Module 4

Analysis of Statically Indeterminate Structures by the Direct Stiffness Method

Lesson 25

The Direct Stiffness Method: Truss Analysis (Continued)

Instructional Objectives

After reading this chapter the student will be able to
1. Transform member stiffness matrix from local to global co-ordinate system.
2. Assemble member stiffness matrices to obtain the global stiffness matrix.
3. Analyse plane truss by the direct stiffness matrix.
4. Analyse plane truss supported on inclined roller supports.

25.1 Introduction

In the previous lesson, the direct stiffness method as applied to trusses was discussed. The transformation of force and displacement from local co-ordinate system to global co-ordinate system were accomplished by single transformation matrix. Also assembly of the member stiffness matrices was discussed. In this lesson few plane trusses are analysed using the direct stiffness method. Also the problem of inclined support will be discussed.

Example 25.1

Analyse the truss shown in Fig. 25.1a and evaluate reactions. Assume EA to be constant for all the members.

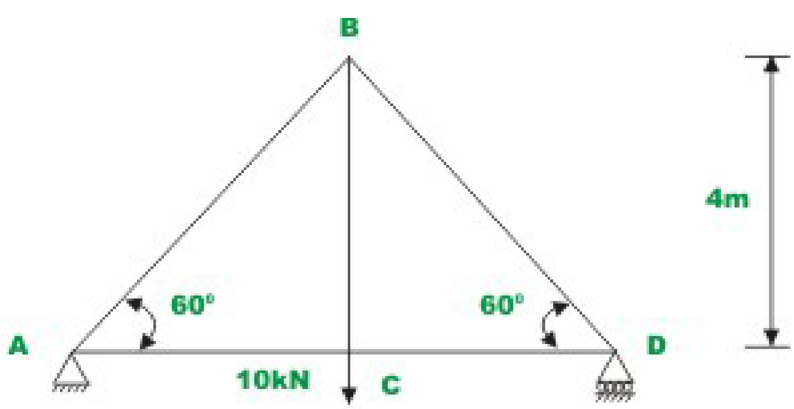

Fig.25.1(a) Plane truss of Example 25.1

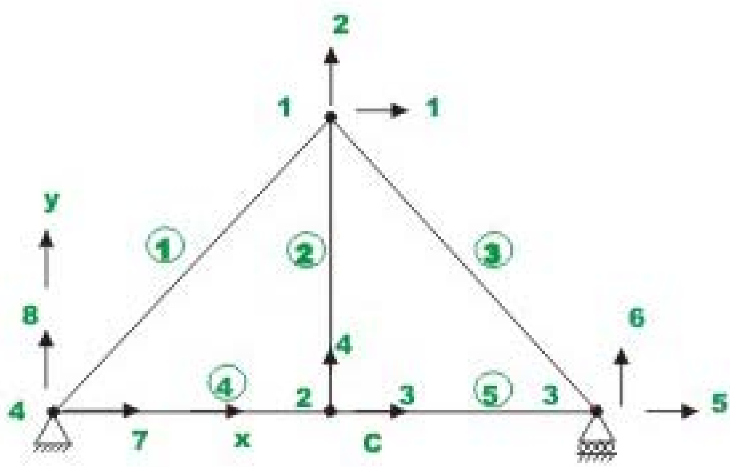

Fig. 25.1(b) Node and member numbering

The numbering of joints and members are shown in Fig. 25.1b. Also, the possible displacements (degrees of freedom) at each node are indicated. Here lower numbers are used to indicate unconstrained degrees of freedom and higher numbers are used for constrained degrees of freedom. Thus displacements 6,7 and 8 are zero due to boundary conditions.

First write down stiffness matrix of each member in global co-ordinate system and assemble them to obtain global stiffness matrix.

Element 1: $\theta = 60°, \quad L = 4.619\ m.$ Nodal points 4-1

$$\left[k^1\right] = \frac{EA}{4.619}\begin{bmatrix} 0.25 & 0.433 & -0.25 & -0.433 \\ 0.433 & 0.75 & -0.433 & -0.75 \\ -0.25 & -0.433 & 0.25 & 0.433 \\ -0.433 & -0.75 & 0.433 & 0.75 \end{bmatrix} \quad (1)$$

Element 2: $\theta = 90°, \quad L = 4.00\ m.$ Nodal points 2-1

$$[k^2] = \frac{EA}{4.0}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \qquad (2)$$

Element 3: $\theta = 120°, \quad L = 4.619\ m.$ Nodal points 3-1

$$[k^3] = \frac{EA}{4.619}\begin{bmatrix} 0.25 & -0.433 & -0.25 & 0.433 \\ -0.433 & 0.75 & 0.433 & -0.75 \\ -0.25 & 0.433 & 0.25 & -0.433 \\ 0.433 & -0.75 & -0.433 & 0.75 \end{bmatrix} \qquad (3)$$

Element 4: $\theta = 0°, \quad L = 2.31\ m.$ Nodal points 4-2

$$[k^4] = \frac{EA}{2.31}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (4)$$

Element 5: $\theta = 0°, \quad L = 2.31\ m.$ Nodal points 2-3

$$[k^5] = \frac{EA}{2.31}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (5)$$

The assembled global stiffness matrix of the truss is of the order 8×8. Now assemble the global stiffness matrix. Note that the element k_{11}^1 of the member stiffness matrix of truss member 1 goes to location $(7,7)$ of global stiffness matrix. On the member stiffness matrix the corresponding global degrees of freedom are indicated to facilitate assembling. Thus,

$$
[K] = EA \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 0.108 & 0 & 0 & 0 & -0.054 & 0.094 & -0.054 & -0.094 \\ 0 & 0.575 & 0 & -0.25 & 0.094 & -0.162 & -0.094 & -0.162 \\ 0 & 0 & 0.866 & 0 & -0.433 & 0 & -0.433 & 0 \\ 0 & -0.25 & 0 & 0.25 & 0 & 0 & 0 & 0 \\ -0.054 & 0.094 & -0.433 & 0 & 0.487 & -0.094 & 0 & 0 \\ 0.094 & -0.162 & 0 & 0 & -0.094 & 0.162 & 0 & 0 \\ -0.054 & -0.094 & -0.433 & 0 & 0 & 0 & 0.487 & 0.0934 \\ -0.094 & -0.162 & 0 & 0 & 0 & 0 & 0.0934 & 0.162 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array}
$$

