Geosynthetics Engineering: In Theory & Practice

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Lecture No - 03
GEOSYNTHETICS ENGINEERING: IN THEORY AND PRACTICE

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Module-1
LECTURE- 3
INTRODUCTION TO REINFORCED EARTH

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RECAP of previous lecture…..

- Sustainability using geosynthetics
- Major applications
- Lessons learned from failure and success
- Key benefits
- Introduction to reinforced earth
- Basic concepts and mechanisms of reinforced earth (partly covered)
Anisotropic Cohesion Concept (Vidal and Schlosser, 1969)

At higher stress level, failure occurs by rupture of the reinforcement.

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For unreinforced soil, $\sigma_1 = \sigma_3 k_p$

The failure envelope for reinforced soil is defined by,

$$\sigma_{1r} = \sigma_1 + \Delta\sigma_1 = \sigma_3 k_p + \Delta\sigma_1$$

They compared the above equation with Rankine-Bell equation for $c$-$\phi$ soils,

$$\sigma_{1r} = \sigma_3 \tan^2 (45^\circ + \phi/2) + 2 c_r \tan (45^\circ + \phi/2) = \sigma_3 k_p + 2 c_r \sqrt{k_p}$$

Where, $c_r = \text{Reinforcement induced cohesion}$

Equating the above two equations, $c_r = \Delta\sigma_1 / 2\sqrt{k_p}$

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Coulomb Analysis (Schlosser and Long, 1973)

\[ \sigma_{1r} = \text{Major principal stress} \]
\[ \sigma_3 = \text{Minor principal stress} \]
\[ R = \text{Resultant force} \]
\[ F = \text{Resultant tensile force within failure plane} \]
\[ A = \text{Cross sectional area of cylindrical sample} \]

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From the force triangle,

\[ F + \sigma_3 A \tan \alpha = \sigma_{1r} A \tan(\alpha - \phi) \]

Assume vertical spacing between reinforcements “h” and tensile strength of a reinforcement = T,

\[ F = \frac{A \tan \alpha}{h} T \]

Combining the above equations,

\[ \sigma_{1r} = \left( \sigma_3 + \frac{T}{h} \right) \tan \alpha \cot(\alpha - \phi) \]

By differentiating, \( \sigma_{1r} \) will be maximum when

\[ \alpha = \left( 45^\circ + \frac{\phi}{2} \right) \]
\[ \sigma_{1r} = \left( \sigma_3 + \frac{T}{h} \right) \tan \left( 45^\circ + \frac{\phi}{2} \right) \cot \left( 45^\circ + \frac{\phi}{2} - \phi \right) \]

\[ \sigma_{1r} = \left( \sigma_3 + \frac{T}{h} \right) \tan \left( 45^\circ + \frac{\phi}{2} \right) \tan \left( 45^\circ + \frac{\phi}{2} \right) \]

\[ \sigma_{1r} = k_p \sigma_3 + k_p \frac{T}{h} \]

Again, \( \sigma_{1r} = \sigma_1 + \Delta \sigma_1 = k_p \sigma_3 + \Delta \sigma_1 \)

Therefore, \[ \Delta \sigma_1 = k_p \frac{T}{h} \]

\[ c_r = \frac{\Delta \sigma_1}{2 \sqrt{k_p}} = \frac{k_p \frac{T}{h}}{2 \sqrt{k_p}} = \frac{\sqrt{k_p}}{2} \frac{T}{h} \]

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NSW Cohesion Theory (Hausmann, 1976)

\[ \sigma_3 + \sigma_r = k_a \sigma_1 \]

\[ \sigma_1 = k_p (\sigma_3 + \sigma_r) = k_p \sigma_3 + k_p \sigma_r \]

From Rankine-Bell equation for c-\(\phi\) soil,

\[ \sigma_1 = k_p \sigma_3 + 2c_r \sqrt{k_p} \]

Comparing the above two equations,

\[ c_r = \frac{\sigma_r \sqrt{k_p}}{2} \]
Considering failure by rupture of reinforcement (Hausmann, 1976),

\[ \sigma = \text{Ultimate tensile strength or yield strength of reinforcement} \]

**Additional Stress in the soil due to inclusion of reinforcement** = \( \sigma_r \)

Therefore, \( \sigma_r \cdot B \cdot H = \sigma \cdot A \)

\[ A = \text{Cross sectional area of the strip} \]

\[ \sigma_r = \frac{\sigma A}{B H} \]

\[ c_r = \frac{\sigma_r \sqrt{k_p}}{2} = \frac{\sigma A}{2BH} \sqrt{k_p} \]

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At low stress level, failure in a reinforced earth tends to occur by slippage. If friction along the reinforcement is proportional to vertical stress and cohesion remains zero,

\[ \sigma_r = F \sigma_1 \]

\[ \sigma_3 + \sigma_r = K_a \sigma_1 \]

\[ \sigma_3 + F \sigma_1 = K_a \sigma_1 \]

\[ \frac{\sigma_3}{\sigma_1} = \frac{K_a}{1 - \sin \varphi_r} \]

\[ \frac{\sigma_3}{\sigma_1} = K_a - F = \frac{1 - \sin \varphi_r}{1 + \sin \varphi_r} \]

\[ \sin \varphi_r = \frac{K_a - F - 1}{F - K_a - 1} \]

Failure conditions for variable \( \sigma_r \) (Hausmann, 1976)

F = friction factor characterizing reinforcement interaction with the cohesionless soil. If \( F = K_a \), \( \varphi_r = 90^\circ \), failure occurs by rupture rather than slippage.

