Module – 9

Seismic Analysis and Design of Various Geotechnical Structures
Seismic Design of Retaining Wall
Pseudo-static analysis

As per Gutenberg and Richter (1956), $a_o = k_h g$ and computed as,

$$\log_{10} a_0 = -2.1 + 0.81 M - 0.027 M^2$$

As per Terzaghi (1950)

$$F_h = \frac{a_h W}{g} = k_h W$$

$$F_v = \frac{a_v W}{g} = k_v W$$
Pseudo Static Analysis
Mononobe-Okabe (1926, 1929)

Failure surface and the forces considered by Mononobe-Okabe

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\[ P_{ae,pe} = \frac{1}{2} \gamma H^2 (1-k_v) K_{ae,pe} \]

\[
K_{ae,pe} = \cos^2 (\phi \mp \beta - \theta) \\
\cos \theta \cos^2 \beta \cos (\delta \pm \beta + \theta) \left[ 1 - \left( \frac{\sin (\phi + \delta) \sin (\phi \mp i-\theta)}{\cos (\delta \pm \beta + \theta) \cos (i-\beta)} \right)^{0.5} \right]^2
\]

\[ \theta = \tan^{-1} \left[ \frac{k_h}{1 - k_v} \right] \]
Madhav and Kameswara Rao (1969) in *Soils and Foundations, JGS, Japan*

\[ P_{pe} = K_{pe} \left[ \frac{1}{2} \gamma (1 - k_v) H^2 \right] \]
Soubra (2000) in Canadian Geotechnical Journal

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Kumar (2001) in Canadian Geotechnical Journal

(a) Convex Failure Surface

(b) Concave Failure Surface
Seismic Passive Earth Pressure / Resistance

Case 1: Positive delta case
Seismic passive force $P_{pd}$ is divided into three components as,
(i) Unit weight component $P_{p\gamma d}$ ($\gamma \neq 0, q = c = 0$)
(ii) Surcharge component $P_{pqd}$ ($q \neq 0, \gamma = c = 0$)
(iii) Cohesion component $P_{pcd}$ ($c \neq 0, \gamma = q = 0$)

$$P_{pd} = P_{p\gamma d} + P_{pqd} + P_{pcd}$$

Variation of parameters
1. Wall batter $\alpha : -30^0 \leq \alpha \leq 30^0$
2. Ground inclination $\beta : -30^0 \leq \beta \leq 30^0$
3. Soil friction angle $\phi : 10^0$ to $50^0$
4. Wall friction angle $\delta : 0$ to $\phi$
5. Wall adhesion $c_a : 0.0$ to $(\tan \delta/\tan \phi)c$
6. Horizontal seismic acceleration coefficient $k_h : 0.0$ to $0.5$
7. Vertical seismic acceleration coefficient $k_v : 0.0k_h, 0.5k_h, 1.0k_h$

Composite Failure surface and forces considered

[Subba Rao and Choudhury, 2005]

\[ \xi = \frac{\pi}{4} - \frac{\phi}{2} + \frac{1}{2} \tan^{-1} \left( \frac{k_h}{1 - k_v} \right) + \frac{\beta}{2} - \frac{1}{2} \sin^{-1} \left[ \sin \left\{ \tan^{-1} \left( \frac{k_h}{1 - k_v} \right) - \beta \right\} \right] \sin \phi \]
Determination of $K_{p_{γd}} (γ ≠ 0, q = c = 0)$

$$M_{P_{p_{γd}}} = M_{W_{1d}} + M_{P_{p_{γR}}} - M_{W_{2kv}} - M_{W_{2kh}}$$

Where,
- $M_{P_{p_{γd}}}$ = moment of $P_{p_{γd}}$.
- $M_{W_{1d}}$ = moment of the soil mass ABDGA, together with the seismic components; $W_{1kh}$ and $W_{1kv}$.
- $M_{P_{p_{γR}}}$ = moment of the Rankine passive force $P_{p_{γR}}$.
- $M_{W_{2kv}}, M_{W_{2kh}}$ = moments of the seismic components of the weight of portion DGE.

$$K_{p_{γd}} = \frac{2P_{p_{γd}} \cos δ}{γ H^2}$$
Determination of $K_{pqd}$ ($q \neq 0, \gamma = c = 0$)

$$M_{Ppqd} = M_{PpqR} + M_{qAG(1-kv)} - M_{qAGkh} - M_{qGEkh} - M_{qGEkv}$$

Where,

$M_{Ppqd} =$ moment of $P_{pqd}$.

$M_{PpqR} =$ moment of the Rankine passive force $P_{pqR}$.

$M_{qAG(1-kv)}, M_{qAGkh} =$ moment of the surcharge load ($q$.AG) together with the seismic components.