(6)

Writing the load-displacement relation for the truss, yields

$$
\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \end{Bmatrix} = EA \begin{bmatrix} 0.108 & 0 & 0 & 0 & -0.054 & 0.094 & -0.054 & -0.094 \\ 0 & 0.575 & 0 & -0.25 & 0.094 & -0.162 & -0.094 & -0.162 \\ 0 & 0 & 0.866 & 0 & -0.433 & 0 & -0.433 & 0 \\ 0 & -0.25 & 0 & 0.25 & 0 & 0 & 0 & 0 \\ -0.054 & 0.094 & -0.433 & 0 & 0.487 & -0.094 & 0 & 0 \\ 0.094 & -0.162 & 0 & 0 & -0.094 & 0.162 & 0 & 0 \\ -0.054 & -0.094 & -0.433 & 0 & 0 & 0 & 0.487 & 0.0934 \\ -0.094 & -0.162 & 0 & 0 & 0 & 0 & 0.0934 & 0.162 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \end{Bmatrix}
$$

(7)

The displacements u_1 to u_5 are unknown. The displacements $u_6 = u_7 = u_8 = 0$.

Also $p_1 = p_2 = p_3 = p_5 = 0$. But $p_4 = -10 \text{ kN}$.

$$
\begin{Bmatrix} 0 \\ 0 \\ 0 \\ -10 \\ 0 \end{Bmatrix} = EA \begin{bmatrix} 0.108 & 0 & 0 & 0 & -0.054 \\ 0 & 0.575 & 0 & -0.25 & 0.094 \\ 0 & 0 & 0.866 & 0 & -0.433 \\ 0 & -0.25 & 0 & 0.25 & 0 \\ -0.054 & 0.094 & -0.433 & 0 & 0.487 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} \qquad (8)
$$

Solving which, the unknown displacements are evaluated. Thus,

$$u_1 = \frac{6.668}{AE}\ ;\ u_2 = \frac{-34.64}{AE};\ u_3 = \frac{6.668}{AE};\ u_4 = \frac{-74.642}{AE};\ u_5 = \frac{13.334}{AE} \qquad (9)$$

Now reactions are evaluated from equation,

$$\begin{Bmatrix} p_6 \\ p_7 \\ p_8 \end{Bmatrix} = EA \begin{bmatrix} 0.094 & -0.162 & 0 & 0 & -0.094 \\ -0.054 & -0.094 & -0.433 & 0 & 0 \\ -0.094 & -0.162 & 0 & 0 & 0 \end{bmatrix} \frac{1}{EA} \begin{Bmatrix} 6.668 \\ -34.64 \\ 6.668 \\ -74.642 \\ 13.334 \end{Bmatrix} \qquad (10)$$

Thus,

$$p_6 = 5.00 \quad \text{kN} \quad ; \quad p_7 = \ 0 \quad ; \quad p_8 = 5.00 \quad \text{kN}\,. \qquad (11)$$

Now calculate individual member forces.

Member 1: $l = 0.50$; $m = 0.866$; $L = 4.619\,m$.

$$\{p'_1\} = \frac{AE}{4.619}\begin{bmatrix} l & m & -l & -m \end{bmatrix} \begin{Bmatrix} u_7 \\ u_8 \\ u_1 \\ u_2 \end{Bmatrix}$$

$$\{p'_1\} = \frac{AE}{4.619}\begin{bmatrix} -0.5 & -0.866 \end{bmatrix} \frac{1}{AE} \begin{Bmatrix} 6.667 \\ -34.64 \end{Bmatrix} = 5.77 \quad \text{kN} \qquad (12)$$

Member 2: $l = 0$; $m = 1.0$; $L = 4.0\,m$.

$$\{p'_1\} = \frac{AE}{4.0}\begin{bmatrix} l & m & -l & -m \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \\ u_1 \\ u_2 \end{Bmatrix}$$

$$\{p'_1\} = \frac{AE}{4.619}\begin{bmatrix} 1 & -1 \end{bmatrix} \frac{1}{AE} \begin{Bmatrix} -74.642 \\ -34.64 \end{Bmatrix} = -10.0 \quad \text{kN} \qquad (13)$$

Member 3: $l = -0.50$; $m = 0.866$; $L = 4.619\,m$.

$$\{p'_1\} = \frac{AE}{4.619}\begin{bmatrix} l & m & -l & -m \end{bmatrix}\begin{Bmatrix} u_5 \\ u_6 \\ u_1 \\ u_2 \end{Bmatrix}$$

$$\{p'_1\} = \frac{AE}{4.619}\begin{bmatrix} -0.5 & 0.5 & -0.866 \end{bmatrix}\frac{1}{AE}\begin{Bmatrix} 13.334 \\ 6.667 \\ -34.64 \end{Bmatrix} = 5.77 \quad \text{kN} \tag{14}$$

Member 4: $l = 1.0$; $m = 0$; $L = 2.31.0\,m$.

$$\{p'_1\} = \frac{AE}{2.31}\begin{bmatrix} l & m & -l & -m \end{bmatrix}\begin{Bmatrix} u_7 \\ u_8 \\ u_3 \\ u_4 \end{Bmatrix}$$

$$\{p'_1\} = \frac{AE}{2.31}\begin{bmatrix} 1 & -1 \end{bmatrix}\frac{1}{AE}\begin{Bmatrix} 0 \\ 6.667 \end{Bmatrix} = -2.88 \quad \text{kN} \tag{15}$$

Member 5: $l = 1.0$; $m = 0$; $L = 2.31.0\,m$.

$$\{p'_1\} = \frac{AE}{2.31}\begin{bmatrix} l & m & -l & -m \end{bmatrix}\begin{Bmatrix} u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix}$$

$$\{p'_1\} = \frac{AE}{2.31}\begin{bmatrix} 1 & -1 \end{bmatrix}\frac{1}{AE}\begin{Bmatrix} 6.667 \\ 13.334 \end{Bmatrix} = -2.88 \quad \text{kN} \tag{16}$$

Example 25.2

Determine the forces in the truss shown in Fig. 25.2a by the direct stiffness method. Assume that all members have the same axial rigidity.

