Geotechnical Earthquake Engineering

by

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Lecture – 32
Module – 8

Site Response Analysis
Site Response - The Problem

Predicts the response of a soil deposit due to earthquake excitation
Site Response

Ideally, a complete ground response analysis should include:

- Rupture mechanism at source of an earthquake (source)
- Propagation of stress waves through the crust to the top of bedrock beneath the site of interest (path)
- How ground surface motion is influenced by the soils that lie above the bedrock (site)
Site Response

In Reality,

- Mechanism of fault rupture is very complicated and difficult to predict in advance
- Crustal velocity and damping characteristics are generally poorly known
- Nature of energy transmission between the source and site is uncertain
Ground Response Analysis

In practice,

- Seismic hazard analyses (probabilistic or deterministic) are used to predict bedrock motions at the location of the site.

- Seismic hazard analyses rely on empirical attenuation relationships to predict bedrock motion parameters.

- Ground response problem becomes one of determining response of soil deposit to the motion of the underlying bedrock.
Ground Response Analysis

Used to:

- Predict ground surface motions, Time histories, Response spectra, Scalar parameters
- Evaluate dynamic stresses and strains, Liquefaction hazards, Foundation loading
- Evaluate ground failure potential, Instability of earth structures, Response of various geotechnical structures like retaining wall, earth dam, pile, various foundations etc.
Ground Response Analysis

Definitions:

Rock outcropping motion → the motion that would occur where rock outcrops at a surface.
Definitions:

Bedrock motion – the motion that occurs at bedrock overlain by a soil deposit. Differs from rock outcrop motion due to lack of free surface effect.
Ground Response Analysis

Definitions:

Free surface motion – the motion that occurs at the surface of a soil deposit
Ground Response Analysis

Common situations # 1

Rock outcrop motion is known – usually obtained from attenuation relationship (based on database of rock outcrop motions)

Free surface motion is to be determined
Ground Response Analysis

Common situations # 2

Free surface motion is known – usually obtained from attenuation relationship (based on database of soil outcrop motions)

Free surface motion is to be determined for site with different soil conditions

![Diagram showing known and unknown soil conditions](image-url)
Ground Response Analysis

Two basic approaches:

- Linear
- Nonlinear
Linear Analysis

- A known time history of bedrock (input) motion is represented as a Fourier series, usually using the FFT.
- Each term in the Fourier series of the bedrock (input) motion is then multiplied by the transfer function to produce the Fourier series of the ground surface (output) motion.
- The ground surface (output) motion can then be expressed in the time domain using the inverse FFT.
The transfer function determines how each frequency in the bedrock (input) motion is amplified, or de-amplified by the soil deposit.

A transfer function may be viewed as a filter that acts upon some input signal to produce an output signal.
Transfer Function Evaluation

Uniform Undamped Soil on Rigid Rock

Assume harmonic base motion,
Then, response should also be harmonic

\[ u(Z, t) = Ae^{i(\omega t - kz)} + Be^{i(\omega t + kz)} \]

Wave traveling in
– z direction
(upward)

Wave traveling in
+ z direction
(downward)
Transfer Function Evaluation

Uniform Undamped Soil on Rigid Rock

Displacement:
\[ u(Z, t) = Ae^{i(\omega t - kz)} + Be^{i(\omega t + kz)} \]

Stress:
\[ \tau(z, t) = G\gamma(z, t) = -GikAe^{i(\omega t - kz)} + GikBe^{i(\omega t + kz)} \]

At \( z = 0 \) (ground surface):
\[ \tau(z, t) = 0 = Gik(B - A)e^{i\omega t} \]

\[ A = B \]
Transfer Function Evaluation

Uniform Undamped Soil on Rigid Rock

\[ u(z,t) = 2A \left[ e^{ikz} + e^{-ikz} \right] \frac{1}{2} e^{i\omega t} \]

\[ u(z,t) = 2A \cos kz e^{i\omega t} \]

Defining a transfer function as the ratio of the displacement at the ground surface to the bedrock displacement

\[ F(\omega) = \begin{vmatrix} u_{0,t} & 0 \\ u & H_{t} \end{vmatrix} = \frac{2Ae^{i\omega t}}{2A \cos kH e^{i\omega t}} = \frac{1}{\cos kH} \]
Transfer Function Evaluation