$M_{qGEkv}, M_{qGEkh} =$ moment of the seismic components of surcharge load ($q$.GE).

$$K_{pqd} = \frac{P_{pqd} \cos \delta}{qH}$$
Determination of $K_{pcd}(c \neq 0, \gamma = q = 0)$

$$M_{P_{pcd}} = M_C = M_{Ca} + M_{P_{pcR}}$$

Where,

$M_{P_{pcd}} = \text{moment of } P_{pcd}$. 

$M_C = \text{moment of the cohesive force } C (\text{on the failure surface BD})$. 

$M_{Ca} = \text{moment of the adhesive force } Ca (\text{on the wall-soil interface AB})$. 

$M_{P_{pcR}} = \text{moment of the Rankine passive force } P_{pcR}$. 

$$K_{pcd} = \frac{P_{pcd} \cos \delta}{2cH}$$
Typical Results
## Typical Results

### Values of $K_{pcd}$

<table>
<thead>
<tr>
<th>Case for $\phi$ (degree)</th>
<th>$\delta/\phi$ for $c_a/c = 0.0$</th>
<th>$\delta/\phi$ for $c_a/c = \tan\delta/\tan\phi$</th>
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<tbody>
<tr>
<td>$\alpha = 0^\circ$</td>
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<tr>
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<tr>
<td>50</td>
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</table>

D. Choudhury, IIT Bombay, India
Variation of earth pressure coefficients with $\alpha$

$\beta = 0^0, k_h = 0.3$
$\phi = 40^0, \delta/\phi = 1.0$

$K_{pdl}$ vs $\alpha$ (degree)

$K_{pdl}$ vs $\alpha$ (degree)

$-30$ $-15$ $0$ $15$ $30$

$10^{-3}$ $10^{-2}$ $10^{-1}$ $10^{0}$

$-30$ $-15$ $0$ $15$ $30$

$10^{-3}$ $10^{-2}$ $10^{-1}$ $10^{0}$

$-- k_v = 0.0k_h$

$-- k_v = 0.5k_h$

$---- k_v = 1.0k_h$

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Variation of earth pressure coefficients with $\beta$

$\alpha = 0^\circ$, $\phi = 40^\circ$

$k_h = 0.1$, $\delta/\phi = 1.0$

$K_{pvd}$ vs $\beta$ (degree)

$K_{pvd}$ vs $\beta$ (degree)
Variation of earth pressure coefficients with $\delta/\phi$

$\alpha = 0^o, \beta = 0^o,$

$\phi = 40^o, k_v = 0.3$

$K_{p/d}$ vs $\delta/\phi$

$K_{p/d}$ vs $\delta/\phi$

$--- k_v = 0.0 k_h$

$--- k_v = 0.5 k_h$

$...... k_v = 1.0 k_h$

$--- k_v = 0.0 k_h$

$--- k_v = 0.5 k_h$

$...... k_v = 1.0 k_h$
Interpolation formula

To obtain seismic passive earth pressure coefficient values for any other parameter, the interpolation formula can be used is proposed as,

$$\log p_{\gamma d}^i = \log p_{\gamma d}^{x_0} + \frac{\log p_{\gamma d}^{x_1} - \log p_{\gamma d}^{x_0}}{x_1 - x_0} (x_i - x_0)$$
Validation of Principle of Superposition

(Data used: $\phi = 40^\circ$, $\delta/\phi = 1.0$, $\gamma = 18$ kN/m$^3$, $q = 15$ kN/m$^2$, $c = 15$ kN/m$^2$, $c_a/c = 0.0$, $\alpha = 0^\circ$, $\beta = 0^\circ$, $H = 5$ m, $k_h = 0.3$)

<table>
<thead>
<tr>
<th>Combination</th>
<th>$k_v$</th>
<th>Independent Failure Surfaces</th>
<th>Single Failure Surface</th>
<th>Error$^\wedge$</th>
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<tr>
<td>(1)</td>
<td>(2)</td>
<td>$K_{pyd}$</td>
<td>$K_{pqd}$</td>
<td>$K_{pcd}$</td>
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<td>11.003</td>
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<td>surcharge</td>
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<td>7.731</td>
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<td>7.685</td>
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$^\wedge$ Error = $\left[\frac{(P_{pd}^* - P_{pd})}{P_{pd}^*}\right] \times 100$
Comparison of $K_{pyd}$ values obtained by present study with available theories in seismic case for $\alpha = 0^\circ$, $\beta = 0^\circ$, $\phi = 30^\circ$

<table>
<thead>
<tr>
<th>$\delta/\phi$</th>
<th>$k_h$</th>
<th>$k_v$</th>
<th>Mononobe-Okabe</th>
<th>Morrison and Ebeling</th>
<th>Soubra</th>
<th>Kumar</th>
<th>Present study</th>
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Negative delta case: Failure surface and forces considered

\[ \xi = \frac{\pi}{4} - \frac{\phi}{2} - \frac{1}{2} \tan^{-1}\left(\frac{k_h}{1-k_v}\right) + \beta + \frac{1}{2} \sin^{-1}\left[\frac{\sin\left\{\tan^{-1}\left(\frac{k_h}{1-k_v}\right) + \beta\right\}}{\sin \phi}\right] \]