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Slippage of reinforcement (Hausmann, 1976)

\[ B H \sigma_r = 2 \tau B^* L' \]

\[ \sigma_r = \frac{\sigma_1 \tan \delta \times 2 B^* L' e}{B H} \]

\[ \tau = \sigma_1 \tan \delta \]

\[ \sigma_r = F \sigma_1 \]

\[ F = \frac{2 \tan \delta B^* L' e}{B H} \]

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Due to lateral expansion, shear stress (\(\tau\)) acts between soil and the reinforcement. Magnitude of the shear stress depends on the angle of wall friction, \(\delta\), between soil and reinforcement as well as on relative movement between soil and reinforcement.

The shear stress is zero at the mid point and at the end of the strip arriving maximum value of \(\sigma_1 \tan \delta\).

The uneven distribution of shear stress and imperfect transfer should be taken into account by considering a reinforcing efficiency (e).

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At higher stress level, failure occurs by rupture of the reinforcement.

At low stress level, failure in a reinforced earth tends to occur by slippage.
BASIC DESIGN OF REINFORCED SOIL WALL
The reinforced earth concept can be used for retaining walls. For basic understanding, a model reinforced soil wall is constructed using paper and gravel/sand.

- For internal stability, we have to determine the length of reinforcement (L), overlap length (L_o) and spacing of the reinforcement (S_v).
- For external stability, we have to check for sliding, overturning and bearing capacity failures.
Failure line (BC) divides the reinforced fill into two zones:

(A) Active zone, and

(B) Restraining zone

Cross section of a reinforced soil wall

- In active zone, the shearing stresses from the soil to the reinforcement act towards the facing element. This means that reinforcement is pulled towards the facing element.

- In restraining zone, the shear stresses from the soil to reinforcement act away from the facing, thereby tend to hold the reinforcement in soil.

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From both experimental and field measurements, it has been observed that the tensile force in reinforcement is low or even zero at the facing, and increases to a maximum value at a distance from the facing with \((+dT/dl)\) and again decreases towards zero at the free end of reinforcement with \((-dT/dl)\).

Mobilization of tensile strength along the length of reinforcement (After Schlosser and Long 1974)

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Coulomb’s wedge failure in reinforced earth wall with force directions (After Schlosser and Vidal 1969)
From the force triangle, \( T_r = W \tan(\theta - \phi) \)

Now, \( W = \frac{1}{2} \gamma H^2 \tan (90^\circ - \theta) \)

Therefore,

\[
T_r = \frac{1}{2} \gamma H^2 \tan (90^\circ - \theta) \tan(\theta - \phi) = \frac{1}{2} \gamma H^2 \cot \theta \tan(\theta - \phi)
\]

\[
\frac{dT_r}{d\theta} = 0
\]

\( T_r \) is maximum when \( \theta = (45^\circ + \phi/2) \)

\[
T_{r(max)} = \frac{1}{2} \gamma H^2 \tan^2 \left( 45^\circ - \frac{\phi}{2} \right) = \frac{1}{2} K_a \gamma H^2
\]

This resultant tensile force is distributed in all the reinforcement layers.

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Triangular distribution where tension increases with depth and becomes maximum at the bottom layer \((i = n)\) (Schlosser and Vidal, 1969).
The maximum tension per meter run of the wall develops at the bottom layer and can be determined using principle of equal angle triangles,

\[
\frac{T_n}{T_i} = \frac{nS_v}{iS_v}, \quad T_i = \frac{iT_n}{n}
\]

Now, \[T_1 + T_2 + T_3 + \ldots + T_n = T_{r(max)} = \frac{1}{2} K_a \gamma H^2\]

Therefore, \[\frac{T_n}{n} (1 + 2 + 3 + \ldots + n) = \frac{1}{2} K_a \gamma H^2\]

\[T_n = T_{max} = \frac{n}{n+1} K_a \gamma HS_v\]

\[n = \frac{H}{S_v}\]
Rankine Force Method:

It is assumed that the $i^{th}$ reinforcement layer resists earth pressure for the zone falls between $S_v(i-1/2)$ and $S_v(i+1/2)$. The concept of Rankine’s method is shown in the Figure below.

(After Schlosser and Vidal, 1969)

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Therefore, the $i^{th}$ layer is subjected to lateral active earth pressure as shown in Figure below.

Active earth pressure in the zone for $i^{th}$ layer reinforcement

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$T_i = \text{Developed tension in reinforcement} = \text{Area of active earth pressure diagram}$

\[ T_i = \frac{1}{2} K_a \gamma S_v \left[ \left( i - \frac{1}{2} \right) + \left( i + \frac{1}{2} \right) \right] S_v = i K_a \gamma S_v^2 \]

Vertical stress at $i^{th}$ layer, $\sigma_v = \gamma i S_v$

Therefore,

$T_i = K_a S_v \sigma_v$
Reinforced Earth (After Vidal, 1966)

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Please let us hear from you

Any question?
Thanks for listening.

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