Module 3

Analysis of Statically Indeterminate Structures by the Displacement Method

Lesson 20

The Moment-Distribution Method: Frames without Sidesway

Instructional Objectives

After reading this chapter the student will be able to
1. Solve plane frame restrained against sidesway by the moment-distribution method.
2. Compute reactions at the supports.
3. Draw bending moment and shear force diagrams.
4. Draw the deflected shape of the plane frame.

20.1 Introduction

In this lesson, the statically indeterminate rigid frames properly restrained against sidesway are analysed using moment-distribution method. Analysis of rigid frames by moment-distribution method is very similar to that of continuous beams described in lesson 18. As pointed out earlier, in the case of continuous beams, at a joint only two members meet, where as in case of rigid frames two or more than two members meet at a joint. At such joints (for example joint C in Fig. 20.1) where more than two members meet, the unbalanced moment at the beginning of each cycle is the algebraic sum of fixed end beam moments (in the first cycle) or the carry over moments (in the subsequent cycles) of the beam meeting at C. The unbalanced moment is distributed to members CB, CD and CE according to their distribution factors. Few examples are solved to explain procedure. The moment-distribution method is carried out on a working diagram.

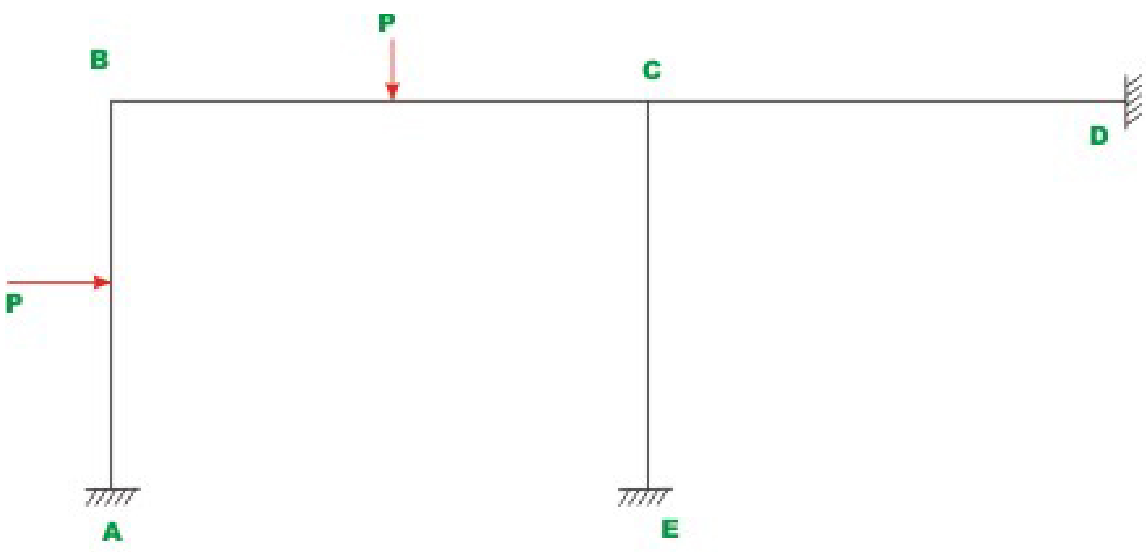

Fig. 20.1 Plane frame

Example 20.1

Calculate reactions and beam end moments for the rigid frame shown in Fig. 20.2a. Draw bending moment diagram for the frame. Assume EI to be constant for all the members.