Typical Results
Typical Results

D. Choudhury, IIT Bombay, India
# Typical Results

## Values of $K_{pcd}$

| Case for | $\phi$ (degree) | $\delta/\phi$ for $c_a/c = 0.0$ | $\delta/\phi$ for $c_a/c = |\tan \delta/\tan \phi|$ |
|----------|-----------------|----------------|------------------|
|          | -0.5           | -0.67         | -0.75          | -1.0     | -0.5 | -0.67 | -0.75 | -1.0 |
| $\alpha = 0^\circ$ | 10         | 1.07           | 1.03           | 1.01       | 0.96   | 0.67   | 0.49   | 0.40   | -     |
|          | 20           | 1.11           | 1.01           | 0.97       | 0.85   | 0.68   | 0.44   | 0.30   | -     |
| $\beta = 0^\circ$ | 30         | 1.09           | 0.94           | 0.87       | 0.70   | 0.62   | 0.29   | -      | -     |
|          | 40           | 1.03           | 0.81           | 0.73       | 0.52   | 0.49   | -      | -      | -     |
|          | 50           | 0.89           | 0.64           | 0.55       | 0.34   | -      | -      | -      | -     |
| $\alpha = 30^\circ$ | 10        | 0.60           | 0.59           | 0.58       | 0.55   | 0.32   | 0.21   | 0.15   | -     |
|          | 20           | 0.58           | 0.54           | 0.52       | 0.48   | 0.27   | 0.14   | 0.10   | -     |
| $\beta = 0^\circ$ | 30          | 0.53           | 0.47           | 0.45       | 0.38   | 0.20   | -      | -      | -     |
|          | 40           | 0.45           | 0.38           | 0.36       | 0.28   | -      | -      | -      | -     |
|          | 50           | 0.36           | 0.29           | 0.26       | 0.18   | -      | -      | -      | -     |

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Variation of earth pressure coefficients with $\alpha$

- $\beta = 0^0$, $k_h = 0.3$
- $\phi = 30^0$, $\delta/\phi = -0.5$

$K_p$ vs $\alpha$ (degree)

- $k_v = 0.0k_h$
- $k_v = 0.5k_h$
- $k_v = 1.0k_h$

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Variation of earth pressure coefficients with $\delta/\phi$

\[ K_{pd} \]

- $\alpha = 0^0$, $\beta = 0^0$
- $\phi = 30^0$, $k_h = 0.3$

\[ K_{pd} \]

- $\alpha = 0^0$, $\beta = 0^0$
- $\phi = 30^0$, $k_h = 0.3$
Validation of Principle of Superposition

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<thead>
<tr>
<th>φ</th>
<th>Case</th>
<th>kₜ, kᵥ</th>
<th>Independent critical failure surfaces</th>
<th>Single critical failure surface</th>
<th>Max. % Error^ in K values</th>
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<td></td>
<td></td>
<td>Kpcd</td>
<td>η</td>
<td>Kpqd</td>
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<td>φ soil with q</td>
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<td>-</td>
<td>0.993</td>
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<tr>
<td>30⁰</td>
<td>φ soil with q</td>
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<td>1.359</td>
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<td>30⁰</td>
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<td>0.5, 0.0</td>
<td>1.027</td>
<td>-⁴⁰</td>
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^ Error = [(Kpd* - Kpd)/Kpd*]x100

D. Choudhury, IIT Bombay, India
Seismic Passive Earth Pressure Distribution

Analytical model proposed by Choudhury et al. (2002)

Formulation of Equations

\[ \frac{dp_y}{dy} = \frac{p_y}{H - y} (1 + aK) + \gamma (1 - k_v - bk_h) \]

\[ p_x = K \left[ \frac{\gamma (1 - k_v - bk_h)}{2 + aK} \right] \left\{ \frac{H^{(2+aK)}}{(H - y)^{(1+aK)}} - (H - y) \right\} \]

\[ a = \frac{(\tan \phi - \cot \theta)(1 + \tan \alpha \tan \delta)}{(\tan \alpha + \cot \theta)(1 + \tan \phi \cot \theta)} + \frac{(\tan \delta - \tan \alpha)}{(\tan \alpha + \cot \theta)} \]

\[ b = \cot (\theta + \phi) \]
## Seismic Passive Earth Pressure Coefficients

<table>
<thead>
<tr>
<th>Case</th>
<th>φ (degree)</th>
<th>k_h, k_v</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.0, 0.0</td>
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<tr>
<td>δ = 0</td>
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<td></td>
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<td>4.579</td>
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<tr>
<td>δ = φ/3</td>
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<tr>
<td></td>
<td>40</td>
<td>7.912</td>
</tr>
</tbody>
</table>

