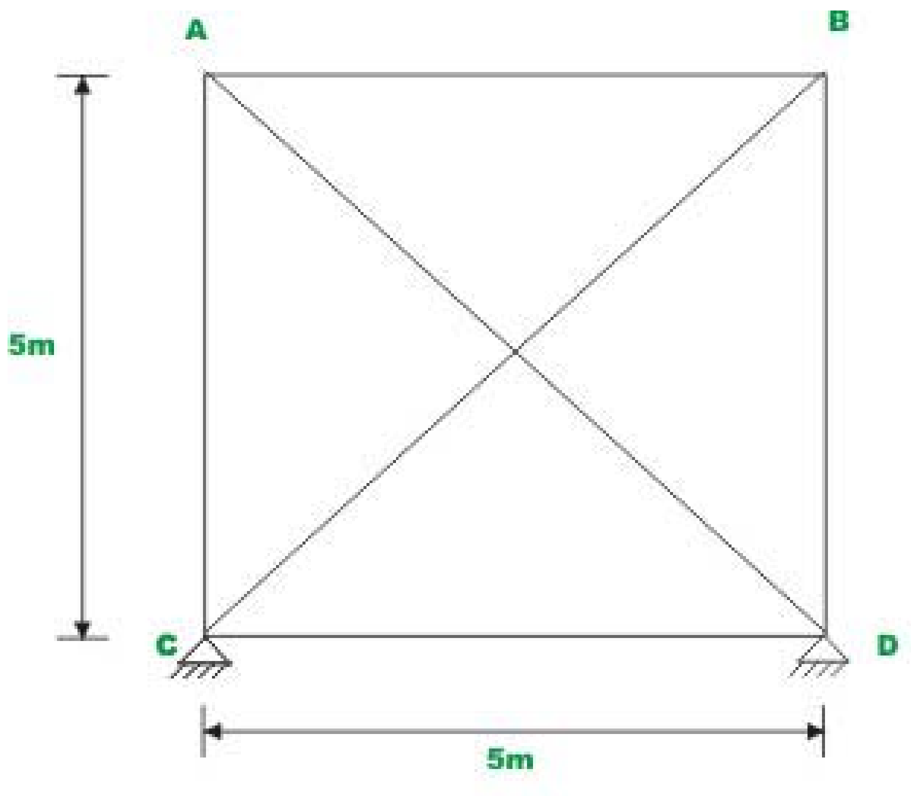

Fig 26.4a (Example 26.1)