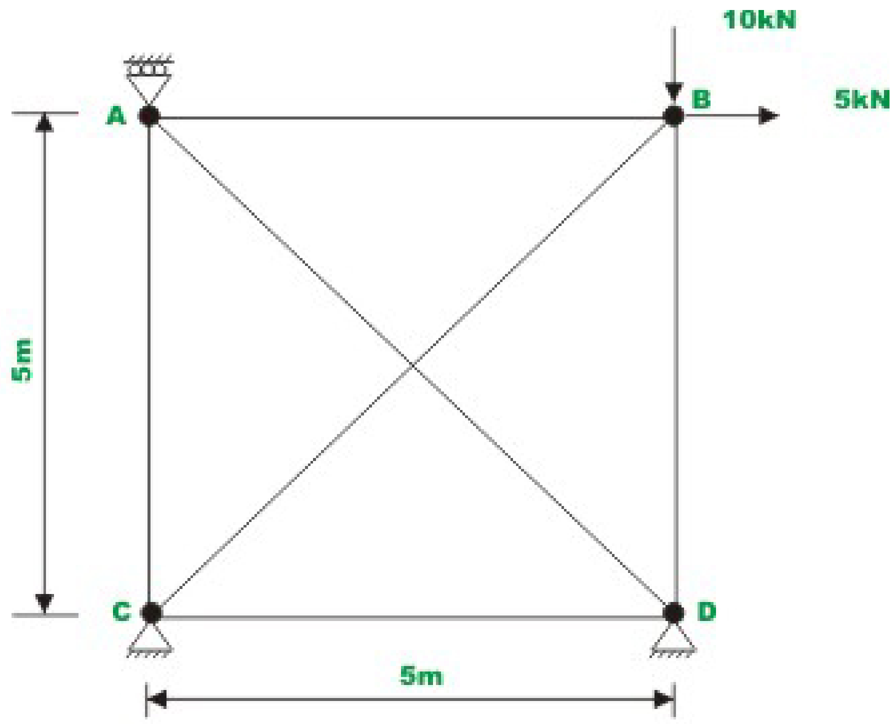

Fig. 25.2a Example 25.2

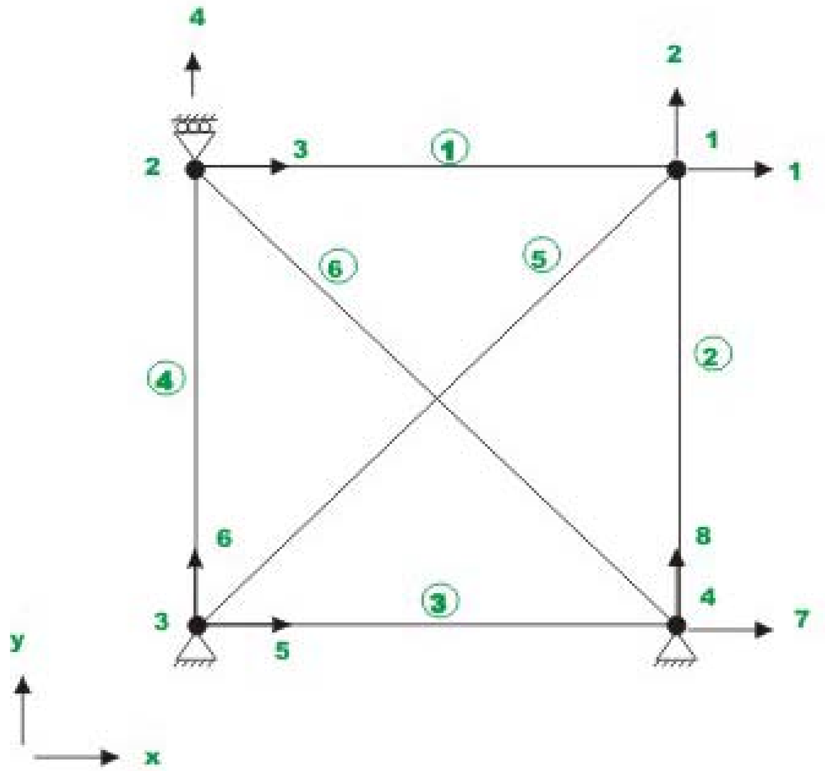

Fig.25.2b Node and member numbering

The joint and member numbers are indicated in Fig. 25.2b. The possible degree of freedom are also shown in Fig. 25.2b. In the given problem u_1, u_2 and u_3 represent unconstrained degrees of freedom and $u_4 = u_5 = u_6 = u_7 = u_8 = 0$ due to boundary condition. First let us generate stiffness matrix for each of the six members in global co-ordinate system.

Element 1: $\theta = 0°, \quad L = 5.00\ m.$ Nodal points 2-1

$$[k^1] = \frac{EA}{5.0} \begin{array}{c} \begin{array}{cccc} 3 & 4 & 1 & 2 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 3 \\ 4 \\ 1 \\ 2 \end{array} \quad (1)$$

Element 2: $\theta = 90°, \quad L = 5.00\ m.$ Nodal points 4-1

$$[k^2] = \frac{EA}{5.0} \begin{array}{c} \begin{array}{cccc} 7 & 8 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ 7 \\ 8 \\ 1 \\ 2 \end{array} \tag{2}$$

Element 3: $\theta = 0°, \quad L = 5.00\ m.$ Nodal points 3-4

$$[k^3] = \frac{EA}{5.0} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \tag{3}$$

Element 4: $\theta = 90°, \quad L = 5.00\ m.$ Nodal points 3-2

$$[k^4] = \frac{EA}{5.0} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ 5 \\ 6 \\ 3 \\ 4 \end{array} \tag{4}$$

Element 5: $\theta = 45°, \quad L = 7.07\ m.$ Nodal points 3-1

$$[k^5] = \frac{EA}{7.07} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 1 & 2 \end{array} \\ \begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix} \end{array} \begin{array}{c} \\ 5 \\ 6 \\ 1 \\ 2 \end{array} \tag{5}$$