D. Choudhury, IIT Bombay, India
## Point of Application of Seismic Passive Earth Resistance

<table>
<thead>
<tr>
<th>Case</th>
<th>$\phi$ (degree)</th>
<th>$k_h$, $k_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta = 0$</td>
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<td>0.333, 0.328, 0.319</td>
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<td>0.333, 0.332, 0.328</td>
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<tr>
<td>$\delta = \phi/3$</td>
<td>30</td>
<td>0.333, 0.331, 0.317</td>
</tr>
<tr>
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<td>40</td>
<td>0.333, 0.330, 0.330</td>
</tr>
</tbody>
</table>
Results

\[ k_h = 0.3, \phi = 30^0, \alpha = 30^0 \]
\[ H = 4.0 \text{ m}, \gamma = 18 \text{ kN/m}^3 \]

(1) \[ k_v = 0.3, \delta = 0 \]
(2) \[ k_v = 0.3, \delta = \phi/3 \]
(3) \[ k_v = 0.0, \delta = 0 \]
Results

\[ k_h = 0.3, \ k_v = 0.3, \ \phi = 30^\circ, \ \delta = \phi/3 \]
\[ H = 4.0 \text{ m}, \ \gamma = 18 \text{ kN/m}^3 \]

(1) \( \alpha = -30^\circ \)
(2) \( \alpha = 0^\circ \)
(3) \( \alpha = 30^\circ \)
Richards and Elms (1979) proposed a method for seismic design of gravity retaining walls which is based on permanent wall displacements. (Displacement based approach)

Displacement should be calculated by following formula, and should be checked against allowable displacement.

\[
d_{pem} = 0.087 \frac{v_{\text{max}}^2 a_{\text{max}}^3}{a_{\text{y}}^4}
\]

where, \(v_{\text{max}}\) is the peak ground velocity, \(a_{\text{max}}\) is the peak ground acceleration, \(a_{\text{y}}\) is the yield acceleration for the wall-backfill system.

Pseudo-static Method

- Major limitations
  - Representation of the complex, transient, dynamic effects of earthquake shaking by single constant unidirectional pseudo-static acceleration is very crude.
  - Relation between K and the maximum ground acceleration is not clear i.e. 1.9 g acceleration does not mean $K = 1.9$

- Advantages
  - Simple and straight-forward
  - No advanced or complicated analysis is necessary.
  - It uses limit state equilibrium analysis which is routinely conducted by Geotechnical Engineers.
Development of Modern Pseudo-Dynamic Approach

Soil amplification is considered.

Frequency of earthquake excitation is considered.

Time duration of earthquake is considered.

Phase differences between different waves can be considered.

Amplitude of equivalent PGA can be considered.

Considers seismic body wave velocities traveling during earthquake.


\[ a_h(z, t) = \{1 + (H - z).(f_a - 1)/H\}a_h \sin [\omega(t - (H - z)/V_s)] \]

\[ a_v(z, t) = \{1 + (H - z).(f_a - 1)/H\}a_v \sin [\omega(t - (H - z)/V_p)] \]

Soil amplification is considered.

The coefficient of seismic passive resistance ($K_{\text{pe}}$) is given by,

$$K_{\text{pe}} = \frac{1}{\tan \alpha} \left( \frac{\sin(\alpha + \phi)}{\cos(\delta + \phi + \alpha)} - \frac{k_h}{2\pi^2 \tan \alpha} \left( \frac{TV_s}{H} \right) \right)$$

$$\times \frac{\cos(\alpha + \phi)}{\cos(\delta + \phi + \alpha)} \times m_1 - \frac{k_v}{2\pi^2 \tan \alpha} \left( \frac{TV_p}{H} \right)$$

$$\times \frac{\sin(\alpha + \phi)}{\cos(\delta + \phi + \alpha)} \times m_2$$

where

$$m_1 = 2\pi \cos 2\pi \left( \frac{t}{T} - \frac{H}{TV_s} \right) + \left( \frac{TV_s}{H} \right)$$

and

$$m_2 = 2\pi \cos 2\pi \left( \frac{t}{T} - \frac{H}{TV_p} \right) + \left( \frac{TV_p}{H} \right)$$

$$\times \left[ \sin 2\pi \left( \frac{t}{T} - \frac{H}{TV_s} \right) - \sin 2\pi \left( \frac{t}{T} \right) \right]$$