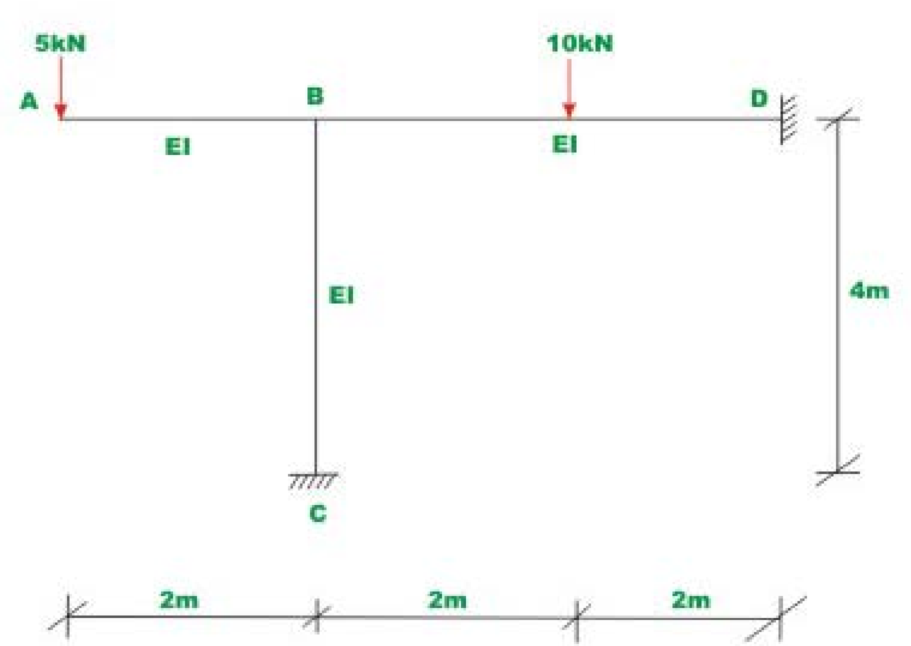

Fig. 20.2a Rigid plane frame of Example 20.1

Solution

In the first step, calculate fixed end moments.

$$M_{BD}^{F} = 5.0 \ kN.m$$

$$M_{DB}^{F} = -5.0 \ kN.m$$

$$M_{BC}^{F} = 0.0 \ kN.m \qquad (1)$$

$$M_{CB}^{F} = 0.0 \ kN.m$$

Also, the fixed end moment acting at B on BA is clockwise.

$$M_{BA}^{F} = -10.0 \ kN.m$$

In the next step calculate stiffness and distribution factors.

$$K_{BD} = \frac{EI}{4} = 0.25EI \quad \text{and} \quad K_{BC} = \frac{EI}{4} = 0.25EI$$

At joint B :

$$\sum K = 0.50EI$$

$$DF_{BD} = \frac{0.25EI}{0.5EI} = 0.5 \; ; \qquad DF_{BC} = 0.5 \qquad (2)$$

All the calculations are shown in Fig. 20.2b. Please note that cantilever member does not have any restraining effect on the joint B from rotation. In addition its stiffness factor is zero. Hence unbalanced moment is distributed between members BC and BD only.

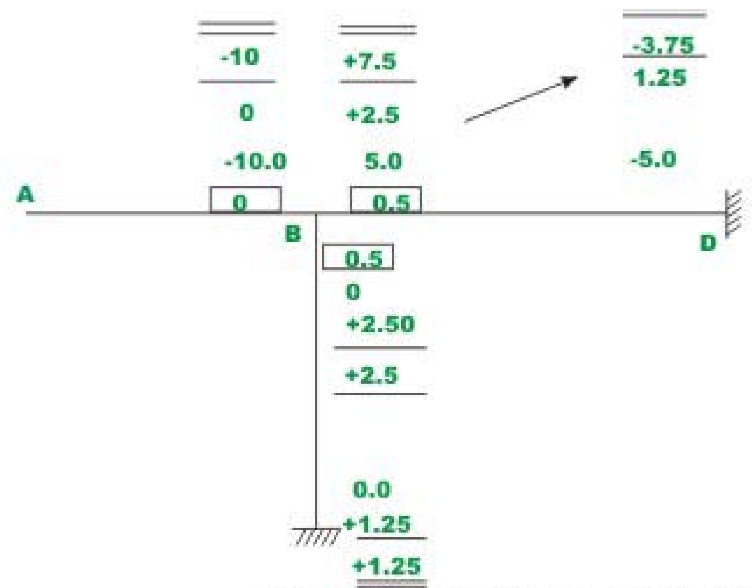

Fig. 20.2b Moment distribution

In this problem the moment-distribution method is completed in only one cycle, as equilibrium of only one joint needs to be considered. In other words, there is only one equation that needs to be solved for the unknown θ_B in this problem. This problem has already been solved by slop- deflection method wherein reactions are computed from equations of statics. The free body diagram of each member of the frame with external load and beam end moments are again reproduced here in Fig. 20.2c for easy reference. The bending moment diagram is shown in Fig. 20.2d.