Element 6: $\theta = 135°, \quad L = 7.07\ m.$ Nodal points 4-2

$$
\begin{array}{ccccc}
7 & 8 & 3 & 4 & \\
\end{array}
$$

$$
\left[k^6\right] = \frac{EA}{7.07}\begin{bmatrix} 0.5 & -0.5 & -0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 & 0.5 \end{bmatrix}\begin{array}{c} 7 \\ 8 \\ 3 \\ 4 \end{array} \qquad (6)
$$

There are eight possible global degrees of freedom for the truss shown in the figure. Hence the global stiffness matrix is of the order (8×8). On the member stiffness matrix, the corresponding global degrees of freedom are indicated to facilitate assembly. Thus the global stiffness matrix is,

$$
[K] = AE\left[\begin{array}{ccc|ccccc} 0.271 & 0.071 & -0.2 & 0 & -0.071 & -0.071 & 0 & 0 \\ 0.071 & 0.271 & 0 & 0 & -0.071 & -0.071 & 0 & -0.20 \\ -0.20 & 0 & 0.271 & -0.071 & 0 & 0 & -0.071 & 0.071 \\ \hline 0 & 0 & -0.071 & 0.271 & 0.071 & -0.20 & 0.071 & -0.071 \\ -0.071 & -0.071 & 0 & 0 & 0.271 & 0.071 & -0.20 & 0 \\ -0.071 & -0.071 & 0 & -0.20 & 0.071 & 0.271 & 0 & 0 \\ 0 & 0 & -0.071 & 0.071 & -0.20 & 0 & 0.271 & -0.071 \\ 0 & -0.20 & 0.071 & -0.071 & 0 & 0 & -0.071 & 0.271 \end{array}\right] \qquad (7)
$$

The force-displacement relation for the truss is,

$$
\begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \end{Bmatrix} = EA\left[\begin{array}{ccc|ccccc} 0.271 & 0.071 & -0.2 & 0 & -0.071 & -0.071 & 0 & 0 \\ 0.071 & 0.271 & 0 & 0 & -0.071 & -0.071 & 0 & -0.20 \\ -0.20 & 0 & 0.271 & -0.071 & 0 & 0 & -0.071 & 0.071 \\ \hline 0 & 0 & -0.071 & 0.271 & 0.071 & -0.20 & 0.071 & -0.071 \\ -0.071 & -0.071 & 0 & 0 & 0.271 & 0.071 & -0.20 & 0 \\ -0.071 & -0.071 & 0 & -0.20 & 0.071 & 0.271 & 0 & 0 \\ 0 & 0 & -0.071 & 0.071 & -0.20 & 0 & 0.271 & -0.071 \\ 0 & -0.20 & 0.071 & -0.071 & 0 & 0 & -0.071 & 0.271 \end{array}\right]\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \end{Bmatrix} \qquad (8)
$$

The displacements u_1, u_2 and u_3 are unknowns.

Here, $p_1 = 5 \quad \text{kN} \quad ; p_2 = -10 \quad ; p_3 = 0$ and $u_4 = u_5 = u_6 = u_7 = u_8 = 0$.

$$\begin{Bmatrix} 5 \\ -10 \\ 0 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \end{Bmatrix} = EA \left[\begin{array}{ccc|ccccc} 0.271 & 0.071 & -0.2 & 0 & -0.071 & -0.071 & 0 & 0 \\ 0.071 & 0.271 & 0 & 0 & -0.071 & -0.071 & 0 & -0.20 \\ -0.20 & 0 & 0.271 & -0.071 & 0 & 0 & -0.071 & 0.071 \\ \hline 0 & 0 & -0.071 & 0.271 & 0.071 & -0.20 & 0.071 & -0.071 \\ -0.071 & -0.071 & 0 & 0 & 0.271 & 0.071 & -0.20 & 0 \\ -0.071 & -0.071 & 0 & -0.20 & 0.071 & 0.271 & 0 & 0 \\ 0 & 0 & -0.071 & 0.071 & -0.20 & 0 & 0.271 & -0.071 \\ 0 & -0.20 & 0.071 & -0.071 & 0 & 0 & -0.071 & 0.271 \end{array}\right] \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{9}$$

Thus,

$$\begin{Bmatrix} 5 \\ -10 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0.271 & 0.071 & -0.20 \\ 0.071 & 0.271 & 0 \\ -0.20 & 0 & 0.271 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \tag{10}$$

Solving which, yields

$$u_1 = \frac{72.855}{AE} \; ; \; u_2 = \frac{-55.97}{AE}; \; u_3 = \frac{53.825}{AE}$$

Now reactions are evaluated from the equation,

$$\begin{Bmatrix} p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & -0.071 \\ -0.071 & -0.071 & 0 \\ -0.071 & -0.071 & 0 \\ 0 & 0 & -0.071 \\ 0 & -0.20 & 0.071 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \tag{11}$$

$p_4 = -3.80$ kN ; $p_5 = -1.19$ kN ; $p_6 = -1.19$ kN ; $p_7 = 3.8\ 0\ kN$; $p_8 = 15.00$ kN

In the next step evaluate forces in members.

Element 1: $\theta = 0°, \quad L = 5.00\ m.$ Nodal points 2-1

$$\{p'_1\} = \frac{AE}{5.0}\begin{bmatrix} l & m & -l & -m \end{bmatrix}\begin{Bmatrix} u_3 \\ u_4 \\ u_1 \\ u_2 \end{Bmatrix}$$

$$\{p'_1\} = \frac{AE}{5.0}\begin{bmatrix} 1 & -1 \end{bmatrix}\frac{1}{AE}\begin{Bmatrix} 53.825 \\ 72.855 \end{Bmatrix} = -3.80 \text{ kN} \tag{12}$$

Element 2: $\theta = 90°, \quad L = 5.00\ m.$ Nodal points 4-1

$$\{p'_1\} = \frac{AE}{5}\begin{bmatrix} l & m & -l & -m \end{bmatrix}\begin{Bmatrix} u_7 \\ u_8 \\ u_1 \\ u_2 \end{Bmatrix}$$

$$\{p'_1\} = \frac{AE}{5}\begin{bmatrix} 1 & -1 \end{bmatrix}\frac{1}{AE}\begin{Bmatrix} 0 \\ -55.97 \end{Bmatrix} = 11.19\text{kN} \tag{13}$$