The seismic passive earth pressure distribution is given by,

$$P_{\text{pe}}(t) = \frac{dP_{\text{pe}}(t)}{dz} = \frac{\gamma z}{\tan \alpha} \left( \frac{\sin(\alpha + \phi)}{\cos(\delta + \phi)} \right)$$

$$- \frac{k_h \gamma z}{\tan \alpha} \left( \frac{\cos(\alpha + \phi)}{\cos(\delta + \phi)} \right) \sin \omega \left( t - \frac{z}{V_s} \right)$$

$$- \frac{k_v \gamma z}{\tan \alpha} \left( \frac{\sin(\alpha + \phi)}{\cos(\delta + \phi)} \right) \sin \omega \left( t - \frac{z}{V_p} \right)$$

Typical non-linear variation of seismic passive earth pressure

Choudhury and Nimbalkar (2005)

Effect of amplification factor on seismic passive earth pressure

\[ a_h(z, t) = \{1 + (H - z).(f_a - 1)/H\}a_h \sin \left[ \omega \left\{ t - (H - z)/V_s \right\} \right] \]

\[ k_h = 0.2, k_v = 0.0, \phi = 30^0, \delta = 16^0 \]

Nimbalkar and Choudhury (2008)

Comparison of proposed pseudo-dynamic method with existing pseudo-static methods – Passive case

\[ \phi = 35^0, \: \delta = \phi/2, \: k_v = 0.5k_v, \: H/\lambda = 0.3, \: H/\eta = 0.16 \]

Choudhury and Nimbalkar (2005) in Geotechnique
Seismic Active Earth Pressure by Pseudo-Dynamic Approach

Choudhury and Nimbalkar (2006)

\[ a_h(z, t) = a_h \sin \left[ \omega \{ t - (H - z)/V_s \} \right] \quad \text{and} \quad a_v(z, t) = a_v \sin \left[ \omega \{ t - (H - z)/V_p \} \right] \]

where \( \omega = \) angular frequency; \( t = \) time elapsed; \( V_s = \) shear wave velocity; \( V_p = \) primary wave velocity.

\[
Q_h(t) = \int_0^H m(z)a_h(z, t)dz = \frac{\lambda \gamma a_h}{4\pi^2 g \tan \alpha} 2\pi H \cos \omega \zeta + \lambda (\sin \omega \zeta - \sin \omega t)
\]

where, \( \lambda = TV_s \) is the wavelength of the vertically propagating shear wave and \( \zeta = t - H/V_s \).

\[
Q_v(t) = \int_0^H m(z)a_v(z, t)dz = \frac{\eta \gamma a_v}{4\pi^2 g \tan \alpha} 2\pi H \cos \omega \psi + \lambda (\sin \omega \psi - \sin \omega t)
\]

where, \( \eta = TV_p \), is the wavelength of the vertically propagating primary wave and \( \psi = t - H/V_p \).

The total (static plus dynamic) active thrust is given by,

\[
P_{ae}(t) = \frac{W \sin(\alpha - \phi) + Q_h(t) \cos(\alpha - \phi) - Q_v(t) \sin(\alpha - \phi)}{\cos(\delta + \phi - \alpha)}
\]
The seismic active earth pressure coefficient, $K_{ae}$ is defined as

$$K_{ae} = \frac{1}{\tan \alpha \cos \delta + \phi - \alpha} \cdot \tan \alpha \cos \delta + \phi - \alpha \cdot 2\pi \tan \alpha \cdot \frac{TV}{H} \cdot \cos \delta + \phi - \alpha \cdot m_1 + \frac{k_1}{2\pi \tan \alpha} \left( \frac{TV}{H} \right) \cdot \cos \delta + \phi - \alpha \cdot m_2$$

where,

$$m_1 = 2\pi \cos 2\pi \left( \frac{t}{T} - \frac{H}{TV} \right) + \frac{TV}{H} \left( \sin 2\pi \left( \frac{t}{T} - \frac{H}{TV} \right) - \sin 2\pi \frac{t}{T} \right)$$

$$m_2 = 2\pi \cos 2\pi \left( \frac{t}{T} - \frac{H}{TV} \right) + \frac{TV}{H} \left( \sin 2\pi \left( \frac{t}{T} - \frac{H}{TV} \right) - \sin 2\pi \frac{t}{T} \right)$$

The seismic active earth pressure distribution is given by,

$$P_{ae}(t) = \frac{\partial P_{ae}(t)}{\partial z} = \frac{\gamma z}{\tan \alpha \cos (\delta + \phi - \alpha)} \sin (\alpha - \phi) + \frac{k_h \gamma z}{\tan \alpha \cos (\delta + \phi - \alpha)} \sin \left[ w \left( \frac{t - \frac{z}{V_s}}{V_s} \right) \right] + \frac{k_v \gamma z}{\tan \alpha \cos (\delta + \phi - \alpha)} \sin \left[ w \left( \frac{t - \frac{z}{V_p}}{V_p} \right) \right]$$
Typical non-linear variation of seismic active earth pressure


\[ a_h(z, t) = \{1 + (H - z)(f_a - 1)/H\} a_h \sin[\omega(t - (H - z)/V_s)] \]