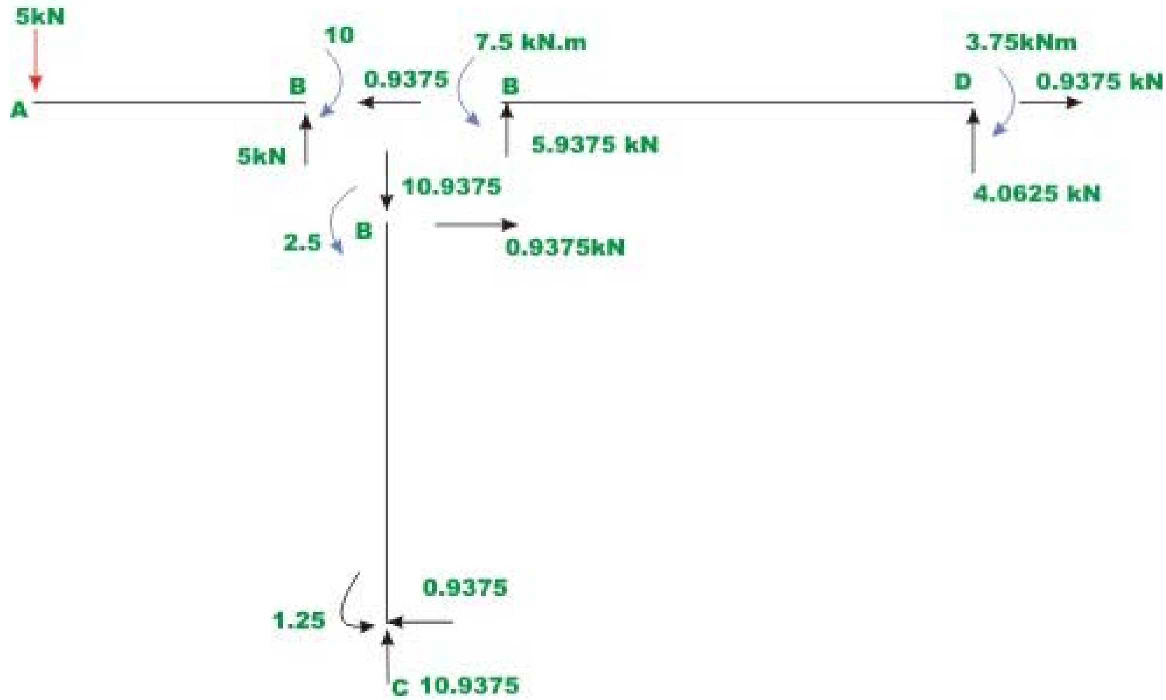

Fig. 20.2c Reactions

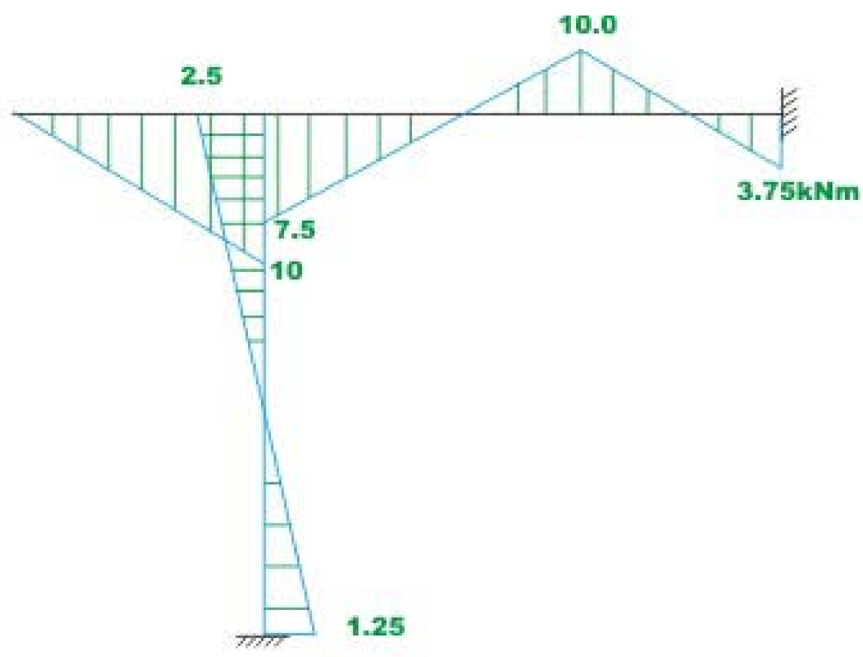

Fig. 20.2 (d) Bending moment diagram

Example 20.2

Analyse the rigid frame shown in Fig. 20.3a by moment-distribution method. Moment of inertia of different members are shown in the diagram.