Element 3: $\theta = 0°, \quad L = 5.00\ m.$ Nodal points 3-4

$$\{p'_1\} = \frac{AE}{5}[0]\{0\} = 0 \tag{14}$$

Element 4: $\theta = 90°, \quad L = 5.00\ m.$ Nodal points 3-2

$$\{p'_1\} = \frac{AE}{5}\begin{bmatrix} l & m & -l & -m \end{bmatrix}\begin{Bmatrix} u_5 \\ u_6 \\ u_3 \\ u_4 \end{Bmatrix}$$

$$\{p'_1\} = \frac{AE}{5}[0]\frac{1}{AE}\{53.825\} = 0 \tag{15}$$

Element 5: $\theta = 45°, \quad L = 7.07\ m.$ Nodal points 3-1

$$\{p'_1\} = \frac{AE}{7.07}\begin{bmatrix} l & m & -l & -m \end{bmatrix}\begin{Bmatrix} u_5 \\ u_6 \\ u_1 \\ u_2 \end{Bmatrix}$$

$$\{p'_1\} = \frac{AE}{7.07}[-0.707 \quad -0.707]\frac{1}{AE}\begin{Bmatrix} 72.855 \\ -55.97 \end{Bmatrix} = -1.688 \quad \text{kN} \qquad (16)$$

Element 6: $\theta = 135°, \quad L = 7.07\ m.$ Nodal points 4-2

$$\{p'_1\} = \frac{AE}{7.07}[l \quad m \quad -l \quad -m]\begin{Bmatrix} u_7 \\ u_8 \\ u_3 \\ u_4 \end{Bmatrix}$$

$$\{p'_1\} = \frac{AE}{7.07}[0.707]\frac{1}{AE}\{53.825\} = 5.38 \quad \text{kN} \qquad (17)$$

25.2 Inclined supports

Sometimes the truss is supported on a roller placed on an oblique plane (vide Fig. 25.3a). At a roller support, the displacement perpendicular to roller support is zero. $i.e.$ displacement along y'' is zero in the present case.

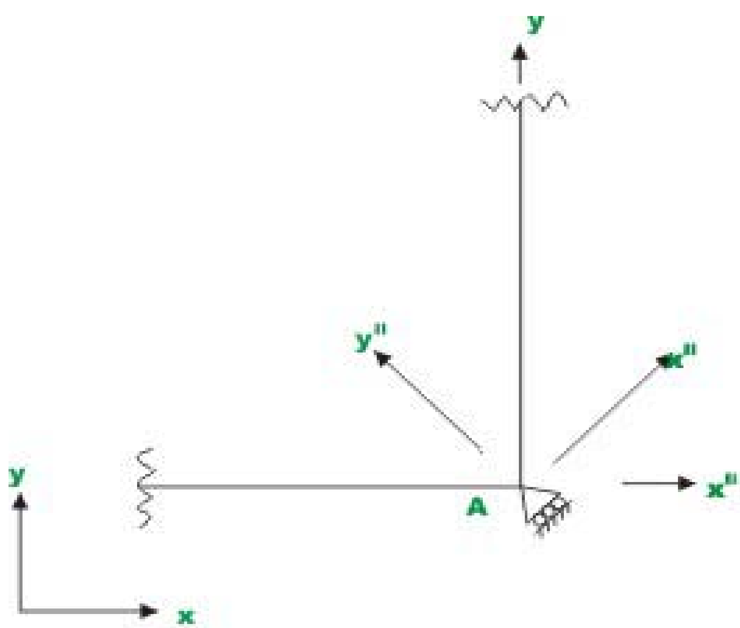

Fig.25.3(a) Inclined support

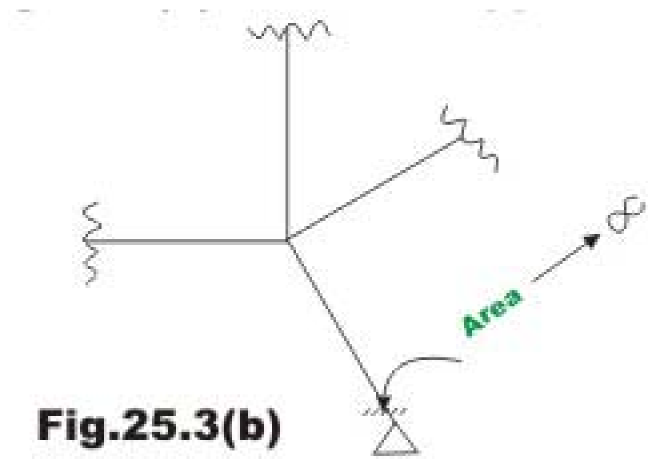

Fig.25.3(b)

If the stiffness matrix of the entire truss is formulated in global co-ordinate system then the displacements along y are not zero at the oblique support. So, a special procedure has to be adopted for incorporating the inclined support in the analysis of truss just described. One way to handle inclined support is to replace the inclined support by a member having large cross sectional area as shown in Fig. 25.3b but having the length comparable with other members meeting at that joint. The inclined member is so placed that its centroidal axis is perpendicular to the inclined plane. Since the area of cross section of this new member is very high, it does not allow any displacement along its centroidal axis of the joint A. Another method of incorporating inclined support in the analysis is to suitably modify the member stiffness matrix of all the members meeting at the inclined support.