Uniform Undamped Soil on Rigid Rock

As \( kH = \frac{wH}{Vs} \) goes to zero, denominator goes to zero Transfer function goes to infinity

\[
\omega_n = V_s \frac{\pi}{2} + n\pi / H
\]

\[
T_s = \frac{2\pi}{\omega_0} = \frac{4H}{Vs}
\]
Transfer Function Evaluation

Uniform Damped Soil on Rigid Rock

How do we handle damping?
Complex shear modulus

\[ G^* = \rho \left( V_S^* \right)^2 = \rho \frac{\omega}{k^*^2} \]

\[ k^* = \left[ \frac{\rho \omega^2}{G^*} \right]^{1/2} \]

\[ V_S^* = \sqrt{\frac{G^*}{\rho}} = V_S \left( 1 + i\xi \right) \]

\[ k^* = \frac{W}{V_S^*} = k \left( 1 - i\xi \right) \]
Transfer Function Evaluation

Uniform Damped Soil on Rigid Rock

Repeat analysis as before
Transfer function becomes

\[
F(\omega) = \begin{vmatrix} u & 0, t \\ u & H, t \end{vmatrix} = \frac{2Ae^{i\omega t}}{2A\cos k^*H e^{i\omega t}} = \frac{1}{\cos k^*H}
\]
Transfer Function Evaluation

Uniform Damped Soil on Rigid Rock

\[ F(w, \xi) = \frac{1}{\cos k^* H} = \frac{1}{\cos \left( \frac{wH}{V^*} \right)} \]

\[ |F(w, \xi)| = \frac{1}{\sqrt{\cos^2 kH + \xi^2 kH^2}} = \frac{1}{\sqrt{\cos^2 \frac{wH}{V_s} + \xi^2 \frac{wH}{V_s}^2}} \]
Transfer Function Evaluation

Uniform Damped Soil on Rigid Rock

Note:
Natural frequencies still exist
Low natural frequencies strongly amplified
High natural frequencies weakly amplified
Very high frequencies de-amplified
Amplification strongly frequency-dependent
Transfer Function Evaluation

Uniform Undamped Soil on Elastic Rock

\[ u_s(z_s, t) = C_s e^{i \omega t - k_s^* t} + D_s e^{i \omega t + k_s^* t} \]

\[ u_r(z_r, t) = C_r e^{i \omega t - k_r^* t} + D_r e^{i \omega t + k_r^* t} \]

\[ u_s(z_s) = H(t) = u_r(z_r) = 0, t \]

\[ \tau_s(z_s) = H(t) = \tau_r(z_r) = 0, t \]

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Transfer Function Evaluation

Uniform Undamped Soil on Elastic Rock

Maintaining equilibrium and compatibility of displacements at the boundary, the amplitude of the transfer function can be written as

\[ |F_{w, \xi = 0}| = \frac{1}{\sqrt{\cos^2 k_s H + \alpha_z^2 \sin^2 k_s H}} \]

\[ \alpha_z = \frac{\rho_s v_{s*}}{\rho_r v_{sr*}} \]

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Transfer Function Evaluation

Uniform Undamped Soil on Elastic Rock

Note:

Even with no soil damping, resonance cannot occur

Why???

Energy removed from soil layer by transmission into rock

Form of radiation damping
Transfer Function Evaluation

Uniform Undamped Soil on Elastic Rock

Maintaining equilibrium and compatibility of displacements at the boundary, the amplitude of the transfer function can be written as

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\left| F_{w, \xi = 0} \right| = \frac{1}{\sqrt{\cos^2 k_s H + \alpha_z^2 \sin^2 k_s H}}
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\alpha_z = \frac{\rho_s v_{ss}^*}{\rho_r v_{sr}^*}
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Transfer Function Evaluation

Uniform Undamped Soil on Elastic Rock

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Even with no soil damping, resonance cannot occur
Why???
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Transfer Function Evaluation

Layered, Damped Soil on Elastic Rock
Transfer Function Evaluation

Layered, Damped Soil on Elastic Rock

For layer $j$

$$u_j = z_j, t = A_j e^{i k_j z_j} + B_j e^{-i k_j z_j} e^{i \omega t}$$

From equilibrium

$$A_{j+1} + B_{j+1} = A_j e^{i k_j h_j} + B_j e^{-i k_j h_j}$$

From compatibility

$$A_{j+1} - B_{j+1} = \frac{G_j^* k_j^*}{G_{j+1}^* k_{j+1}^*} A_j e^{i k_s h_j} - B_j e^{-i k_s h_j}$$
Transfer Function Evaluation

Layered, Damped Soil on Elastic Rock

For layer $j$

$$u_j = z_j, t = A_j e^{ik_j z_j} + B_j e^{-ik_j z_j} e^{i\omega t}$$

From equilibrium

$$A_{j+1} + B_{j+1} = A_j e^{ik_j h_j} + B_j e^{-ik_j h_j}$$

From compatibility

$$A_{j+1} - B_{j+1} = \frac{G_j k_j^*}{G_{j+1} k_{j+1}^*} A_j e^{ik_s h_j} - B_j e^{-ik_s h_j}$$