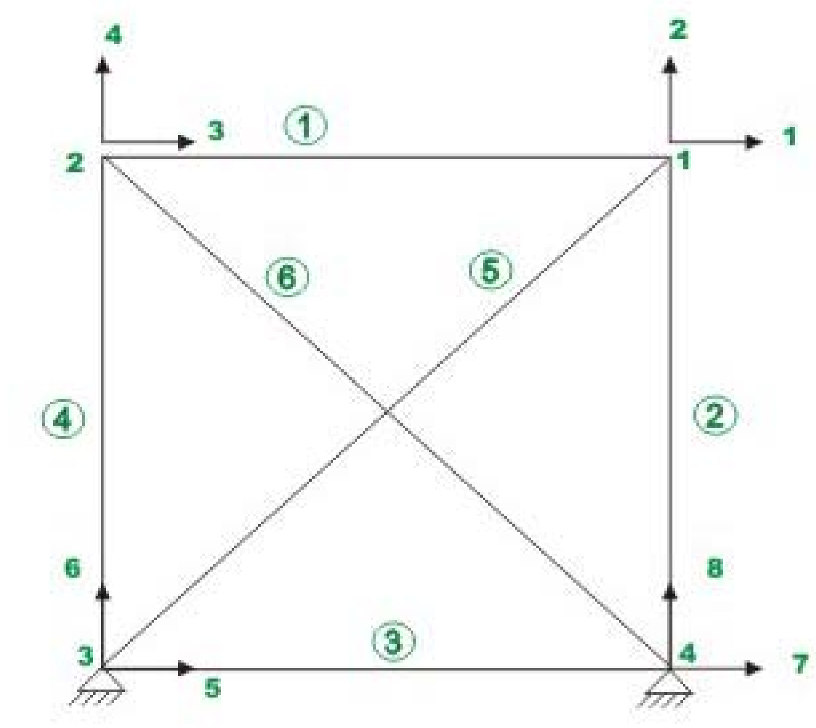

Fig 26.4b Node and member numbering

Solution

For this problem assembled stiffness matrix is available in Fig.26.4b.The joints and members are numbered as shown in Fig.26.4b. In the given problem u_1, u_2, u_3, u_4 and u_5 represent unconstrained degrees of freedom. Due to support conditions, $u_6 = u_7 = u_8 = 0$.

The temperature of the member (2) is raised by 50° C.Thus,

$$\Delta L^2 = \alpha L \Delta T = \frac{1}{75000} \times 5 \times 50 = 3.333 \times 10^{-3} m \quad (1)$$

The forces are developed in member (2), as it was prevented from expansion.

$$\begin{Bmatrix} (p_7)_f \\ (p_8)_f \\ (p_1)_f \\ (p_2)_f \end{Bmatrix} = 300 \times 3.333 \times 10^{-3} \begin{Bmatrix} \cos\theta \\ \sin\theta \\ -\cos\theta \\ -\sin\theta \end{Bmatrix}$$

$$= \begin{Bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{Bmatrix} \qquad (2)$$

The displacement of the member (5) was raised by $40^\circ C$. Thus,

$$\Delta L^5 = \alpha L \Delta T = \frac{1}{75{,}000} \times 5\sqrt{2} \times 40 = 3.771 \times 10^{-3}\, m$$

The forces developed in member (5) as it was not allowed to expand is

$$\begin{Bmatrix} (p_5)_t \\ (p_6)_t \\ (p_7)_t \\ (p_8)_t \end{Bmatrix} = 300 \times 3.771 \times 10^{-3} \begin{Bmatrix} 0.707 \\ 0.707 \\ -0.707 \\ -0.707 \end{Bmatrix}$$

$$= 0.8 \begin{Bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{Bmatrix} \qquad (3)$$

The global force vector due to thermal load is

$$\begin{Bmatrix} (p_1)_t \\ (p_2)_t \\ (p_3)_t \\ (p_4)_t \\ (p_5)_t \\ (p_6)_t \\ (p_7)_t \\ (p_8)_t \end{Bmatrix} = \begin{Bmatrix} -0.8 \\ -1.8 \\ 0 \\ 0 \\ 0.8 \\ 0.8 \\ 0 \\ 1 \end{Bmatrix} \qquad (4)$$

Writing the load-displacement relation for the entire truss is given below.

$$\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \end{Bmatrix} = AE \begin{bmatrix} 0.271 & 0.071 & -0.20 & 0 & -0.071 & -0.071 & 0 & 0 \\ 0.071 & 0.271 & 0 & 0 & -0.071 & -0.071 & 0 & -0.2 \\ -0.20 & 0 & 0.271 & -0.071 & 0 & 0 & -0.071 & 0.071 \\ 0 & 0 & -0.071 & 0.129 & 0 & -0.2 & 0.071 & 0.071 \\ -0.071 & -0.071 & 0 & 0 & 0.271 & 0.071 & -0.2 & 0 \\ -0.071 & -0.071 & 0 & -0.2 & 0.071 & 0.271 & 0 & 0 \\ 0 & 0 & -0.071 & 0.071 & -0.2 & 0 & 0.271 & -0.071 \\ 0 & -0.2 & 0.071 & -0.071 & 0 & 0 & -0.071 & 0.271 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \end{Bmatrix} + \begin{Bmatrix} -0.8 \\ -1.8 \\ 0 \\ 0 \\ 0.8 \\ 0.8 \\ 0 \\ 1 \end{Bmatrix} \quad (5)$$

In the above problem $p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = p_7 = p_8 = 0$ and $u_6 = u_7 = u_8 = 0$.