Effect of soil amplification on seismic active earth pressure


D. Choudhury, IIT Bombay
Experimental Validation

Using

Geotechnical Dynamic Centrifuge Facility

at University of California, Davis, CA, USA
BART at SFO, CA, USA

Courtesy: BART
Geotechnical Centrifuge at UC Davis

Speci: 240g-ton capacity, 9.1m radius, max payload 4500 kg, bucket area 4m²
UC Davis Dynamic Centrifuge Facility

Biaxial Shaker: 2700 kg max payload, 30g max freq. 200 Hz, actuator force 400 kN
Dynamic Centrifuge Tests

- **Atik and Sitar (2010)** have conducted an experimental and analytical program was designed and conducted to evaluate the magnitude and distribution of seismically induced lateral earth pressures on cantilever retaining structures with dry medium dense sand backfill.

Model used for centrifuge test by Atik and Sitar (2010)

Dynamic Centrifuge Test Results

- Zeng (1998) described the behaviour of gravity quay walls under earthquake loading using data from three centrifuge tests.
- Using a modified pseudo-static approach, ground settlement in the backfill, influence of pore pressure on the wedge angle has also been studied.


Validation of Analytical Results of Pseudo-Dynamic Approach with Dynamic Centrifuge Test Results

Dynamic moment increment,

\[ \sqrt[3]{\frac{M}{\gamma H^3}} \],

where

\[ M(Z, t) = \int_{0}^{Z} p(z, t) \cos(\delta (Z - z)) dz \]

\[ \phi = 37^0, \delta = 20^0, k_h = 0.184, k_v = 0, f_a = 2, \]

\[ G = 57 \text{ MPa}, T = 1.0 \text{ s} \]

- - - Mononobe-Okabe method
- - - Present method
+ Centrifuge test results
(Steedman and Zeng, 1990)
Model proposed by Nimbalkar and Choudhury (2008) for Seismic Design of Retaining Wall considering wall-soil inertia

Active earth pressure condition

Proposed Design Factors for Retaining Wall

by Nimbalkar and Choudhury (2008)

Soil thrust factor, \( F_T = \frac{K_{ae}}{K_a} \)

Wall inertia factor, \( F_I = \frac{C_{IE(t)}}{C_{Ia}} \)

where, \( C_{IE(t)} = \frac{\cos \delta - \sin \delta \tan \phi_p}{\tan \phi_b} + \frac{Q_{hw}(t) + Q_{vw}(t) \tan \phi_p}{P_{ae}(t) \tan \phi_b} \)

\( C_{Ia} = \frac{\cos \delta - \sin \delta \tan \phi_b}{\tan \phi_b} \)

Combined dynamic factor, \( F_w = F_T F_I = \frac{W_w(t)}{W_w} \)
Typical Variation of Soil thrust factor $F_T$, Wall inertia factor $F_I$ and Combined dynamic factor $F_W$

$k_v=0.5k_h$, $\phi = 30^0$, $\delta = 15^0$, $H/TV_s = 0.3$, $H/TV_p = 0.16$, $H/TV_{sw}=0.012$, $H/TV_{pw}=0.0077$

Factors $F_W, F_I, F_T$
Typical Results

**Effect of angle of internal friction ($\phi$)**

$$k_v = 0.5k_h, \delta = \phi/2, H/TV_s = 0.3, H/TV_p = 0.16,$$

$$H/TV_{sw} = 0.012, H/TV_{pw} = 0.0077$$

**Effect of wall friction angle ($\delta$)**

$$k_v = 0.5k_h, \phi = 30^\circ, H/TV_s = 0.3, H/TV_p = 0.16,$$

$$H/TV_{sw} = 0.012, H/TV_{pw} = 0.0077$$

D. Choudhury, IITB

Nimbalkar and Choudhury (2008)
## Comparison of Soil thrust factor $F_T$, Wall inertia factor $F_I$ and Combined Dynamic Factor $F_W$

<table>
<thead>
<tr>
<th>$k_h$</th>
<th>$k_v$</th>
<th>Present study</th>
<th>Richards and Elms (1979)</th>
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<td></td>
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<td>$F_T$</td>
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</table>
Model proposed by Nimbalkar and Choudhury (2007) for Seismic Design of Retaining Wall considering wall-soil inertia

Passive earth pressure condition

Proposed Design Factors for Retaining Wall

by Nimbalkar and Choudhury (2007)

\[ F_T = \frac{K_{ae,pe}}{K_{a,p}} \]

\[ F_I = \frac{C_{IE}(t)}{C_{la,lp}} \]

\[ C_{la,lp} = \pm \frac{\cos \delta - \sin \delta \tan \phi_b}{\tan \phi_b} \]

\[ F_w = F_T F_I = \frac{W_w(t)}{W_w} \]
Variation of soil passive resistance factor $F_T$, wall inertia factor $F_I$ and combined dynamic factor $F_W$.