If we know response at layer $j$ ($A_j$ and $B_j$ are known), then we have two equations with two unknowns ($A_{j+1}$ and $B_{j+1}$). We can relate $A_{j+1}$ and $B_{j+1}$ to $A_j$ and $B_j$ by means of recursive relationships.
Transfer Function Evaluation

Layered, Damped Soil on Elastic Rock

Solving for the unknowns

\[ A_{j+1} = \frac{1}{2} A_j \left( 1 + \alpha_j e^{ik_j h_j} \right) + \frac{1}{2} B_j \left( 1 - \alpha_j e^{-ik_j h_j} \right) \]

\[ B_{j+1} = \frac{1}{2} A_j \left( 1 - \alpha_j e^{ik_j h_j} \right) + \frac{1}{2} B_j \left( 1 + \alpha_j e^{-ik_j h_j} \right) \]

Or, relating the coefficients to those at the ground surface

\[ A_{j+1} = a_{j+1} \omega A_1 \]

\[ B_{j+1} = b_{j+1} \omega B_1 \]

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Transfer Function Evaluation

Layered, Damped Soil on Elastic Rock

Then, a transfer function relating the motion in layer $i$ to the motion in layer $j$ can be written as

$$F_{ij} \omega = \frac{a_i \omega + b_i \omega}{a_j \omega + b_j \omega}$$

If we know the motion at any layer, we can use this transfer function to compute the corresponding motion at any other layer.
Equivalent Linear Approach

The actual nonlinear hysteretic stress – strain behavior of cyclically loaded soils can be approximated by equivalent linear properties

Assume some initial strain and use to estimate $G$ and $\xi$

Determine peak strain and effective strain $\gamma_{\text{eff}} = R_{\gamma} \gamma_{\text{max}}$
Select properties based on updated strain level

Compute response with new properties and determine resulting effective shear strain
Repeat until computed effective strains are consistent with assumed effective strains
Non – linear Approach

Solve Wave equation incrementally

\[ \frac{\partial \tau}{\partial z} = \rho \frac{\partial^2 u}{\partial t^2} = \rho \frac{\partial u}{\partial t} \]

Approximate partial derivatives

\[ \frac{\partial \tau}{\partial z} \approx \frac{\tau_{i+1,t} - \tau_{i,t}}{\Delta z} \]
\[ \frac{\partial u}{\partial t} \approx \frac{u_{i,t+\Delta t} - u_{i,t}}{\Delta t} \]

Finite difference form

\[ \frac{\tau_{i+1,t} - \tau_{i,t}}{\Delta z} \approx \rho \frac{u_{i+1,t} - u_{i,t}}{\Delta t} \]
Non – linear Approach

Solve Wave equation incrementally

then

\[ \dot{u}_{i,t+\Delta t} = \dot{u}_{i,t} + \frac{\Delta t}{\rho \Delta z} (\tau_{i+1,t} - \tau_{i,t}) \]

Velocity at time \( t+\Delta t \) can be calculated from velocity and shear stress at time \( t \)
Non-Linear Approach

Solve wave equation incrementally

- Start with initial stiffness, $G_{\text{max}}$
- Compute response for small time step, $\Delta t$
- Compute shear strain amplitude at end of time step
- Use stress-strain model to find $G_{\tan}$ for next time step
- Compute shear strain amplitude at end of next time step
- Continue stepping through time for entire input motion
Non – linear Approach

Solve Wave equation incrementally

Nonlinear response is simulated in incrementally linear fashion

Material damping is taken care by hysteretic response

Approach requires good model for description of soil stress – strain behaviour
Non-Linear Stress-Strain Models

Two main types

- Cyclic nonlinear models
- Advanced constitutive models
Non – Linear Stress – Strain Models

Cyclic nonlinear models

Requires:
- Backbone curve
- Unloading – reloading rules
- Pore pressure model
Non – Linear Stress – Strain Models

Advanced constitutive models

Requires:

• Yield surfaces
• Hardening rule
• Failure surface
• Flow rule
Non-Linear Stress-Strain Models

Cyclic nonlinear models

Advantages:
- Relatively simple
- Small number of parameters

Disadvantages:
- Simplistic representation of soil behavior
- Cannot capture dilatancy effects

Advanced constitutive models

Advantages:
- Can better represent mechanics of yield, failure

Disadvantages:
- Many parameters
- Difficult to calibrate