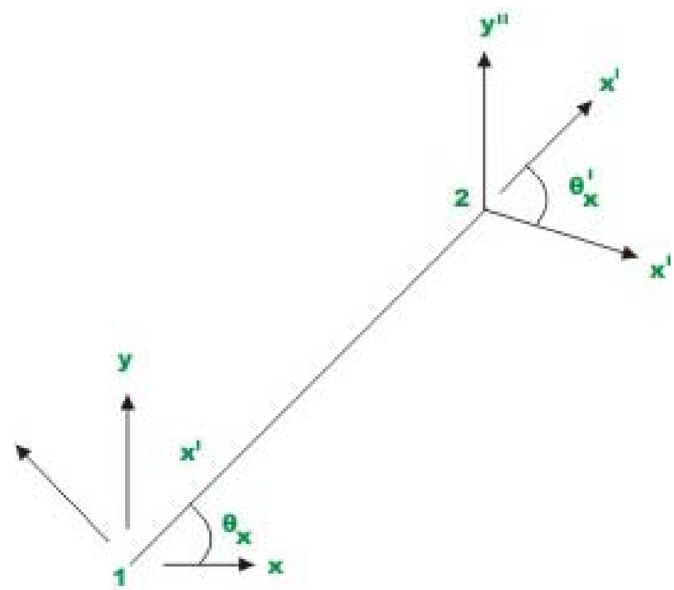

Fig.25.4 Truss member in global and local co-ordinate system

Consider a truss member as shown in Fig. 25.4. The nodes are numbered as 1 and 2. At 2, it is connected to a inclined support. Let $x'y'$ be the local co-ordinate axes of the member. At node 1, the global co-ordinate system xy is also shown. At node 2, consider nodal co-ordinate system as $x''y''$, where y'' is perpendicular to oblique support. Let u'_1 and u'_2 be the displacements of nodes 1 and 2 in the local co-ordinate system. Let u_1, v_1 be the nodal displacements of node 1 in global co-ordinate system xy. Let u''_2, v''_2 be the nodal displacements along x''- and y''- are in the local co-ordinate system $x''y''$ at node 2. Then from Fig. 25.4,

$$u'_1 = u_1 \cos\theta_x + v_1 \sin\theta_x$$

$$u'_2 = u''_2 \cos\theta_{x''} + v''_2 \sin\theta_{x''} \tag{25.1}$$

This may be written as

$$\begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix} = \begin{bmatrix} \cos\theta_x & \sin\theta_x & 0 & 0 \\ 0 & 0 & \cos\theta_{x''} & \sin\theta_{x''} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u''_2 \\ v''_2 \end{Bmatrix} \tag{25.2}$$

Denoting $l = \cos\theta_x \;; m = \sin\theta_x \;; l'' = \cos\theta_{x''} \;; m'' = \sin\theta_{x''}$

$$\begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix} = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l'' & m'' \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u''_2 \\ v''_2 \end{Bmatrix} \tag{25.3a}$$

or $\{u'\} = [T']\{u\}$

where $[T']$ is the displacement transformation matrix.

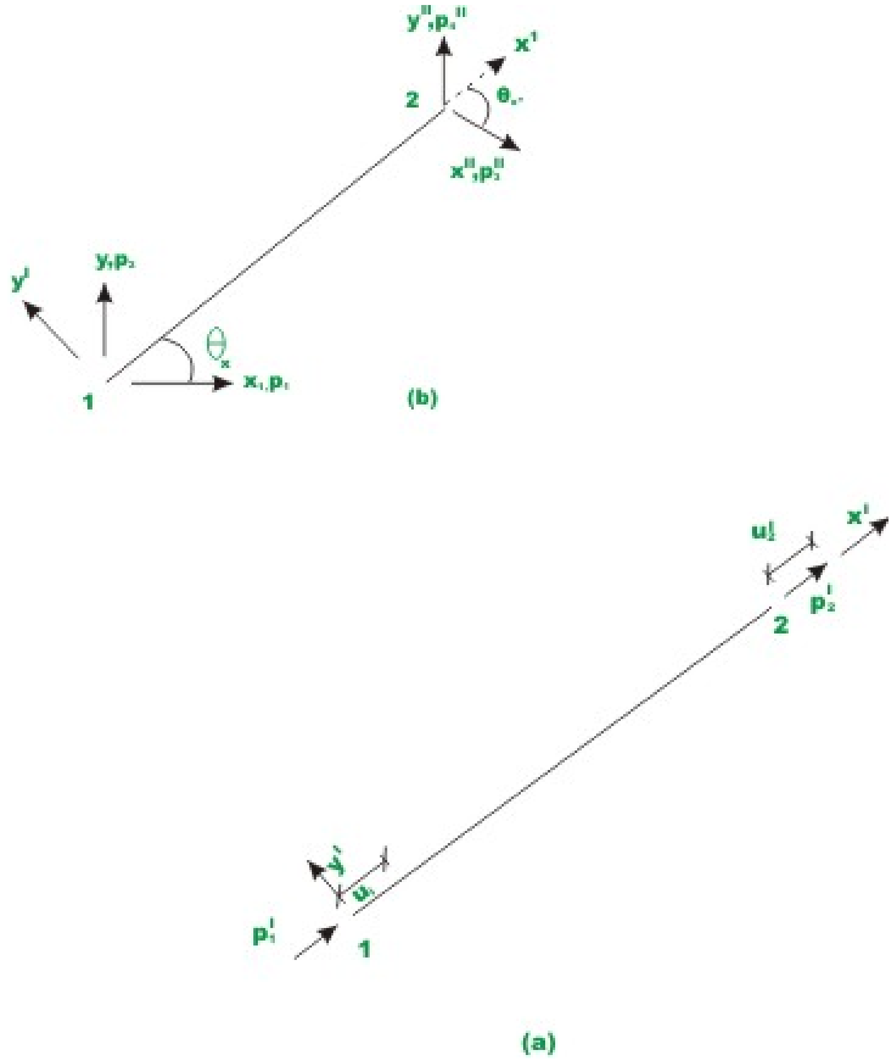

Fig.25.5 Displacement and force transformation

Similarly referring to Fig. 25.5, the force p'_1 has components along x and y axes. Hence

$$p_1 = p'_1 \cos\theta_x \qquad (25.4a)$$

$$p_2 = p'_1 \sin\theta_x \qquad (25.4b)$$

Similarly, at node 2, the force p'_2 has components along x'' and y'' axes.