$\kappa = 0.5k_h$, $\phi = 30^0$, $\delta = 15^0$, $H/TV_s = 0.3$, $H/TV_p = 0.16$

$H/TV_{sw} = 0.012$, $H/TV_{pw} = 0.0086$
Pseudo-dynamic Method in Displacement –based analysis

- Choudhury and Nimbalkar (2007) proposed pseudo-dynamic method to compute the seismic rotational displacements of retaining wall for passive earth pressure condition. *(Soil Dynamics and Earthquake Engg., 2007)*

Pseudo-dynamic forces acting on soil–wall system for rotational stability

Variation of rotational displacement (θ) with \( k_h \)

Pseudo-dynamic Method

- Ghosh (2008) presented study on seismic active earth pressure behind a non-vertical cantilever retaining wall using pseudo-dynamic analysis.

Model considered by Ghosh (2008) by pseudo-dynamic method for active case

Variation of active pressure coefficient $K_{ae}$ with $\alpha_h$ for $f = 30^\circ$, $H/\lambda = 0.3$, $H/\eta = 0.16$, $f_a = 1$ by Ghosh (2008)

Pseudo-dynamic Method

- Basha and Babu (2010) have presented use of pseudo-dynamic method to compute the rotational displacements of gravity retaining walls under passive condition when subjected to seismic loads.
- Authors have combined the concept of Newmark’s sliding block method (1965) for computing the rotational displacements under seismic condition by using the limit equilibrium analysis under seismic conditions.
- Major conclusion was that major factor which controls the amount of rotation of wall during an earthquake is the soil friction angle.

Pseudo-dynamic Method

- Ghosh and Sharma (2010) presented the pseudo-dynamic analysis for calculating seismic active earth pressure for non-vertical retaining wall supporting c-\(\phi\) backfill.

- Bellezza et al. (2012) claim that a more rational pseudo-dynamic approach has been developed for fully submerged soil under the assumption that a restrained or free water condition exists within the backfill.

- Bellezza et al. (2012) have also extended their study to study the effect of amplification phenomena.


Provisions in Design Codes

- **Indian Design Code**
  - *IS 1893 - Part 5 (1984)*, provides information regarding earthquake resistant design for retaining wall for active and passive case. Use of M-O method.
  - Point of application at mid-height for dynamic component.
  - Pseudo-static is used, which excludes the deformation criteria.

Codal Provisions

- Indian Design Code
- As per IS 1893 - Part 5 (1984), active earth pressure exerted against wall can be,
  \[ P_a = \left(\frac{1}{2}\right) W \cdot H^2 C_a \]
- where \( C_a \) is given by,

\[
C_a = \frac{(1 \pm \alpha_v) \cos^2(\phi - \lambda - \alpha)}{\cos \lambda \cos^2 \alpha \cos(\delta + \lambda + \alpha)} \times \left[ \frac{1}{1 + \left\{ \frac{\sin(\phi + \delta) \sin(\phi - i - \delta)}{\cos(\alpha - i) \cos(\delta + \alpha + \lambda)} \right\}^{1/2}} \right]^2
\]

where, \( \alpha_v \) vertical seismic coefficient - its direction being taken consistently throughout the stability analysis of wall and equal to \( (1/2) \alpha_h \), where \( \alpha_h \) horizontal seismic coefficient.
Codal Provisions

- Indian Design Code
- **IS 1893 - Part 5 (1984)**, passive earth pressure exerted against wall can be,
  \[ P_p = \frac{1}{2} W \cdot H^2 C_p \]
- where \( C_p \) is given by,

\[
C_p = \frac{(1 + \alpha_v) \cos^2(\delta + \alpha - \lambda)}{\cos \lambda \cos^2 \alpha \cos(\delta - \alpha + \lambda)} \times \left[ \frac{1}{1 + \left\{ \frac{\sin(\phi + \delta) \sin(\phi + i - \delta)}{\cos(\alpha - i) \cos(\delta - \alpha + \lambda)} \right\}^{\frac{1}{2}}} \right]^2
\]

where \( f \) is soil friction angle, \( \delta \) friction angle for soil and wall.
Provisions in Design Codes

• European Design Code
  – Eurocode 8 (2003) explains the design of structures for earthquake resistance, wherein part 5 explains the procedure for foundations, retaining structures and geotechnical aspects.
  
  – It is based on pseudo-static method and follows displacement (for translation and rocking mode) based approach given by Richards and Elms (1979).
  