Thus solving for unknown displacements,

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} = \frac{1}{AE} \begin{bmatrix} 0.271 & 0.071 & -0.2 & 0 & -0.071 \\ 0.071 & 0.271 & 0 & 0 & -0.071 \\ -0.20 & 0 & 0.271 & -0.071 & 0 \\ 0 & 0 & -0.071 & 0.129 & 0 \\ -0.071 & -0.071 & 0 & 0 & 0.271 \end{bmatrix} \left(\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} -0.8 \\ -1.8 \\ 0 \\ 0 \\ 0.8 \end{Bmatrix} \right) \quad (5)$$

Solving equation (5), the unknown displacements are calculated as

$$u_1 = 0.0013m, u_2 = 0.0020m, u_3 = -0.0005m, u_4 = 0$$
$$u_5 = -0.0013m \quad (6)$$

Now, reactions are computed as,

$$\begin{Bmatrix} p_6 \\ p_7 \\ p_8 \end{Bmatrix} = \begin{bmatrix} -0.071 & -0.071 & 0 & -0.2 & 0.071 \\ 0 & 0 & -0.071 & 0.071 & -0.2 \\ 0 & -0.2 & 0.071 & -0.071 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} + \begin{Bmatrix} 0.8 \\ 0 \\ 1 \end{Bmatrix} \quad (7)$$

All reactions are zero as truss is externally determinate and hence change in temperature does not induce any reaction. Now member forces are calculated by using equation (26.10b)

Member (1): L=5m, $\theta = 0^\circ$

$$p_2^{'} = \frac{AE}{5}[-1\ 0\ 1\ 0]\begin{Bmatrix} u_3 \\ u_4 \\ u_1 \\ u_2 \end{Bmatrix} \quad (8)$$

$p_2^{'} =$ 0.1080 Kn

Member 2: L=5m, $\theta = 90^\circ$,nodal points 4-1

$$p_2^{'} = \frac{AE}{5}[0\ -1\ 0\ 1]\begin{Bmatrix} u_7 \\ u_8 \\ u_1 \\ u_2 \end{Bmatrix} - 300 \times 3.771 \times 10^{-5} \quad (9)$$

=0.1087 kN

Member (3): L=5m, $\theta = 0^\circ$,nodal points 3-4

$$p_2^{'} = \frac{300}{5}[-1\ 0\ 1\ 0]\begin{Bmatrix} u_5 \\ u_6 \\ u_7 \\ u_8 \end{Bmatrix} \quad (10)$$

=0.0780kN

Member (4): $\theta = 90^\circ, L = 5m,$ nodal points 3-2

$$p_2^{'} = \frac{300}{5}[0\ -1\ 0\ 1]\begin{Bmatrix} u_5 \\ u_6 \\ u_3 \\ u_4 \end{Bmatrix} = 0 \quad (11)$$

Member (5): $\theta = 45^\circ, L = 5\sqrt{2}$,nodal points 3-1

$$p_2^{'} = \frac{300}{5\sqrt{2}}[-0.707\ -0.707\ 0.707\ 0.707]\begin{Bmatrix} u_5 \\ u_6 \\ u_1 \\ u_2 \end{Bmatrix} - 300 \times 3.333 \times 10^{-3} \quad (12)$$

=-0.8619 kN

Member (6) : $\theta = 135^\circ, L = 5\sqrt{2}$,nodal points 4-2

$$p_2^{'} = \frac{300}{5\sqrt{2}} [0.707\ -0.707\ -0.707\ 0.707] \begin{Bmatrix} u_7 \\ u_8 \\ u_3 \\ u_4 \end{Bmatrix} = 0.0150 \text{ kN.} \qquad (13)$$

Summary

In the last four lessons, the direct stiffness method as applied to the truss analysis was discussed. Assembly of member stiffness matrices, imposition of boundary conditions, and the problem of inclined supports were discussed. Due to the change in temperature the truss members either expand or shrink. However, in the case of statically indeterminate trusses, the length of the members is prevented from either expansion or contraction. Thus, the stresses are developed in the members due to changes in temperature. Similarly the errors in fabricating truss members also produce additional stresses in the trusses. In this lesson, these effects are accounted for in the stiffness analysis. A couple of problems are solved.