$$p''_3 = p'_2 \cos\theta''_x \tag{25.5a}$$

$$p''_4 = p'_2 \sin\theta''_x \tag{25.5b}$$

The relation between forces in the global and local co-ordinate system may be written as,

$$\begin{Bmatrix} p_1 \\ p_2 \\ p''_3 \\ p''_4 \end{Bmatrix} = \begin{bmatrix} \cos\theta_x & 0 \\ \sin\theta_x & 0 \\ 0 & \cos\theta''_x \\ 0 & \sin\theta''_x \end{bmatrix} \begin{Bmatrix} p'_1 \\ p'_2 \end{Bmatrix} \tag{25.6}$$

$$\{p\} = [T']^T \{p'\} \tag{25.7}$$

Using displacement and force transformation matrices, the stiffness matrix for member having inclined support is obtained.

$$[k] = [T']^T [k'][T']$$

$$[k] = \begin{bmatrix} l & 0 \\ m & 0 \\ 0 & l'' \\ 0 & m'' \end{bmatrix} \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l'' & m'' \end{bmatrix} \tag{25.8}$$

Simplifying,

$$[k] = \frac{EA}{L} \begin{bmatrix} l^2 & lm & -ll'' & -lm'' \\ lm & m^2 & -ml'' & -mm'' \\ -ll'' & -ml'' & l''^2 & l''m'' \\ -lm'' & -mm'' & l''m'' & m''^2 \end{bmatrix} \tag{25.9}$$

If we use this stiffness matrix, then it is easy to incorporate the condition of zero displacement perpendicular to the inclined support in the stiffness matrix. This is shown by a simple example.

Example 25.3

Analyse the truss shown in Fig. 25.6a by stiffness method. Assume axial rigidity EA to be constant for all members.

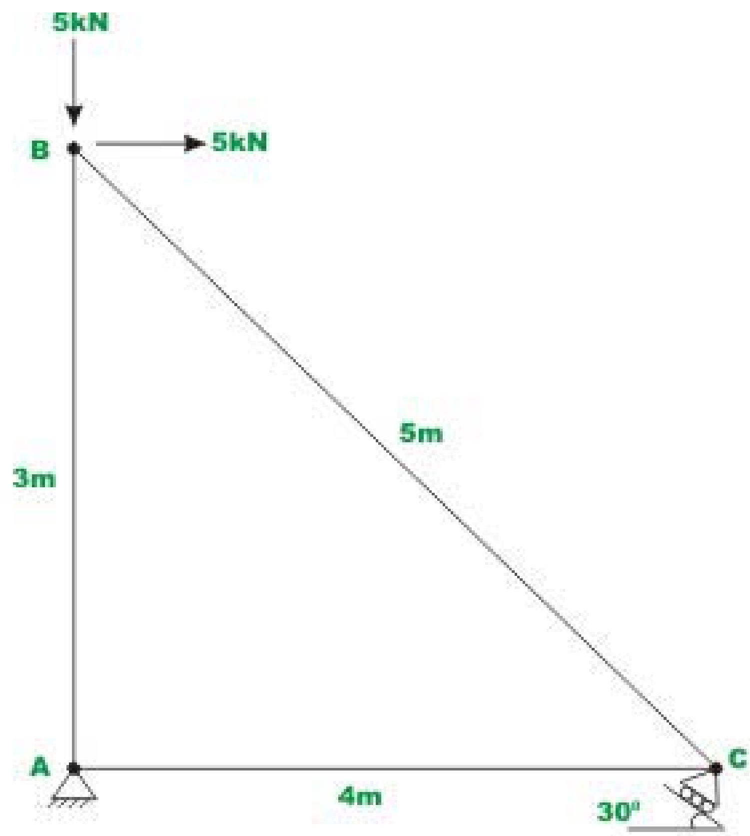

Fig.25.6(a) Plane truss with inclined support

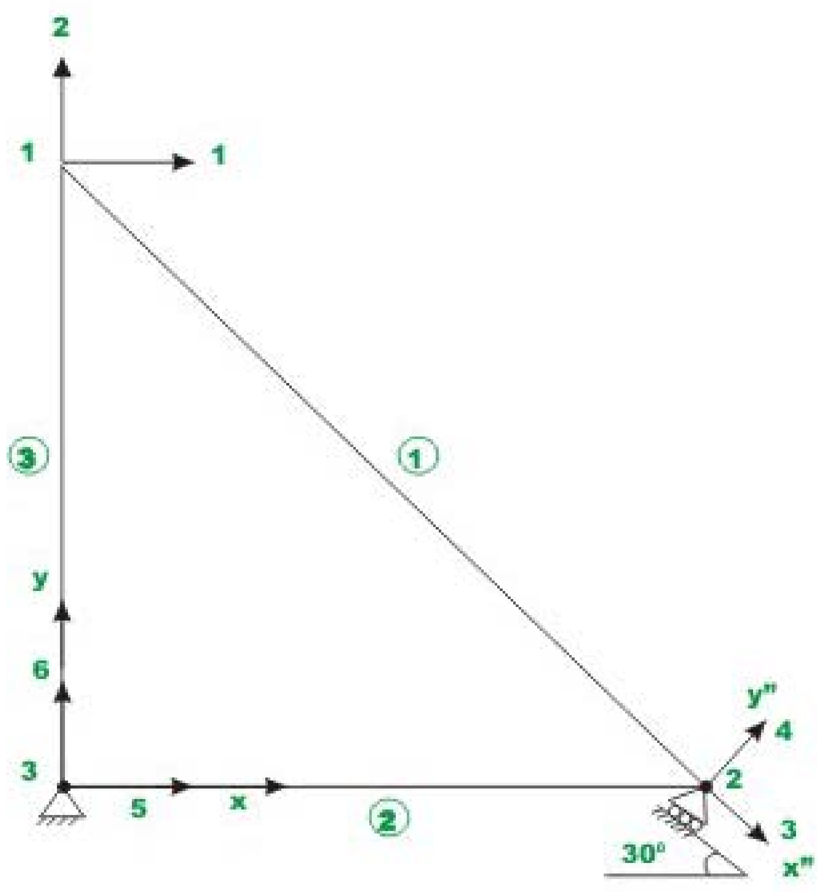

Fig. 25.6(b) Member and node numbering

The nodes and members are numbered in Fig. 25.6b. The global co-ordinate axes are shown at node 3. At node 2, roller is supported on inclined support. Hence it is required to use nodal co-ordinates $x''-y''$ at node 2 so that u_4 could be set to zero. All the possible displacement degrees of freedom are also shown in the figure. In the first step calculate member stiffness matrix.