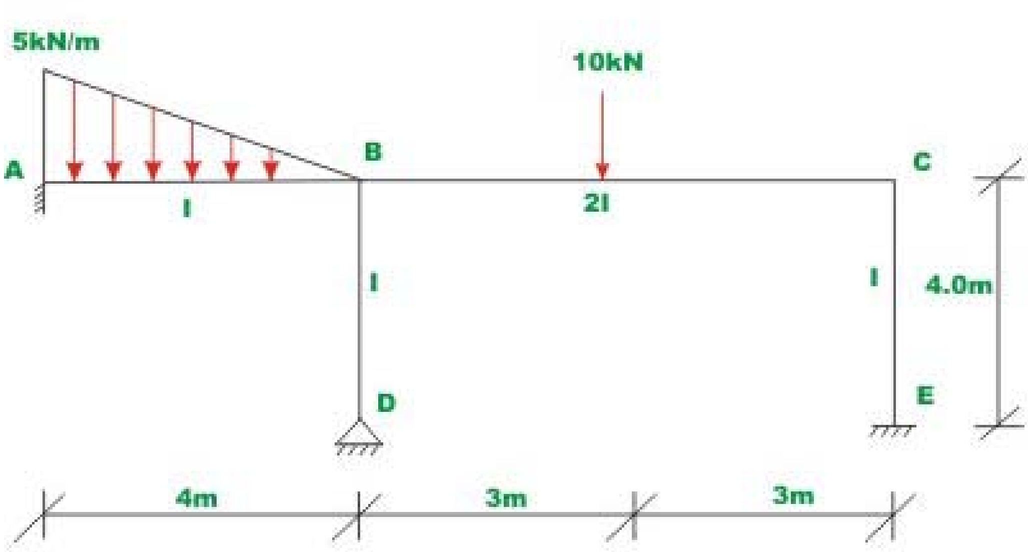

Fig. 20.3 (a) Example 20.2

Solution:

Calculate fixed end moments by locking the joints A, B, C, D and E

$$M_{AB}^{F} = \frac{5 \times 4^2}{20} = 4.0 \text{ kN.m}$$

$$M_{BA}^{F} = -2.667 \text{ kN.m}$$

$$M_{BC}^{F} = 7.5 \text{ kN.m}$$

$$M_{CB}^{F} = -7.5 \text{ kN.m}$$

$$M_{BD}^{F} = M_{DB}^{F} = M_{CE}^{F} = M_{EC}^{F} = 0 \qquad (1)$$

The frame is restrained against sidesway. In the next step calculate stiffness and distribution factors.

$$K_{BA} = 0.25EI \quad \text{and} \quad K_{BC} = \frac{2EI}{6} = 0.333EI$$

$$K_{BD} = \frac{3}{4}\frac{EI}{4} = 0.1875EI \;; \qquad K_{CE} = 0.25EI \tag{2}$$

At joint B:

$$\sum K = K_{BA} + K_{BC} + K_{BD}$$
$$= 0.7705EI$$

$$DF_{BA} = 0.325 \;; \qquad DF_{BC} = 0.432$$

$$DF_{BD} = 0.243 \tag{3}$$

At joint C:

$$\sum K = 0.583EI$$

$$DF_{CB} = 0.571 \;; \qquad DF_{CD} = 0.429$$

In Fig. 20.3b, the complete procedure is shown on a working diagram. The moment-distribution method is started from joint C. When joint C is unlocked, it will rotate under the action of unbalanced moment of $7.5\ \text{kN.m}$. Hence the $7.5\ \text{kN.m}$ is distributed among members CB and CE according to their distribution factors. Now joint C is balanced. To indicate that the joint C is balanced a horizontal line is drawn. This balancing moment in turn developed moments $+2.141\ \text{kN.m}$ at BC and $+1.61\ \text{kN.m}$ at EC. Now unlock joint B. The joint B is unbalanced and the unbalanced moment is $-(7.5 + 2.141 - 2.67) = -6.971\ \text{kN.m}$. This moment is distributed among three members meeting at B in proportion to their distribution factors. Also there is no carry over to joint D from beam end moment BD as it was left unlocked. For member BD, modified stiffness factor is used as the end D is hinged.