  – Eurocode 8 (2003) highlights guidelines to take into account values of $k_h$ and $k_v$ in absence of any study.
Codal Provisions

- Eurocode 8 (2003) mentions -
  1. For the purpose of the pseudo-static analysis, the seismic action shall be represented by a set of horizontal and vertical static forces equal to the product of the gravity forces and a seismic coefficient.
  2. The vertical seismic action shall be considered as acting upward or downward so as to produce the most unfavourable effect.
  3. The intensity of such equivalent seismic forces depends, for a given seismic zone, on the amount of permanent displacement which is both acceptable and actually permitted by the adopted structural solution.
• **Eurocode 8 (2003)**
  
  It mentions that, in the **absence** of specific studies, the horizontal \((k_h)\) and vertical \((k_v)\) seismic coefficients affecting all the masses shall be taken as:

\[ k_h = \alpha \frac{S}{r}, \]

\[ k_v = \pm 0.5k_h, \text{ if } \frac{a_{vg}}{a_g} \text{ is larger than 0.6} \]

\[ k_v = \pm 0.33k_h, \text{ otherwise} \]

where, \(k_h\) and \(k_v\) are seismic horizontal and vertical coefficients, \(\alpha\) ratio of the design ground acceleration on type A ground, \(a_g\), to the acceleration of gravity \(g\), \(a_{vg}\) is design ground acceleration in the vertical direction, \(a_g\) is design ground acceleration on type A ground.
Provisions in Design Codes

• **International Building Code**
  
  – **IBC (2006)** categorizes sites into categories namely A, B, C, D, E, F based on soil profile, shear wave velocity, SPT values and undrained shear strength values.
  
  – Based on that, the design seismic category should be selected.
  
  – It mentions that retaining walls shall be designed to ensure stability against overturning, sliding, excessive foundation pressure and water uplift.

International Building Code (2006),
INTERNATIONAL CODE COUNCIL, INC.
Eurocode 8 (2003) mentions –

1. Earth retaining structures shall be designed to **fulfil their function during and after an earthquake, without** suffering significant structural damage.

2. **Permanent displacements**, in the form of combined sliding and tilting, the latter due to irreversible deformations of the foundation soil, may be acceptable if it is shown that they are **compatible** with functional and/or aesthetic requirements.
Requalification of Geotechnical Earth Retaining Structures

- Eurocode 8 (2003) gives general guidelines for retaining wall design as:
  1. The choice of the structural type shall be based on normal service conditions, following the general principles of EN 1997-1:2004, Section 9.
  2. Proper attention shall be given to the fact that conformity to the additional seismic requirements may lead to adjustment and, occasionally, to a more appropriate choice of structural type.
  3. The backfill material behind the structure shall be carefully graded and compacted in situ, so as to achieve as much continuity as possible with the existing soil mass.
Requalification of Geotechnical Earth Retaining Structures

- Eurocode 8 (2003) gives general guidelines

4. **Drainage systems** behind the structure shall be capable of absorbing transient and **permanent** movements without impairment of their functions.

5. Particularly in the case of **cohesionless** soils containing water, the **drainage** shall be **effective** to well below the potential failure surface behind the structures.

6. It shall be ensured that the **supported soil** has an **enhanced safety margin** against **liquefaction** under the design earthquake.
Requalification of Geotechnical Earth Retaining Structures

Choudhury et al. (2004) mentioned following points, as a requalification measure for a retaining wall against seismic activity-

1. Method which gives maximum active earth pressure and minimum passive earth pressure should be considered.
2. The point of application of seismic earth pressure must be considered based on some logical analysis, rather than some arbitrary selection of values.
Requalification of Geotechnical Earth Retaining Structures

- Choudhury et al. (2004) continued...

3. In **displacement** based analysis, wall dimensions should be determined for given factor of safety (sliding 1.5, overturning 1.5, bearing capacity 2.5, eccentricity = 1/6\(^{th}\) of base size)

4. **Cumulative displacements** and **rotations** of the wall then must be compared for different loadings, based on magnitude of earthquake.

5. In displacement based analysis, **computed** displacements should be compared with **permissible** displacements. If computed displacements are **more** than permissible displacements, section should be **redesigned**.
Calculation of seismic earth pressure is very important for design of retaining structures in the earthquake prone areas.

Pseudo-static method was the first attempt for analyzing structures in seismic areas. Though it has serious limitations, it is widely used till date because of its simplicity.

Pseudo-dynamic method considers the time dependent nature of the earthquake force. Hence has more accuracy as compared with pseudo-dynamic methods.
Summary (contd.)

• In earthquake-resistant design of retaining wall, displacement-based analysis should be used for a better design rather than using the force-based analysis.

• Other methods which use tools like FEM, method of horizontal slices can also be considered for analysis of retaining structures in seismic areas.

• Seismic requalification techniques like those explained in Eurocode 8 (2003) should be studied for a safe design of earth retaining structures.