Member 1: $\theta_x = 143.13°$, $\theta_{x''} = 6.87°$, $L = 5.00\ m.$ Nodal points 1-2

$l = -0.80$; $m = 0.6$; $l'' = 0.993$; $m'' = 0.12$.

$$[k^1] = \frac{EA}{5.0} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.64 & -0.48 & 0.794 & 0.096 \\ -0.48 & 0.36 & -0.596 & -0.072 \\ 0.794 & -0.596 & 0.986 & 0.119 \\ 0.096 & -0.072 & 0.119 & 0.014 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \quad (1)$$

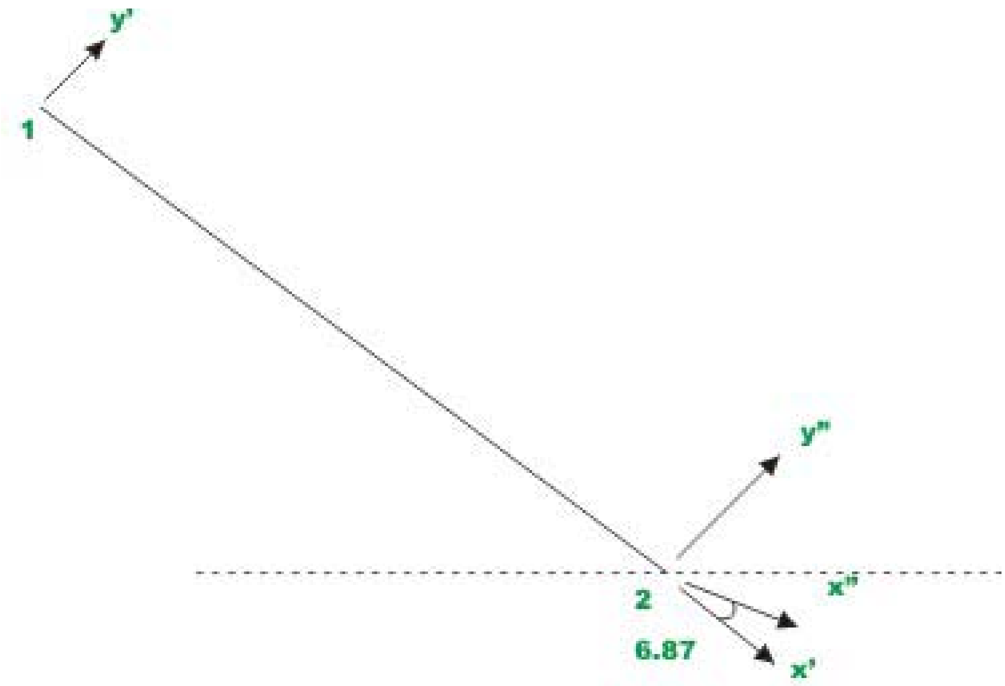

Fig.25.6c Member 1

Member 2: $\theta_x = 0°, \theta_{x''} = 30°, \quad L = 4.00\ m.$ Nodal points 2-3

$l = 1\ ; \quad m = 0 \quad ; \quad l'' = 0.866 \ ; \quad m'' = 0.50$.

Fig.25.6(d) Member 2

$$[k^2] = \frac{EA}{4.0} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.64 & -0.48 & 0.794 & 0.096 \\ -0.48 & 0.36 & -0.596 & -0.072 \\ 0.794 & -0.596 & 0.986 & 0.119 \\ 0.096 & -0.072 & 0.119 & 0.014 \end{bmatrix} \end{array} \begin{array}{c} \\ 5 \\ 6 \\ 3 \\ 4 \end{array} \qquad (2)$$

Member 3: $\theta_x = 90°, \quad L = 3.00\ m., l = 0\ ; \quad m = 1$ Nodal points 3-1

.

$$[k^3] = \frac{EA}{3.0} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ 5 \\ 6 \\ 1 \\ 2 \end{array} \qquad (3)$$

For the present problem, the global stiffness matrix is of the order (6×6). The global stiffness matrix for the entire truss is.

$$[k] = EA \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{ccc:ccc} 0.128 & -0.096 & 0.159 & 0.019 & 0 & 0 \\ -0.096 & 0.405 & -0.119 & -0.014 & 0 & -0.333 \\ 0.159 & -0.119 & 0.385 & 0.132 & -0.217 & 0 \\ \hdashline 0.019 & .0.014 & 0.132 & 0.065 & -0.125 & 0 \\ 0 & 0 & -0.217 & -0.125 & 0.25 & 0 \\ 0 & -0.333 & 0 & 0 & 0 & 0.333 \end{array} \right] \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \qquad (4)$$

Writing load-displacement equation for the truss for unconstrained degrees of freedom,

$$\begin{Bmatrix} -5 \\ 5 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0.128 & -0.096 & 0.159 \\ -0.096 & 0.405 & -0.119 \\ 0.159 & -0.119 & 0.385 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \qquad (5)$$

Solving ,

$$u_1 = \frac{-77.408}{AE}\ ; \ u_2 = \frac{3.728}{AE}; \ u_3 = \frac{33.12}{AE} \qquad (6)$$

Now reactions are evaluated from the equation

$$\begin{Bmatrix} p_4 \\ p_5 \\ p_6 \end{Bmatrix} = AE \begin{bmatrix} 0.019 & .0.014 & 0.132 \\ 0 & 0 & -0.217 \\ 0 & -0.333 & 0 \end{bmatrix} \frac{1}{AE} \begin{Bmatrix} -77.40 \\ 3.728 \\ 33.12 \end{Bmatrix} \tag{6}$$

$$p_4 = 2.85 \quad \text{kN} \ ; \quad p_5 = -7.19 \quad \text{kN} \quad ; p_6 = -1.24 \quad \text{kN}$$

Summary

Sometimes the truss is supported on a roller placed on an oblique plane. In such situations, the direct stiffness method as discussed in the previous lesson needs to be properly modified to make the displacement perpendicular to the roller support as zero. In the present approach, the inclined support is handled in the analysis by suitably modifying the member stiffness matrices of all members meeting at the inclined support. A few problems are solved to illustrate the procedure.