Example 20.3

Analyse the rigid frame shown in Fig. 20.4a by moment-distribution method. Draw bending moment diagram for the rigid frame. The flexural rigidities of the members are shown in the figure.

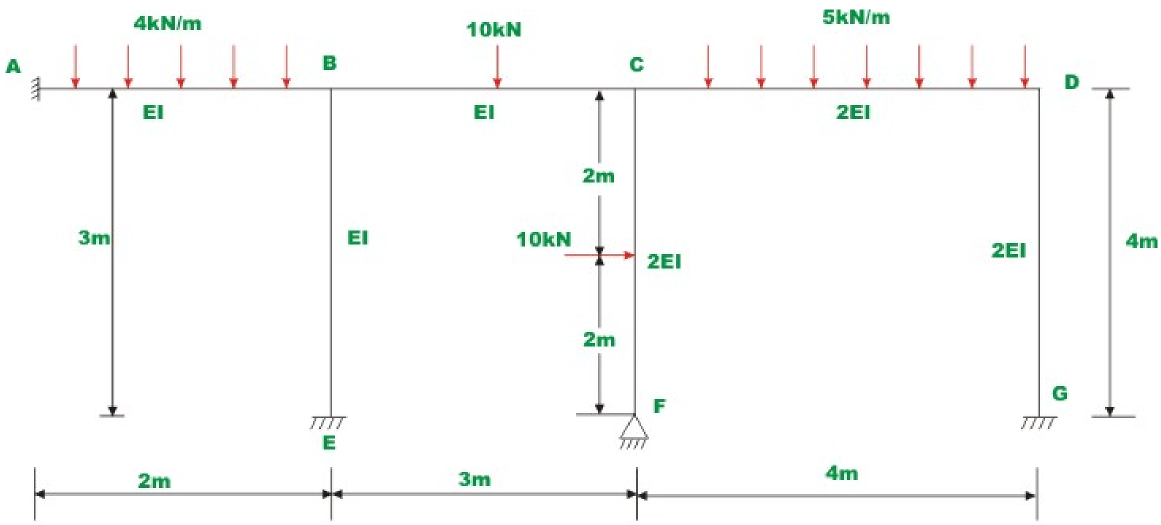

Fig. 20.4a Example 20.3

Solution:

Assuming that the joints are locked, calculate fixed end moments.

$$M_{AB}^{F} = 1.333 \quad \text{kN.m} \;\; ; \;\; M_{BA}^{F} = -1.333 \quad \text{kN.m}$$

$$M_{BC}^{F} = 4.444 \quad \text{kN.m}; \quad M_{CB}^{F} = -2.222 \quad \text{kN.m}$$

$$M_{CD}^{F} = 6.667 \quad \text{kN.m} \;\; ; \; M_{DC}^{F} = -6.667 \quad \text{kN.m}$$

$$M_{BE}^{F} = 0.0 \quad \text{kN.m} \;\; ; \quad M_{EB}^{F} = 0.0 \quad \text{kN.m}$$

$$M_{CF}^{F} = 5.0 \quad \text{kN.m} \;\; ; \quad M_{FC}^{F} = -5.0 \quad \text{kN.m} \qquad (1)$$

The frame is restrained against sidesway. Calculate stiffness and distribution factors.

$$K_{BA} = 0.5EI \;; \qquad K_{BC} = 0.333EI \;; \qquad K_{BE} = 0.333EI$$

$$K_{CB} = 0.333EI \;; \qquad K_{CD} = 0.5EI \;; \qquad K_{CF} = \frac{3}{4}\frac{2EI}{4} = 0.375EI$$

$$K_{DC} = 0.5EI \;; \qquad K_{DG} = 0.5EI \qquad (2)$$

Joint B **:**

$$\sum K = 0.5EI + 0.333EI + 0.333EI = 1.166EI$$

$$DF_{BA} = 0.428\ ; \qquad DF_{BC} = 0.286$$

$$DF_{BE} = 0.286$$

Joint C:

$$\sum K = 0.333EI + 0.5EI + 0.375EI = 1.208EI$$

$$DF_{CB} = 0.276\ ; \qquad DF_{CD} = 0.414$$

$$DF_{CF} = 0.31$$

Joint D:

$$\sum K = 1.0EI$$

$$DF_{DC} = 0.50\ ; \qquad DF_{DG} = 0.50 \qquad (3)$$

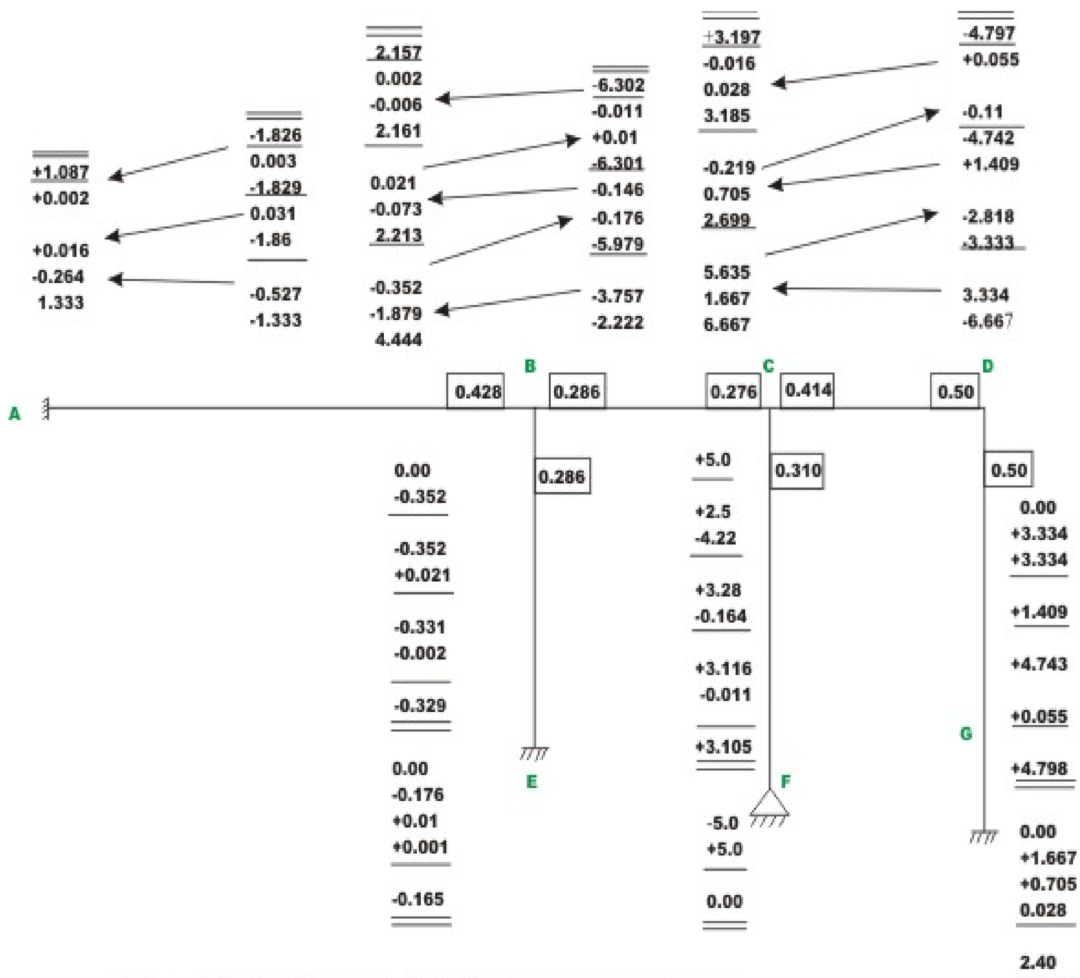

Fig. 20.4b Moment distribution computation

The complete moment-distribution method is shown in Fig. 20.4b. The moment-distribution is stopped after three cycles. The moment-distribution is started by releasing and balancing joint D. This is repeated for joints C and B respectively in that order. After balancing joint F, it is left unlocked throughout as it is a hinged joint. After balancing each joint a horizontal line is drawn to indicate that joint has been balanced and locked. When moment-distribution method is finally stopped all joints except fixed joints will be left unlocked.

Summary

In this lesson plane frames which are restrained against sidesway are analysed by the moment-distribution method. As many equilibrium equations are written as there are unknown displacements. The reactions of the frames are computed from equations of static equilibrium. The bending moment diagram is drawn for the frame. A few problems are solved to illustrate the procedure. Free-body diagrams are drawn wherever required.